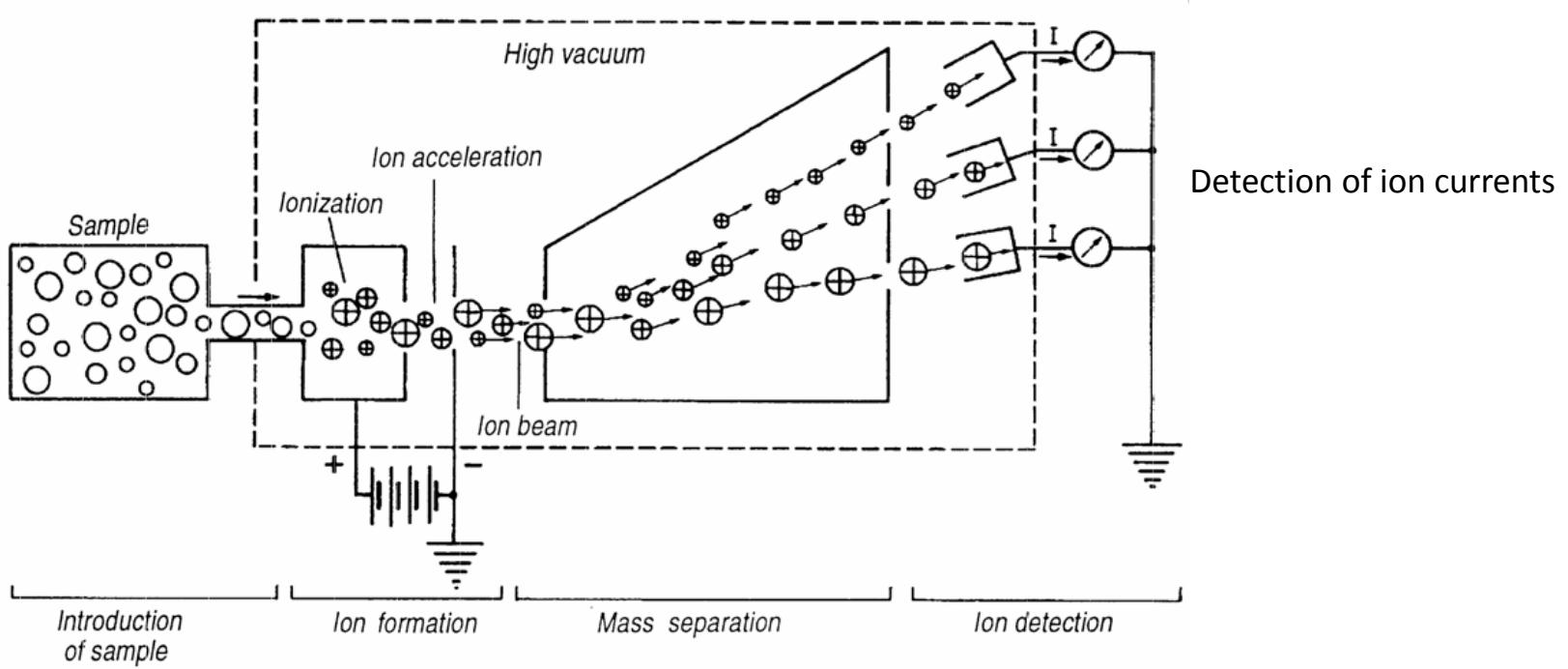
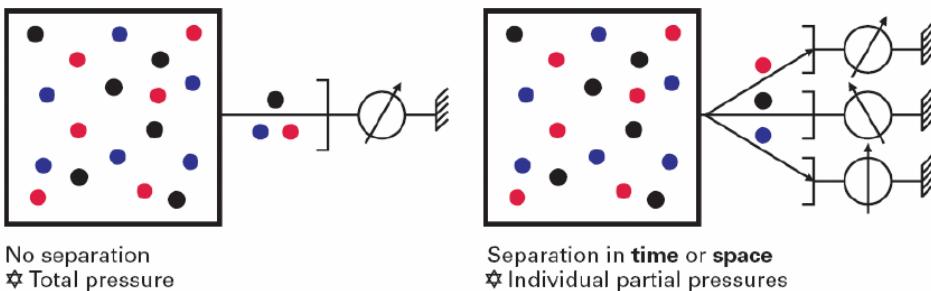




Practical aspects of mass spectrometry

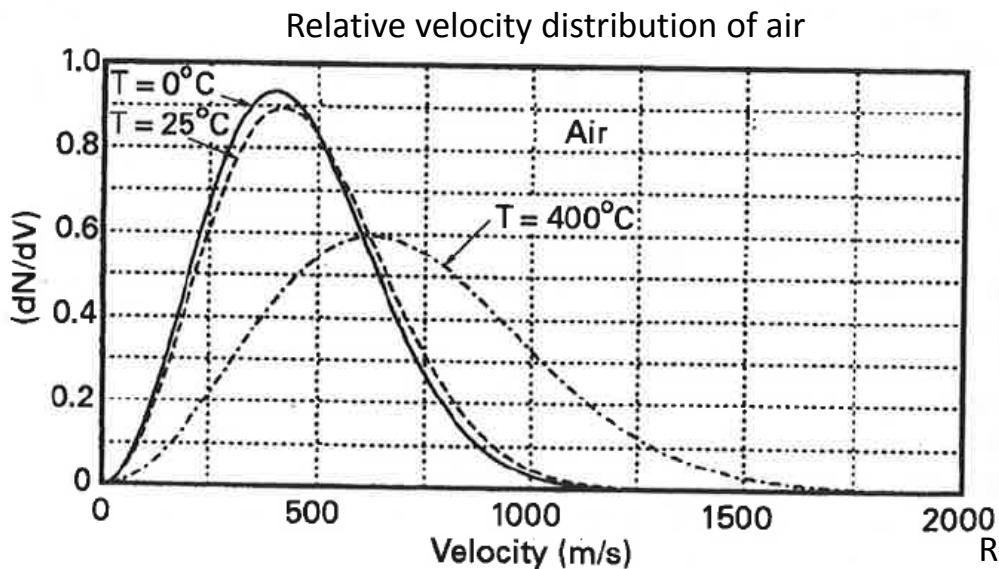
Andrey Tarasov
Lecture on mass-spectrometry
30.10.2015



Mass spectrometer

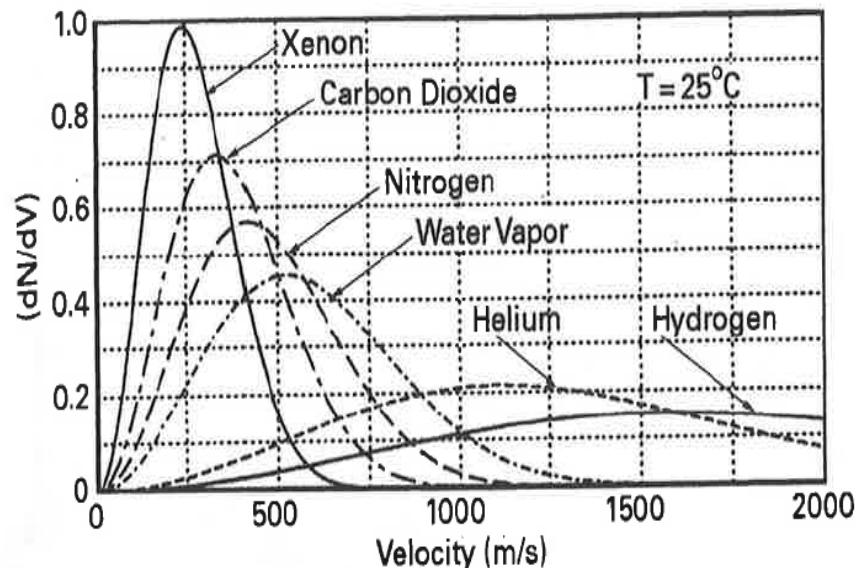
The method is not inherently quantitative

Kinetic picture of gas



@22°C, 10^5 Pa, $2.48 \cdot 10^{25}$ molecules, $3.4 \cdot 10^{-9}$ m spacing
 10^{-7} Pa, $2.48 \cdot 10^{13}$ molecules, $3.0 \cdot 10^{-5}$ m spacing

Relative velocity distribution of several gases



Maxwell-Boltzmann distribution for ideal gas:

$$\frac{dn}{dv} = \frac{2N}{\pi^{1/2}} \left(\frac{m}{2kT} \right)^{3/2} v^2 e^{-mv^2/(2kT)}$$

$$v = \left(\frac{8kT}{\pi m} \right)^{1/2} \quad v_p = (2kT/m)^{1/2}$$

v - mean average velocity, v_p - peak velocity

N - total number of particles

T - absolute temperature

m - particle mass

k - Boltzmann constant

J. F. O'Hanlon, A users guide to vacuum Technology, WILEY, 2003



MAX-PLANCK-GESELLSCHAFT

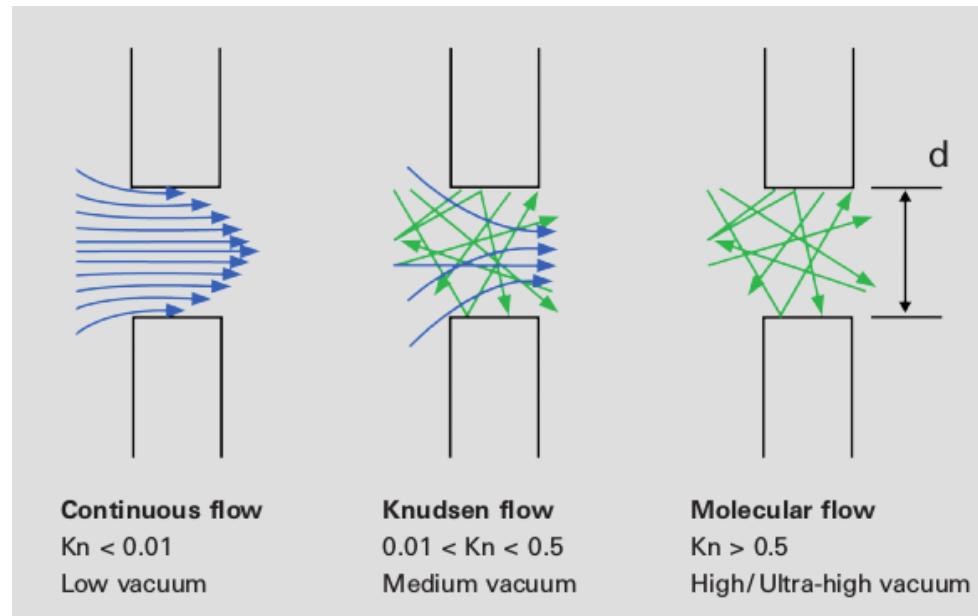
Flow Regimes

$$Kn = l/d$$

l – mean free path [m]

d – Diameter of flow channel [m]

Kn - Knudsen number dimensionless



Continuous flow

$Kn < 0.01$

Low vacuum

Knudsen flow

$0.01 < Kn < 0.5$

Medium vacuum

Molecular flow

$Kn > 0.5$

High/Ultra-high vacuum



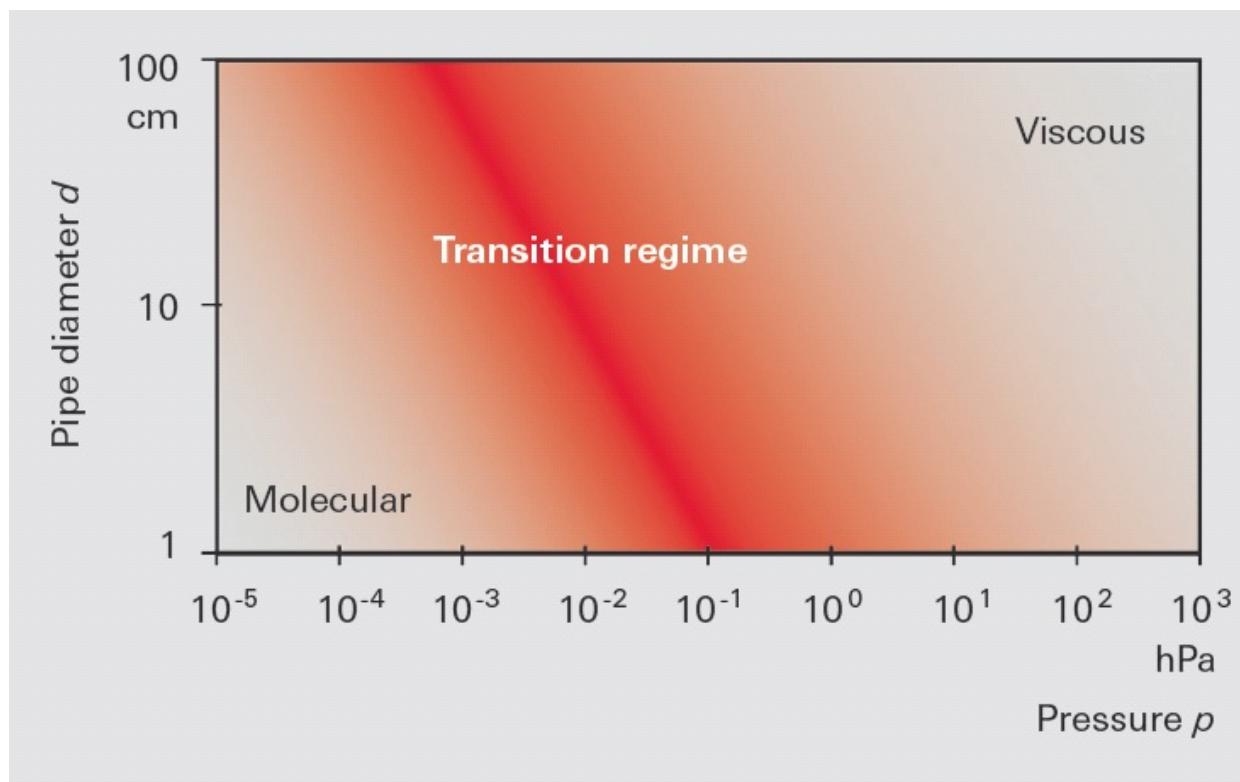
Flow Regimes

$$Kn = l/d$$

l – mean free path [m]

d – Diameter of flow channel [m]

Kn - Knudsen number dimensionless



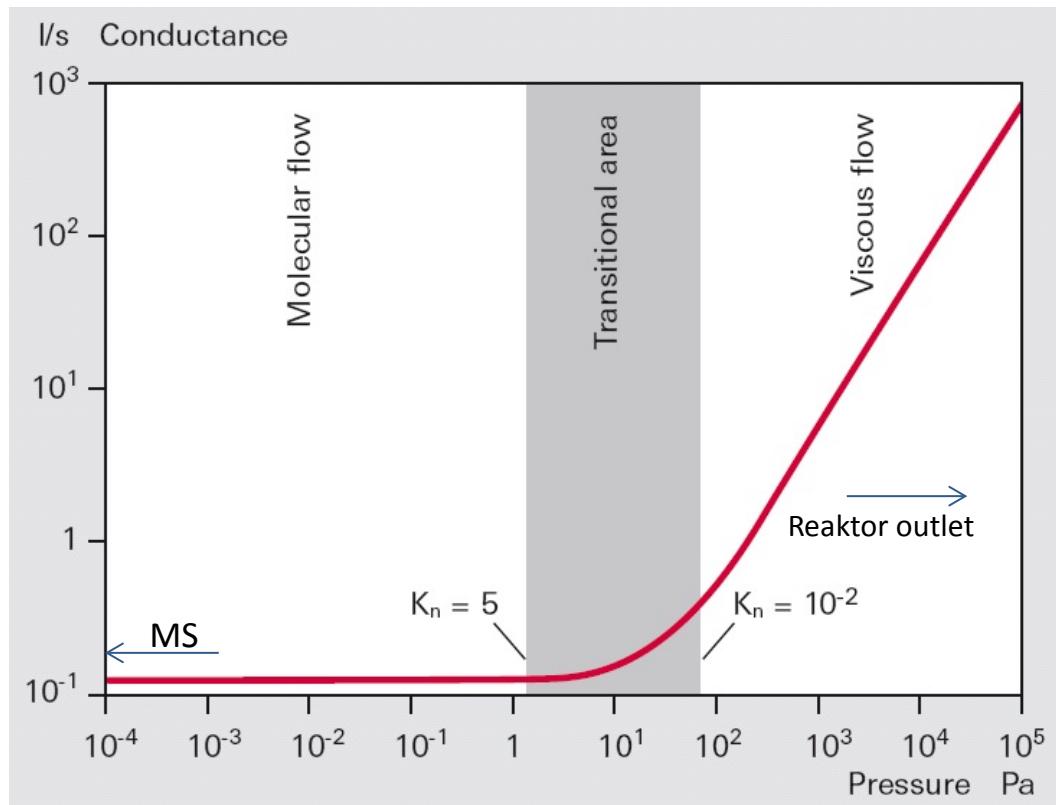


MAX-PLANCK-GESELLSCHAFT

Flow Regimes

$$Kn = l/d$$

Gass flowing through piping produces a pressure difference at the ends of the piping:



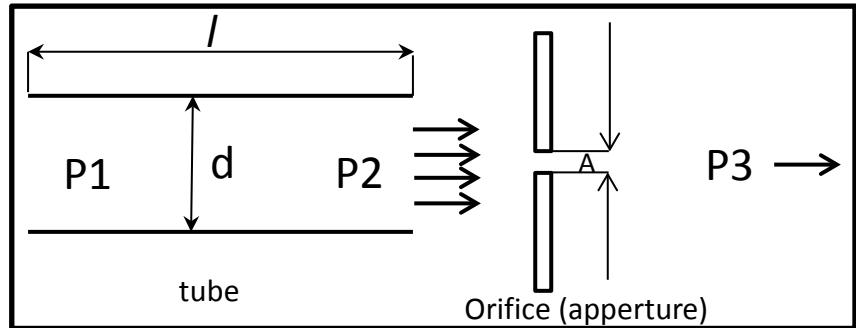
$$C = Q/P_2 - P_1$$

C – conductance [m^3/s]

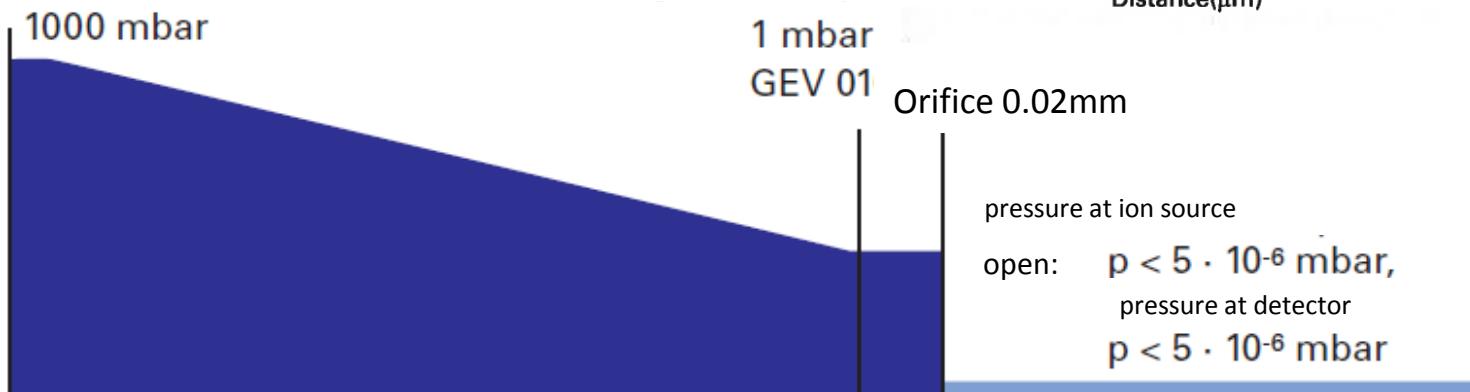
Q – throughput $d(PV)/dt$ [$\text{Pa} * \text{m}^3/\text{s}$]

$P_2 - P_1$ – pressure drop at the ends of the pipe

Inlet System



Continues pressure reduction through the capillary
 $d=0.15\text{mm}$. $L=1.5\text{m}$

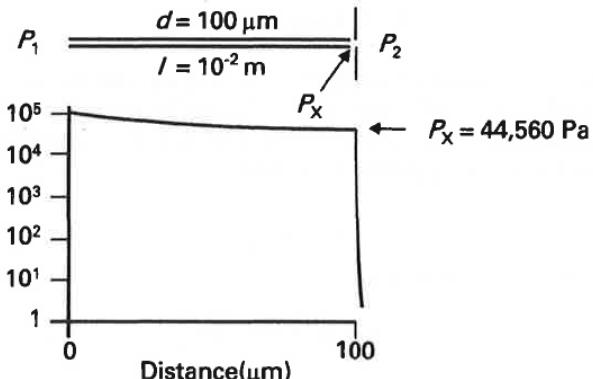


$$Q_{\text{or, mol}} = A \cdot \frac{\bar{v}}{4} \cdot (p_1 - p_2) = A \cdot \sqrt{\frac{kT}{2\pi m_0}} \cdot (p_1 - p_2)$$

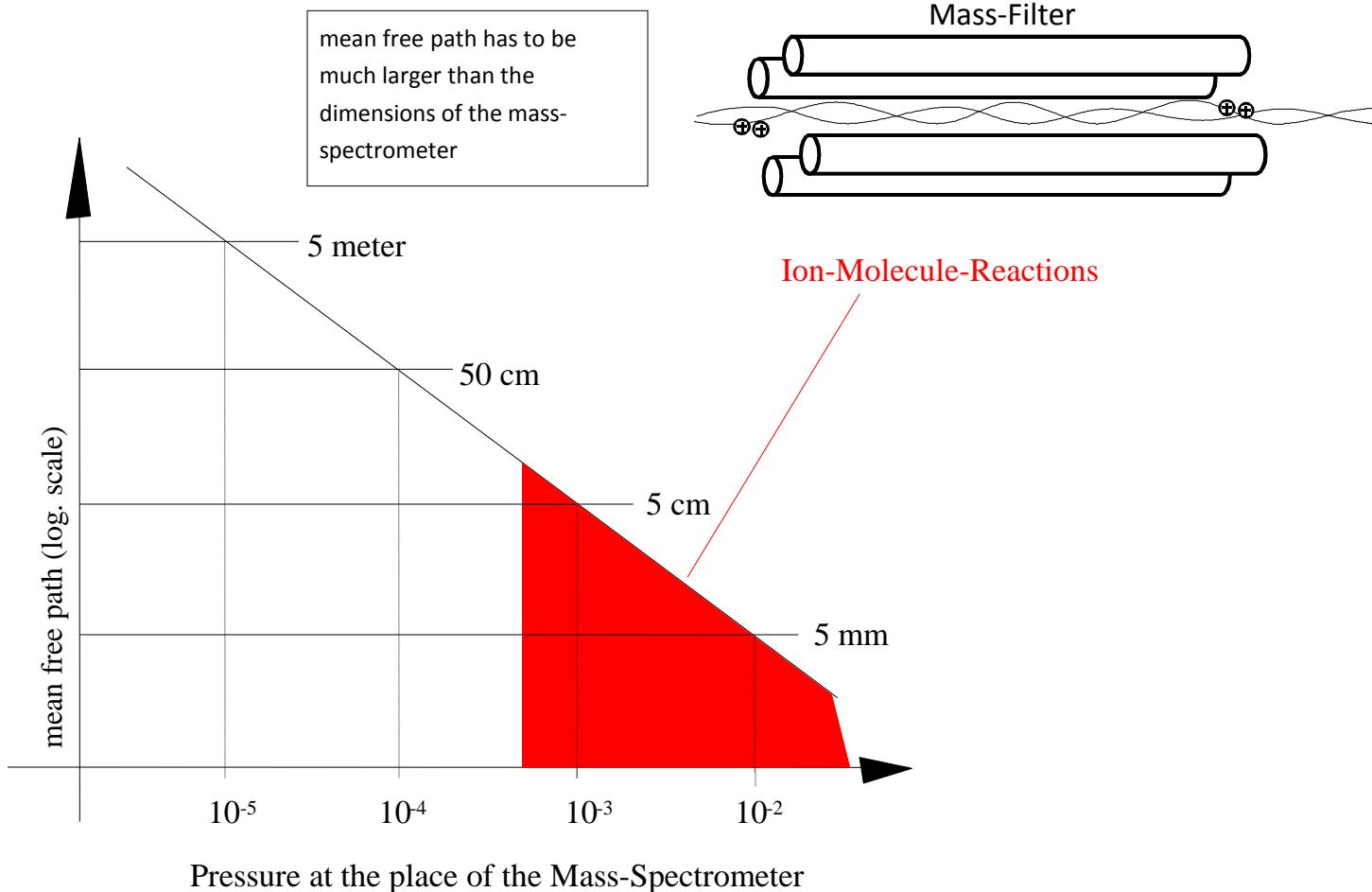
$$Q_{\text{pipe, mol}} = \frac{\pi \cdot d^4}{256\vartheta \cdot l} \cdot (p_1 + p_2) \cdot (p_1 - p_2)$$

ϑ – viscosity

\bar{v} – mean thermal velocity, (peak velocity v_p)



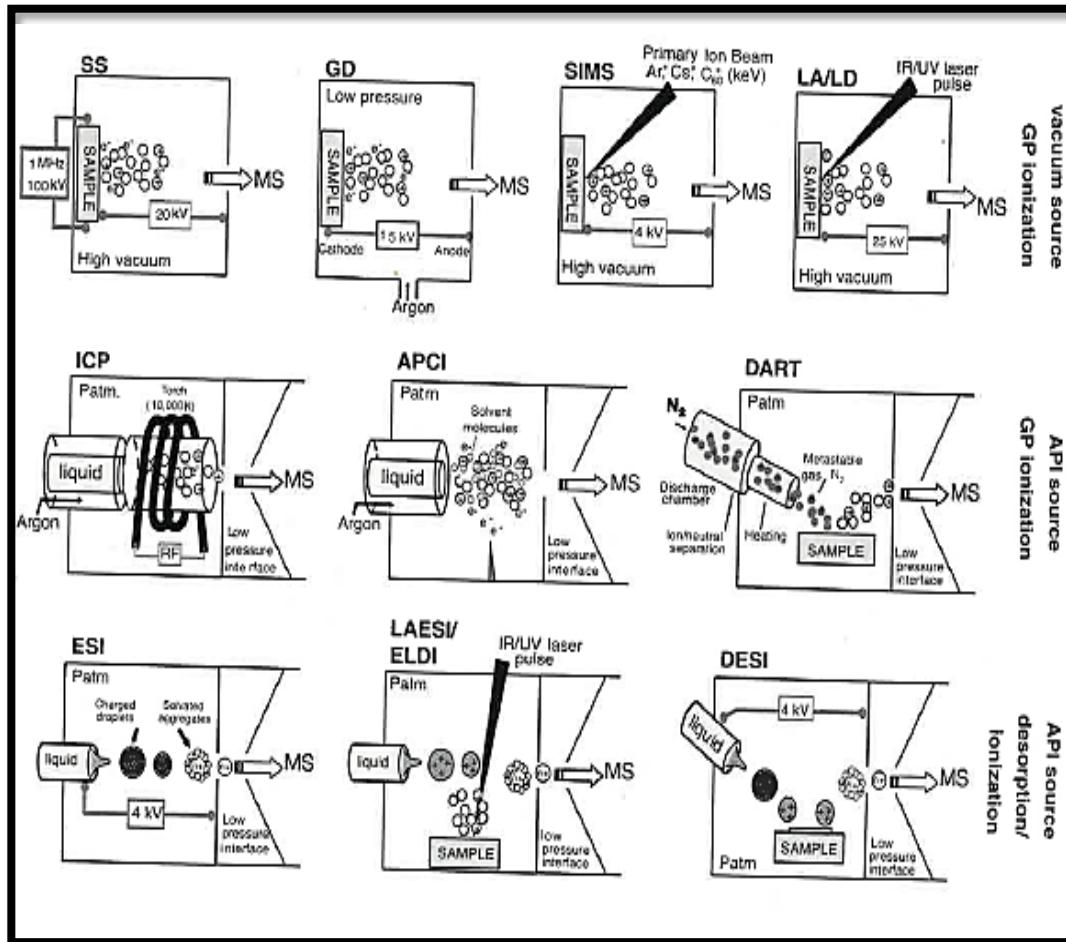
Why is it necessary to work under vacuum?



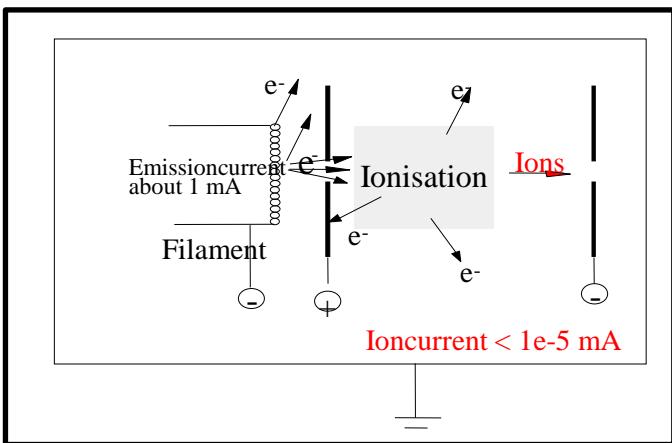


MAX-PLANCK-GESELLSCHAFT

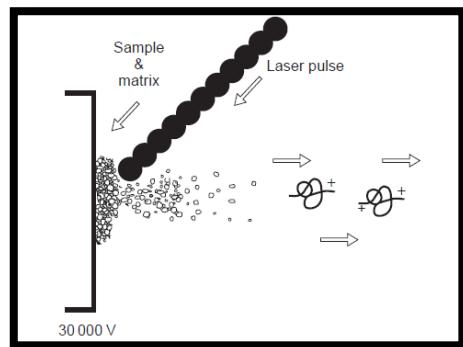
Ionization

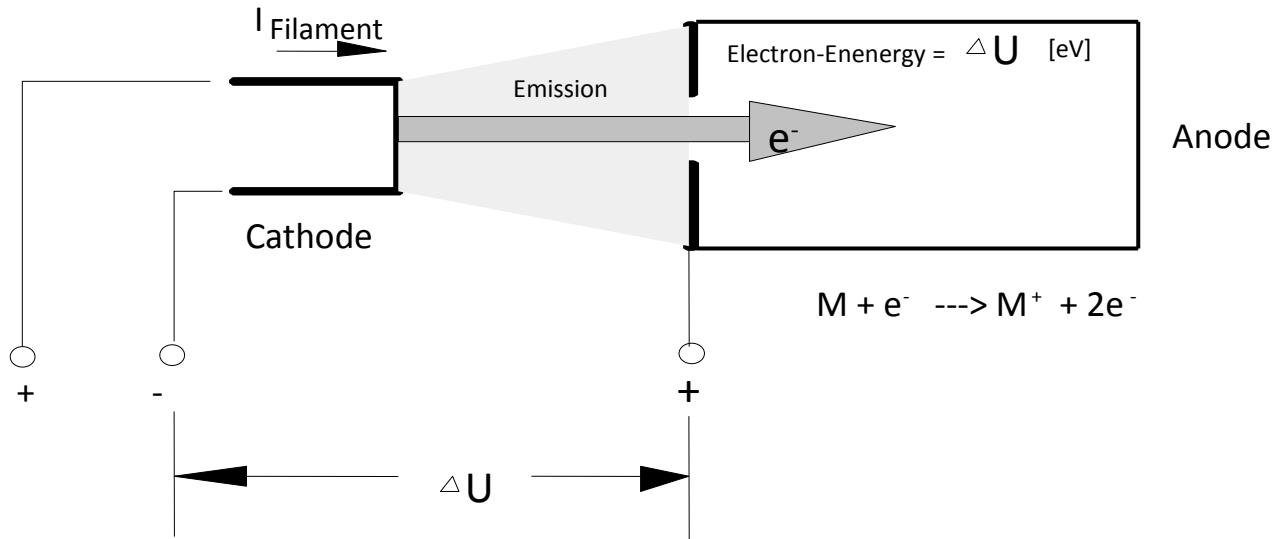


EII



MALDI





The emission is a function of the filament temperature, the material (work-function), the applied voltage and the geometry (local field strength).

The filament is heating up the ion source by radiation, take the vapour pressure of the material into account!

$$W_{Re} = 4.72 \text{ eV}, W_w = 4.55 \text{ eV}, W_y = 3.1 \text{ eV}, W_{Th} = 3.4 \text{ eV}, W_{Ir} = 5.00-5.67 \text{ eV},$$



Ionization

Filament materials

Rhenium:

rather high vapour pressure

used mainly in High Vacuum

Tungsten

used in the UHV, has a longer

lifetime than Rhenium, Tungsten

gets brittle WC_x

Yt-Oxide on Iridium

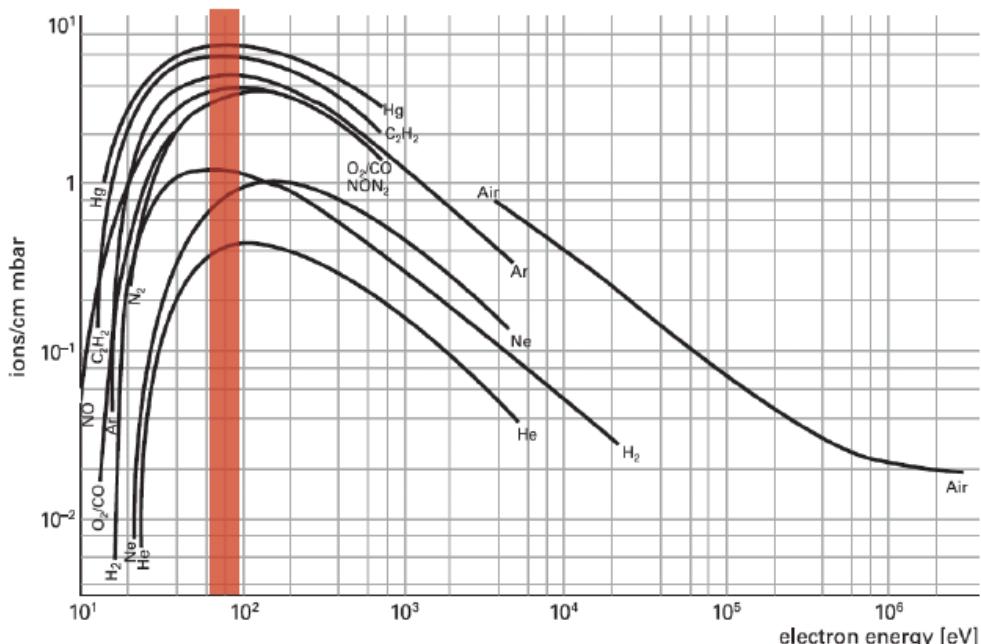
rather resistant against air,

replaced thoriated Iridium

temperature is much lower compared to pure metall

Ionization

- Small fraction of sample is converted in ionized state (0.1%)
- 20% of all ions are extracted from the ionisation chamber
- Collision energy influences the number and type of ions (50-150eV)



60-100 eV

$$I = k * n * \sigma$$

$$I \left[\frac{\text{ion}}{\text{sec}} \right] = \alpha \left[\frac{\text{ion}}{\text{molec}} \right] \cdot \rho \left[\frac{\text{molec}}{\text{cm}^2 \text{sec}} \right] \cdot S_{\text{ef}} [\text{cm}^2]$$

α – fraction of the molecules which has been undergone ionization and reached collector i.e.

$$\alpha = \omega_1 \cdot \omega_2 \cdot \omega_3$$

ω_1 - probability that the molecule reaches the ionization zone

ω_2 - probability of the ionization of the molecule in the ionization zone

ω_3 - probability that the formed ion reaches collector

ρ – molecules flux through the effective area S_{ef}

k – Instrument sensitivity

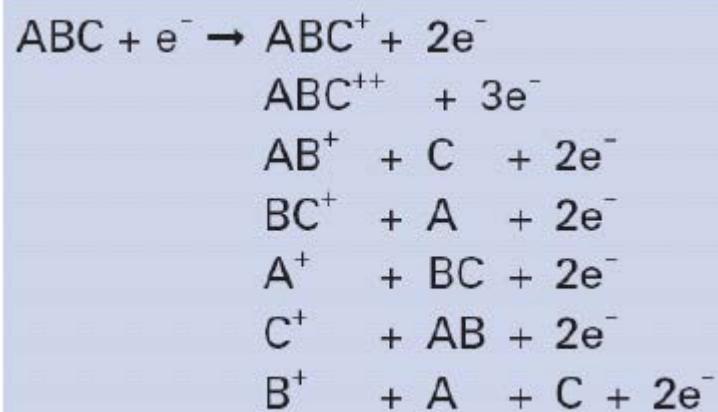
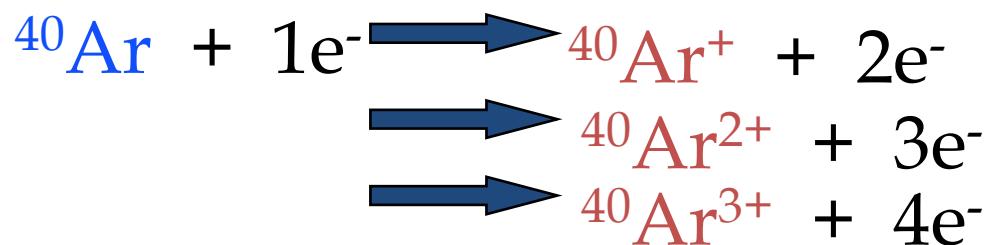
n – molecules/ cm^3

σ – ionization cross section, \AA^2

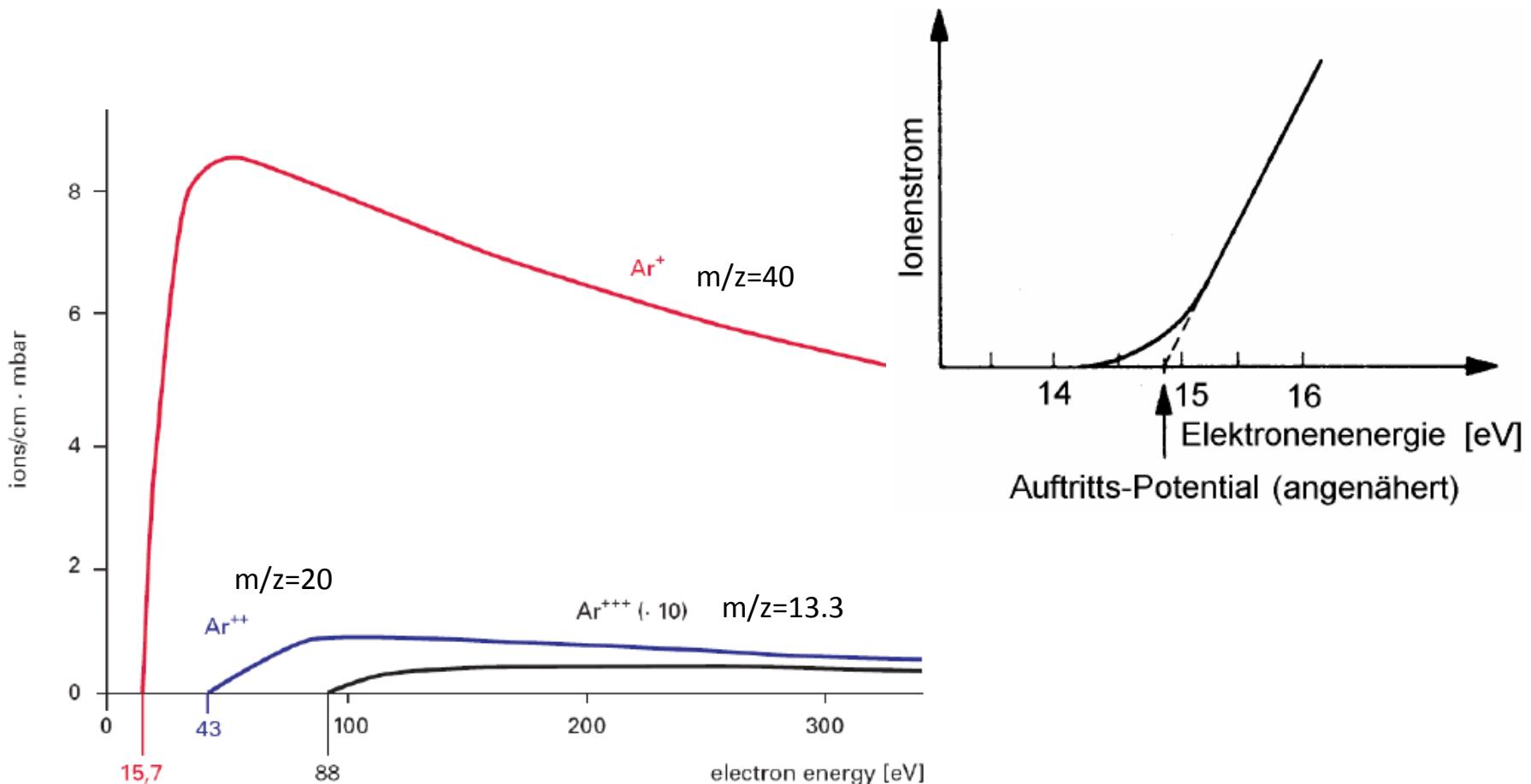
Ionization

Electron-Impact Cross Sections for Ionization at 60eV

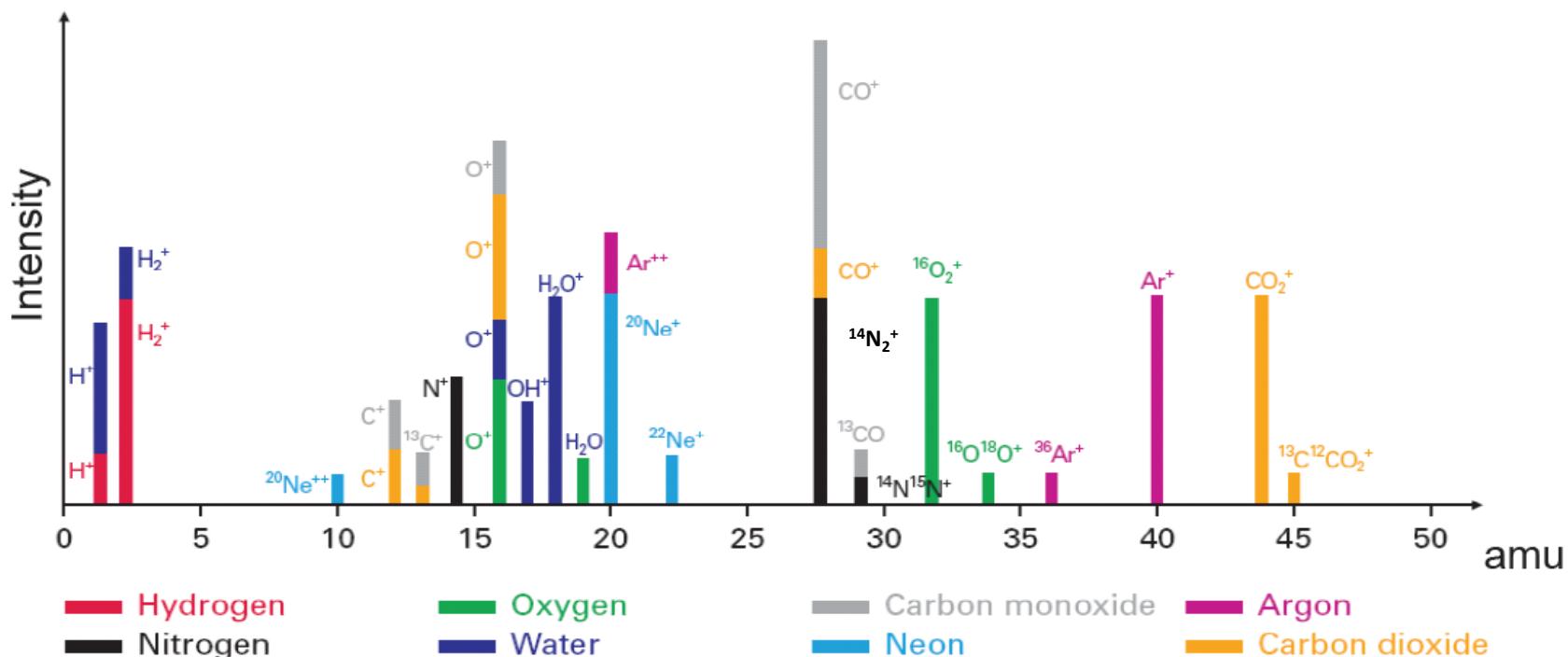
Molecule / ion	$\sigma, \text{\AA}^2$
H ₂	1.015
N ₂	2.381
O ₂	2.287
CO	2.395
CO ⁺	0.813
CO ₂	3.317
CH ₄	3.471
CH ₄ ⁺	0.964
CH ₃ ⁺	0.951
N ₂ O	3.513



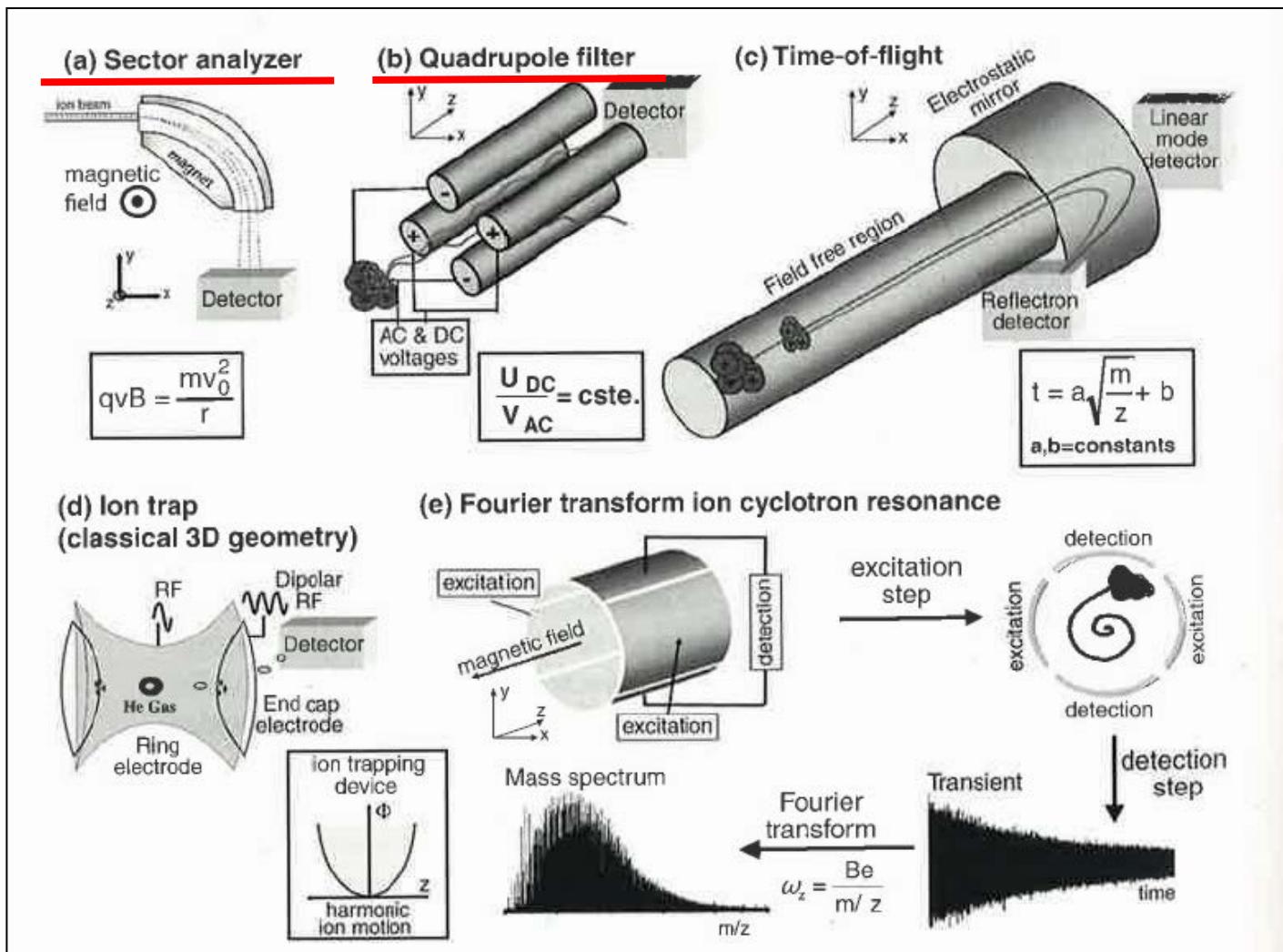
Ionization



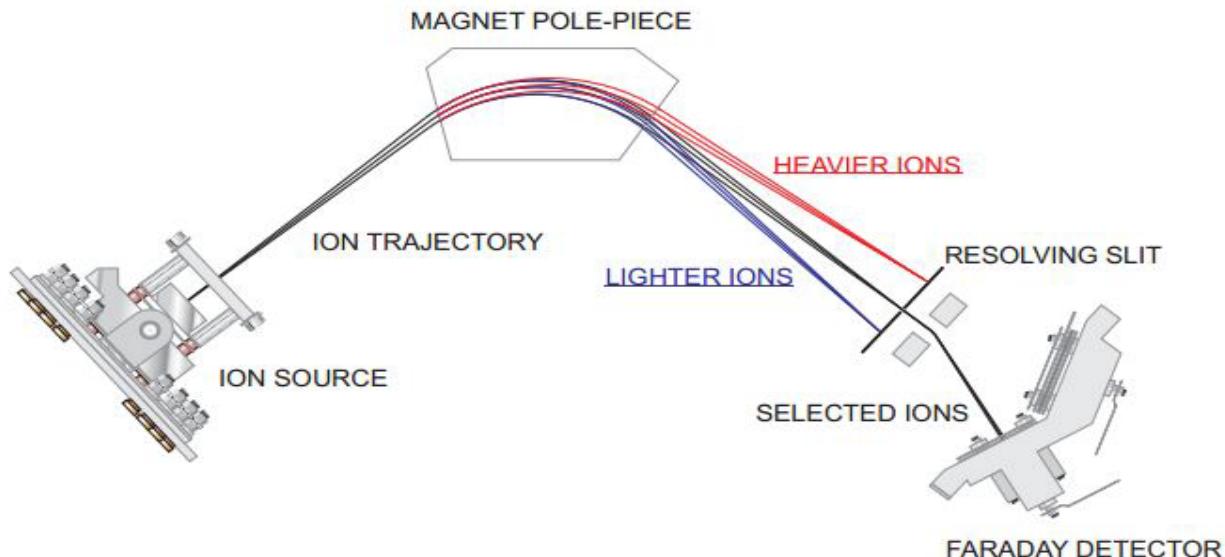
- Decreasing of electron energy in order to reduce overlapping signals from fragments
- $\text{HF}^+/\text{Ar}^{++}$, $m/z = 20$, I_{ion} 14eV/43eV
- Multiple charged fragments cause the signals on non-integer mass/charge ration Ar^{3+}



Mass separation



Mass separation Magnetic sector mass analyzer

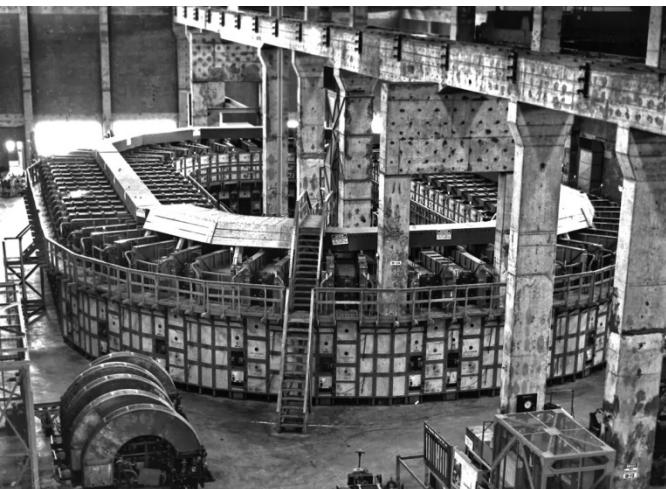
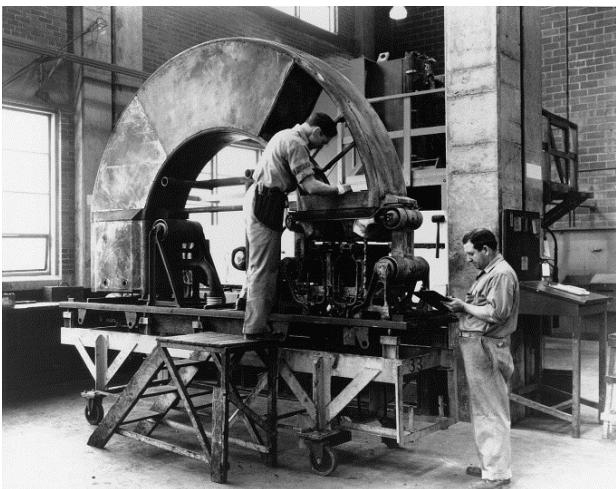


$$E_k = E_i \quad \frac{M \cdot v^2}{2} = q \cdot U \quad \text{Kinetic energy of ions is determined by the accelerating potential (1.5 – 15kV)}$$

$$F_L = F_C \quad \frac{M \cdot v^2}{R} = q \cdot v \cdot H \quad \text{Lorenz force is in equilibrium with centrifugal force}$$

Hence: $\frac{M}{q} = \frac{R^2 \cdot H^2}{2 \cdot U}$ The trajectory radius remains constant e.g. 200mm. To detect the definite M/z one varies the magnetic field induction.

Manhattan projekt (1942-1946)



Calutron

Uranium enrichment

^{238}U 99.2%, ^{235}U 0.72%

$^{238}\text{UF}_5^+$ m/z 328

$^{235}\text{UF}_5^+$ m/z 325

Reactor grade 3-4% U-235

Weapon grade 90% U-235

Later on in Soviet Russia (1969 – 1972)



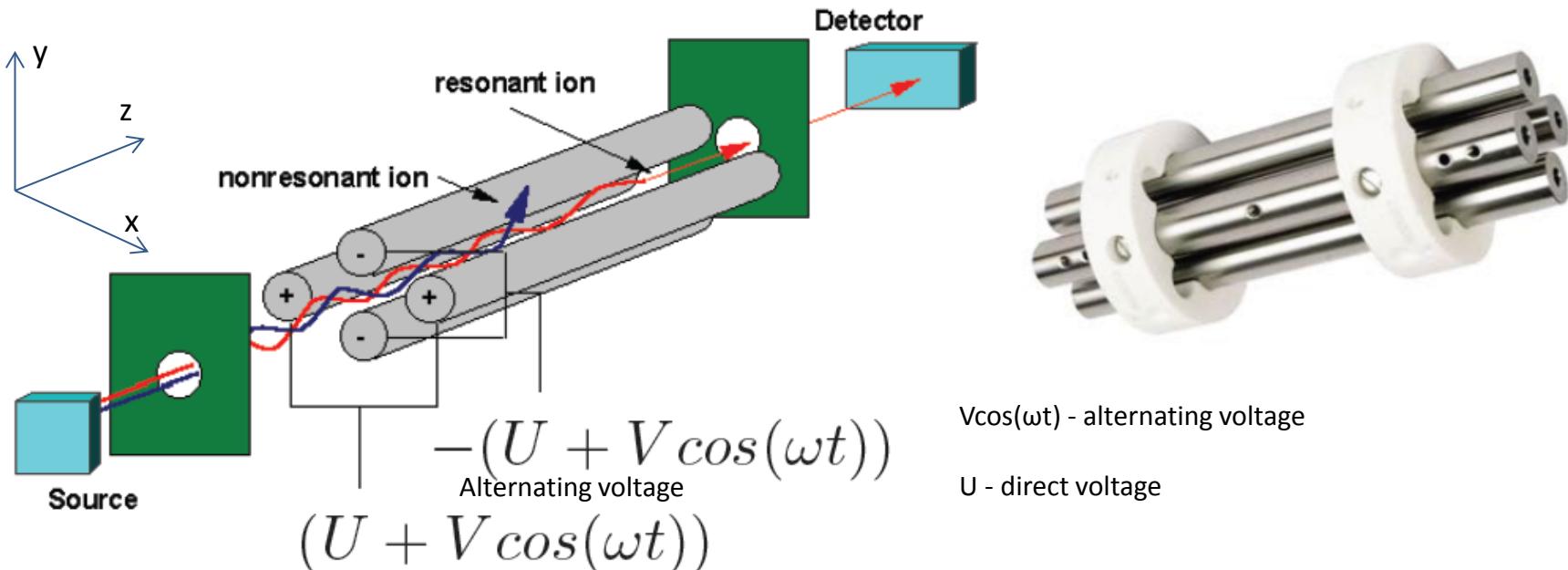
MS1301

High temperature mass spectrometer for the analysis of the gas phase over hard volatile substances, namely oxides.

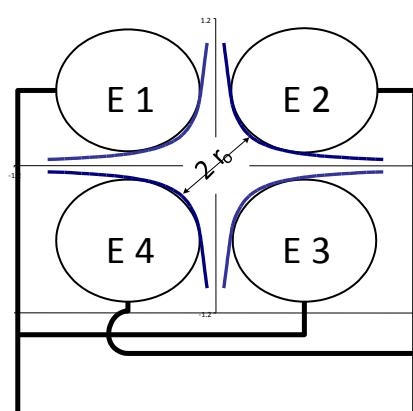


MAX-PLANCK-GESELLSCHAFT

Mass separation Quadrupole analyzer

 $V \cos(\omega t)$ - alternating voltage

U - direct voltage



$$\Phi = (U + V \cos \omega t)(x^2 - y^2) / r_0^2$$

Potential between the electrodes

$$Ex = d\Phi/dx \quad Ey = d\Phi/dy \quad Ez = d\Phi/dz$$

$$eE = ma$$

$$m\ddot{x} + 2e(U + V \cos \omega t) \frac{x}{r_0^2} = 0$$

$$m\ddot{y} - 2e(U + V \cos \omega t) \frac{y}{r_0^2} = 0$$

$$m\ddot{z} = 0$$



Mass separation Quadrupole analyzer

Use of the transformations

$$a = \frac{8eU}{mr_o^2\omega^2}$$

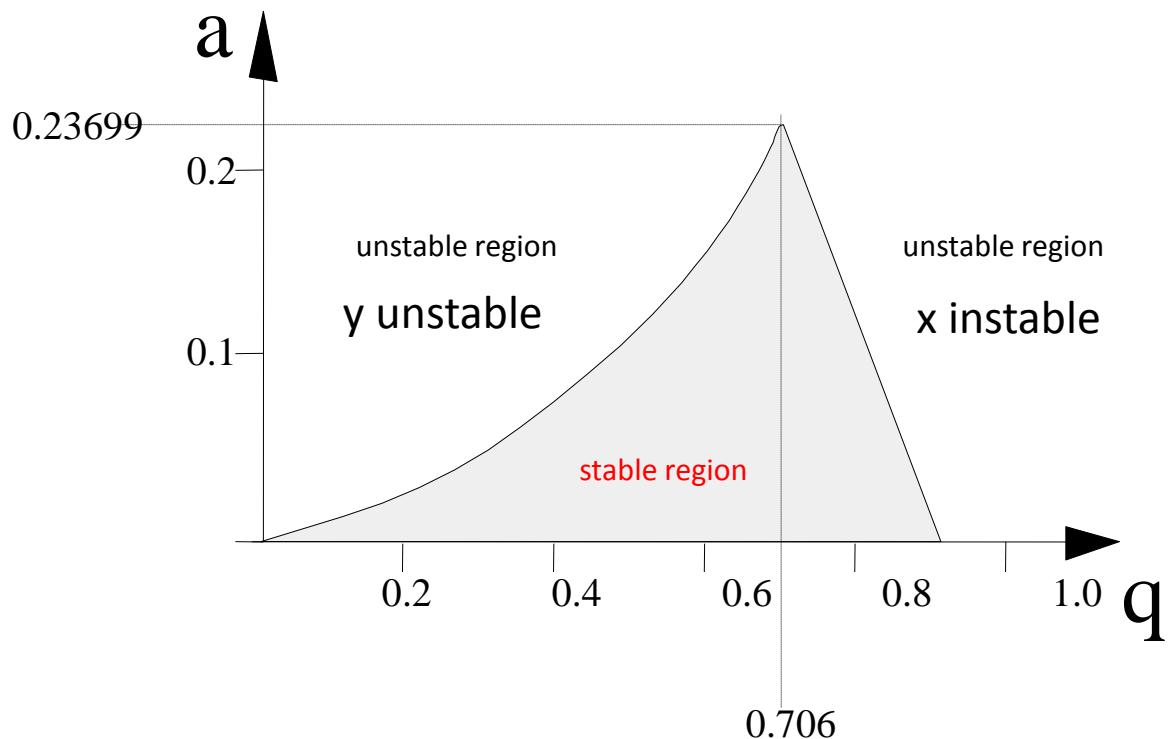
$$q = \frac{4eV}{mr_o^2\omega^2} \quad \omega t = 2\xi$$

gives the following differential equations (Mathieu's equations)

$$\left. \begin{array}{l} \frac{d^2x}{dt^2} + (a + 2q \cos 2\xi)x = 0 \\ \frac{d^2y}{dt^2} - (a + 2q \cos 2\xi)y = 0 \\ \frac{d^2z}{dt^2} = 0 \end{array} \right\}$$

Describes the motion of charged particle in the quadrupole field

Mass Filter stability diagram



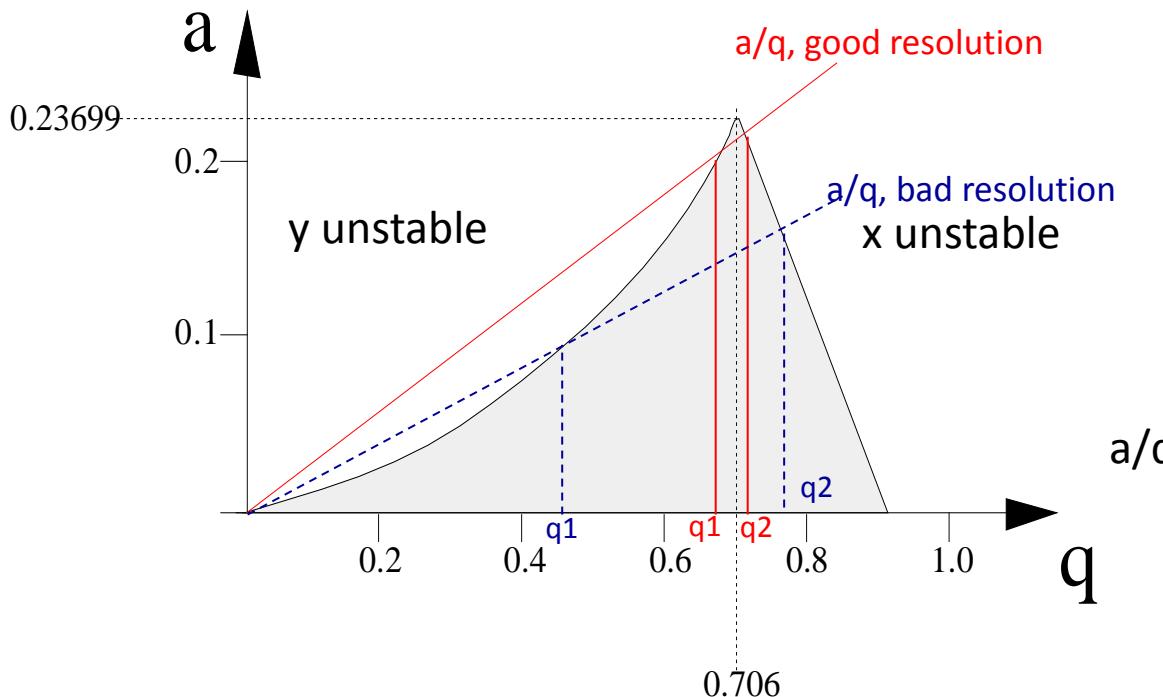
$$a = \frac{8 * e * U}{m * l_0^2 * \omega^2}$$

$$q = \frac{4 * e * V}{m * l_0^2 * \omega^2}$$



MAX-PLANCK-GESELLSCHAFT

Mass separation Quadrupole analyzer



$$a = \frac{8 * e * U}{m * r_0^2 * \omega^2}$$

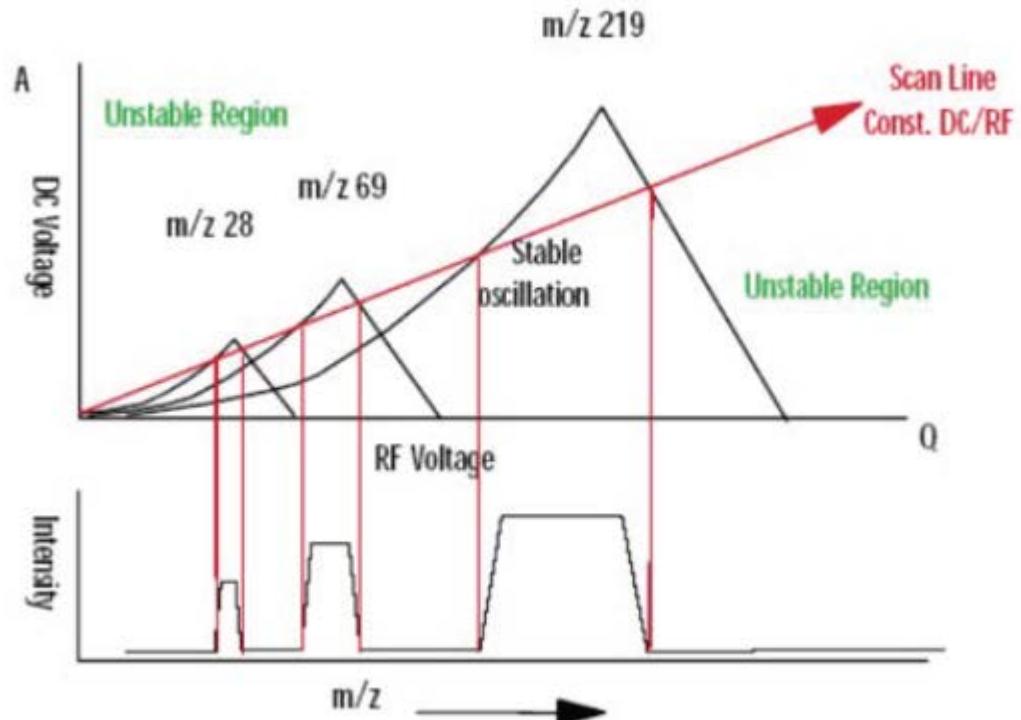
$$q = \frac{4 * e * V}{m * r_0^2 * \omega^2}$$

$$a/q = 2U/V = \text{const} = 0.1768$$

- The ratio a/q (which is proportional to U/V) is kept constant and the absolute values of U and V are increased.
- For $a/q < 0.1678$ (which is the top of the stability-diagram) the stability region is entered periodically for the different masses.
- The slope a/q determines the mass resolution.

$$m_1 = \frac{4eV}{r_0^2 \omega^2 q_1} > m > m_2 = \frac{4eV}{r_0^2 \omega^2 q_2}$$

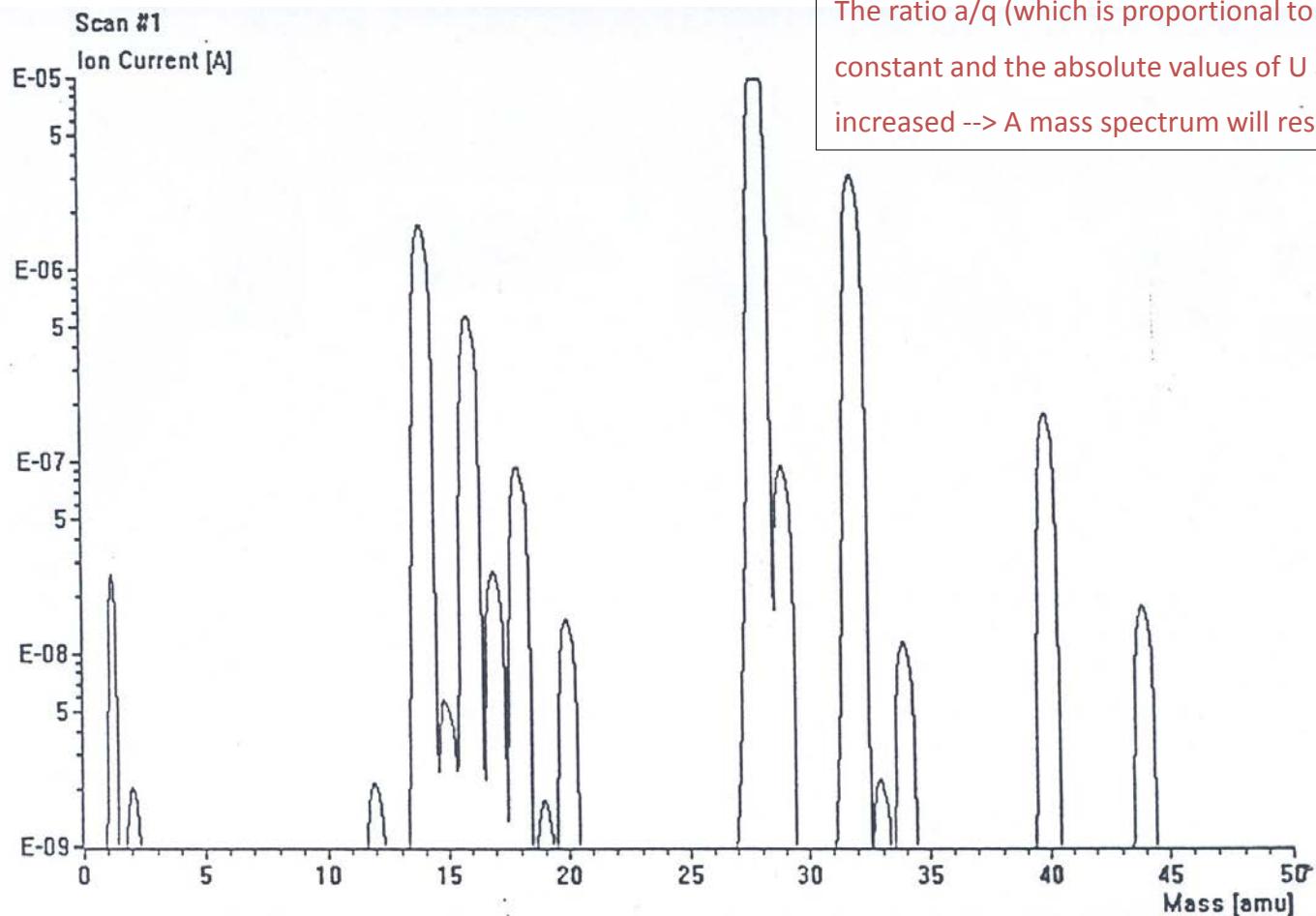
Mass separation Quadrupole analyzer



- Scanning the U/V amplitude ratio gives a mass spectrum
- The scan parameters determine the resolution and transmission
- Lower resolution at higher masses



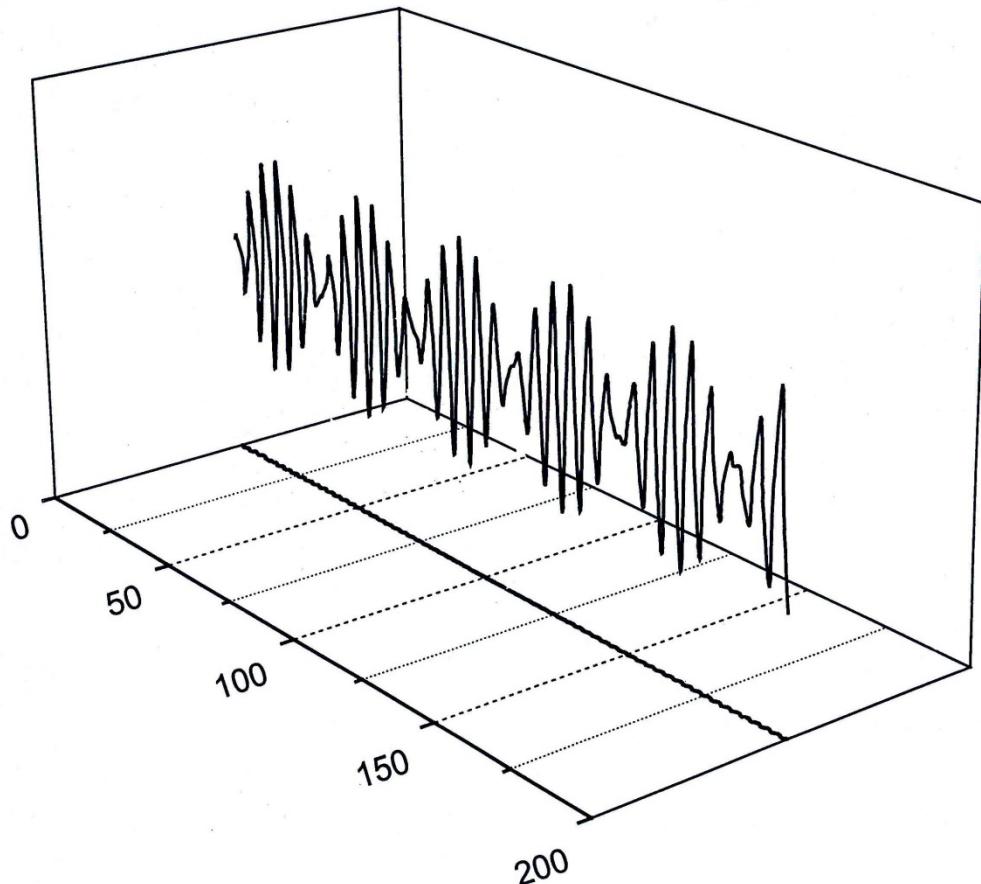
Mass separation Quadrupole analyzer



Mass separation Quadrupole analyzer

Flight path of ions through the mass analyzer

→ Ion mass identical to set mass (transmission full)



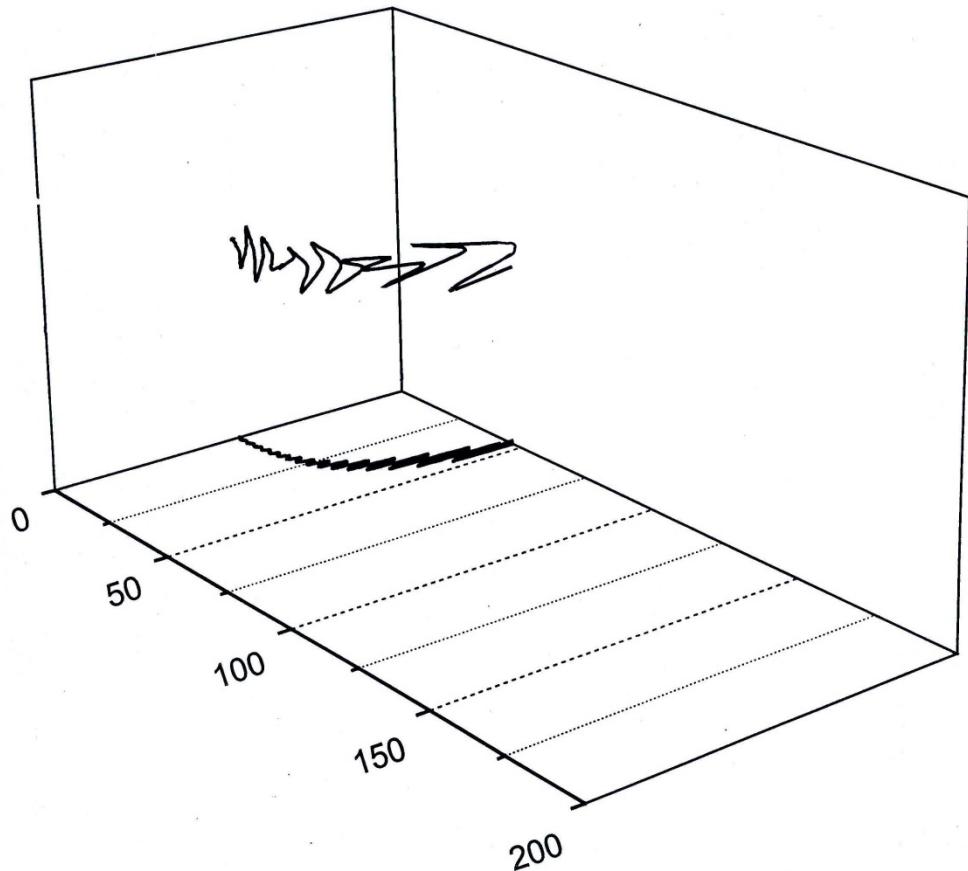
$$\left. \begin{array}{l} \frac{d^2x}{dt^2} + (a + 2q \cos 2\xi)x = 0 \\ \frac{d^2y}{dt^2} - (a + 2q \cos 2\xi)y = 0 \\ \frac{d^2z}{dt^2} = 0 \end{array} \right\}$$



Quadrupole Mass Spectrometer

Flight path of ions through the mass analyzer

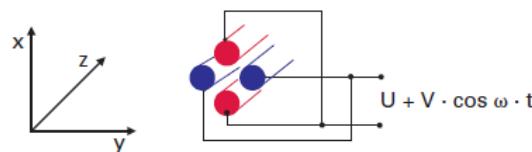
→ Ion mass higher than set mass (transmission none)





MAX-PLANCK-GESELLSCHAFT

Mass separation Quadrupole analyzer

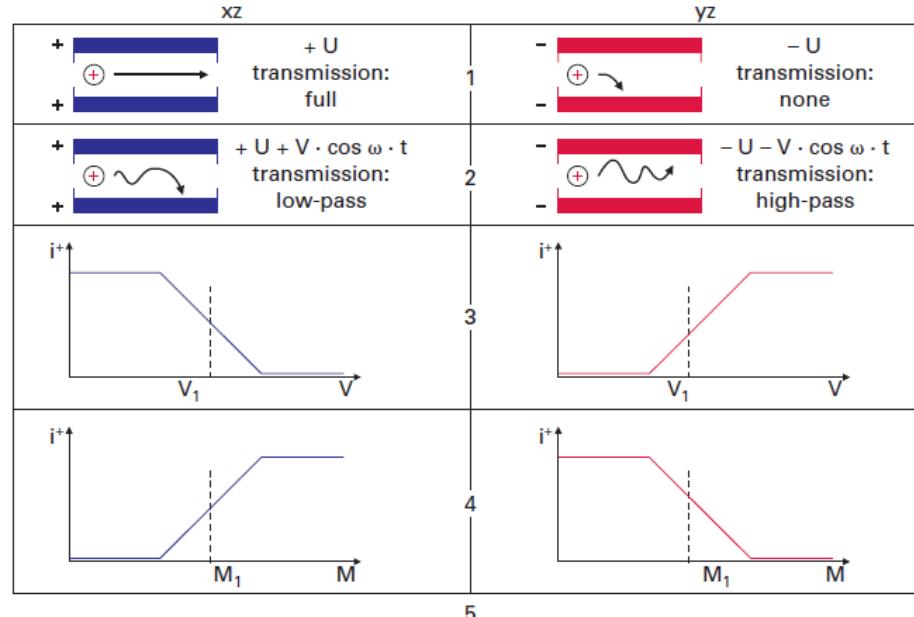


Ions reach the detector

ions are neutralized

Fixed M/q
 $V > V_1$ no transmission

Fixed U/V
 $M > M_1$ transmission

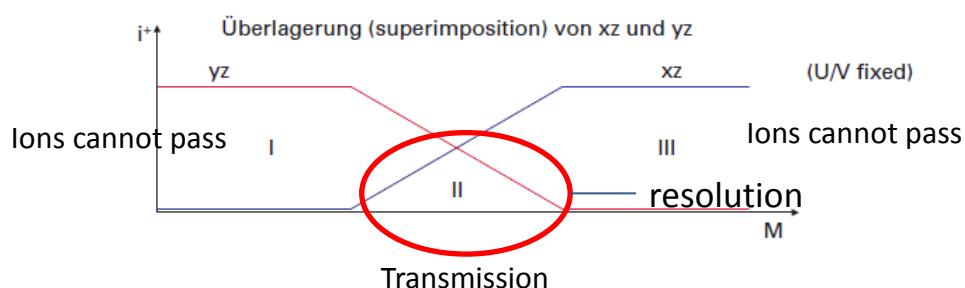


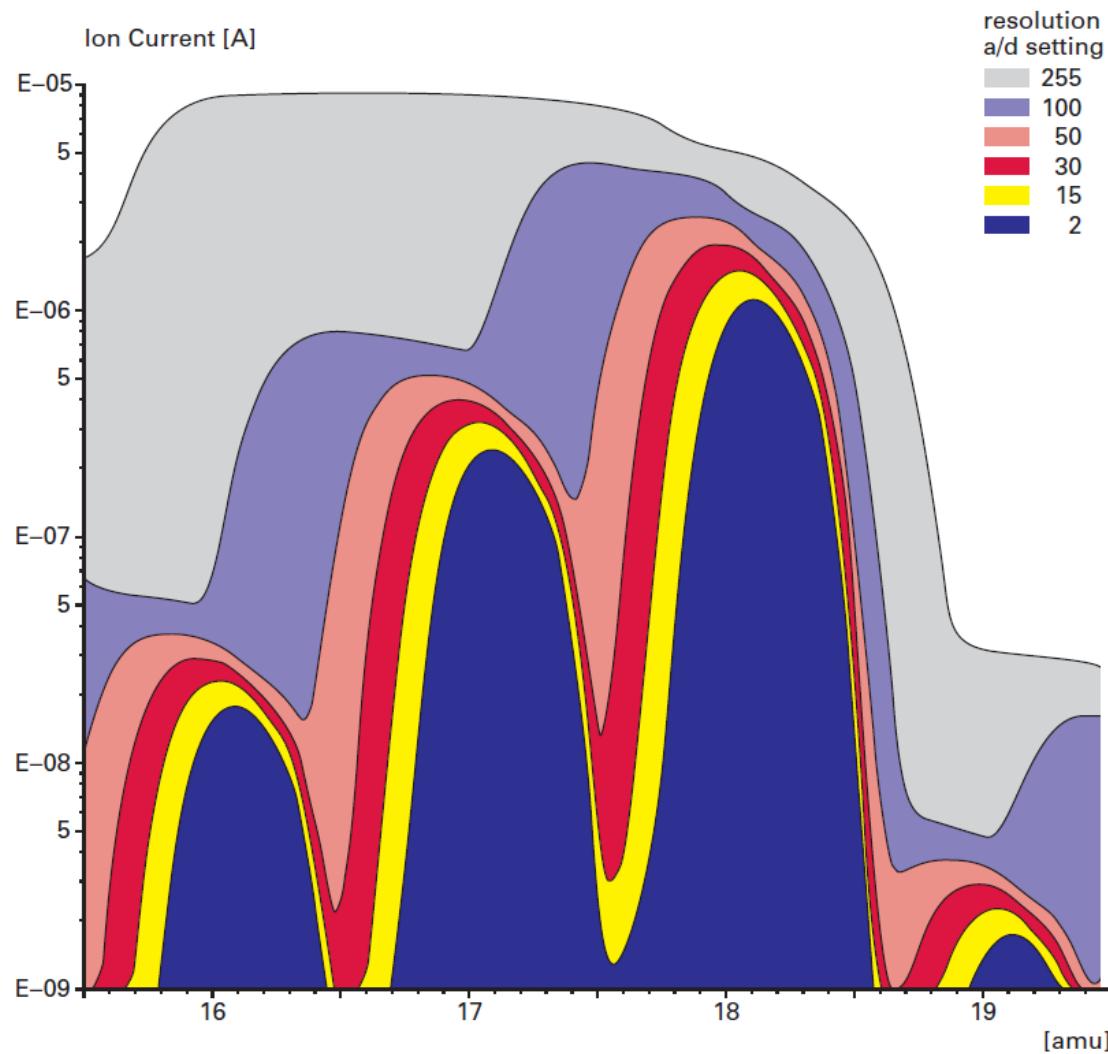
ions are neutralized

ions reach the detector

Fixed M/q
 $V > V_1$ transmission

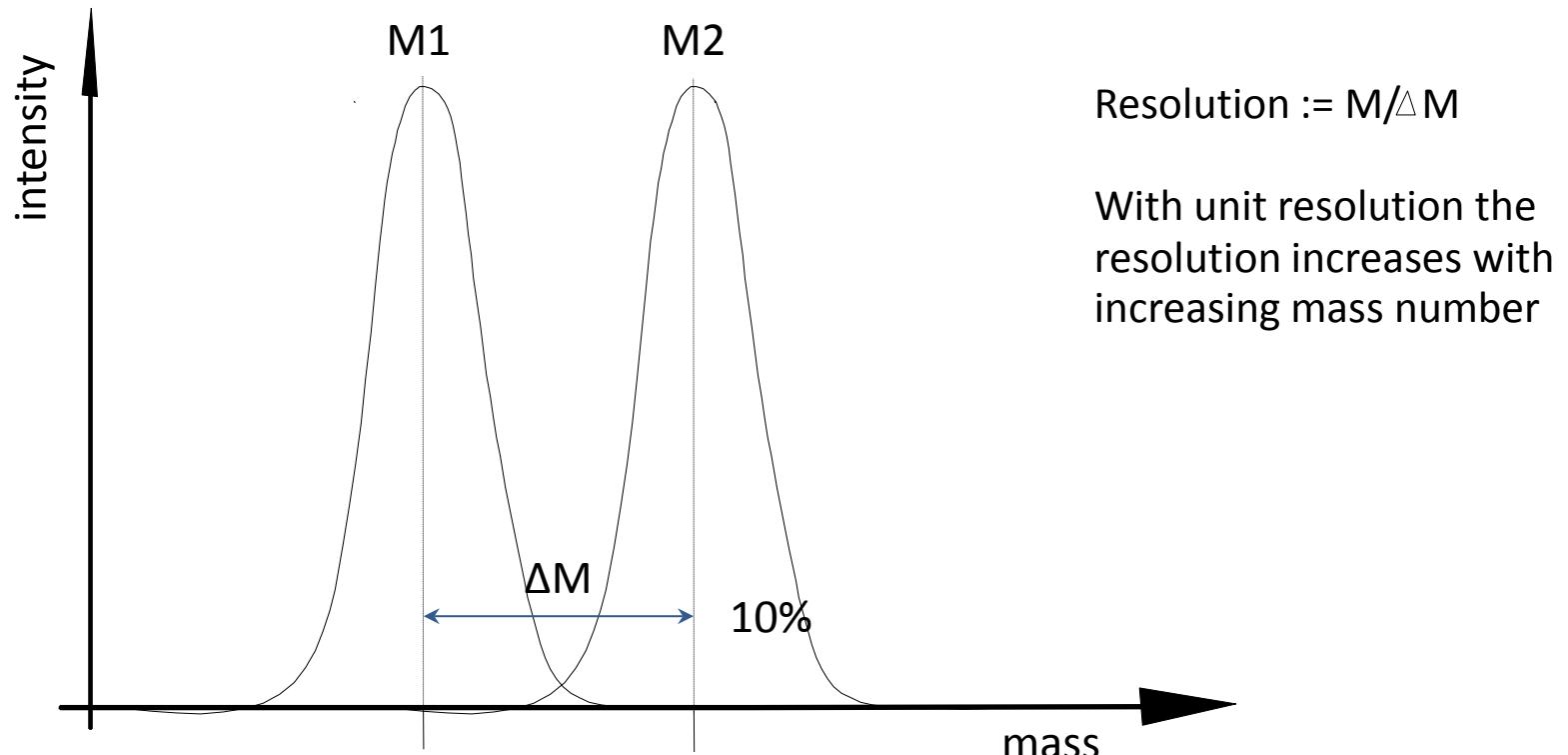
Fixed U/V
 $M > M_1$ transmission suppressed





Mass separation Quadrupole analyzer

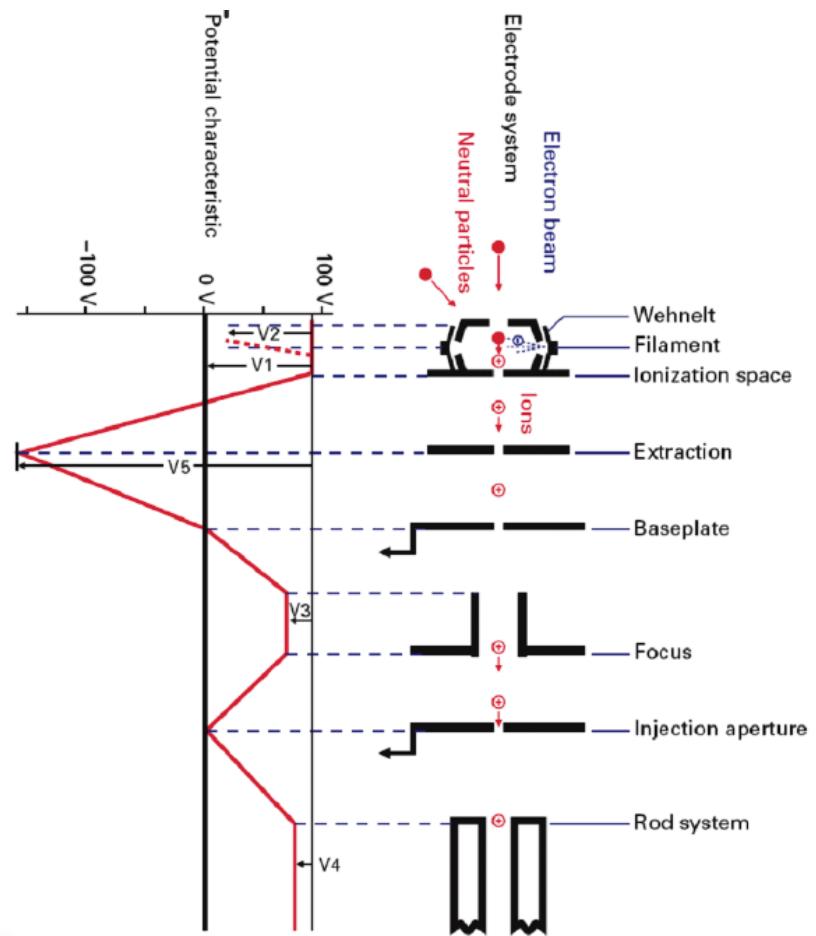
Unit Resolution (Relative resolution on 10% peak high)



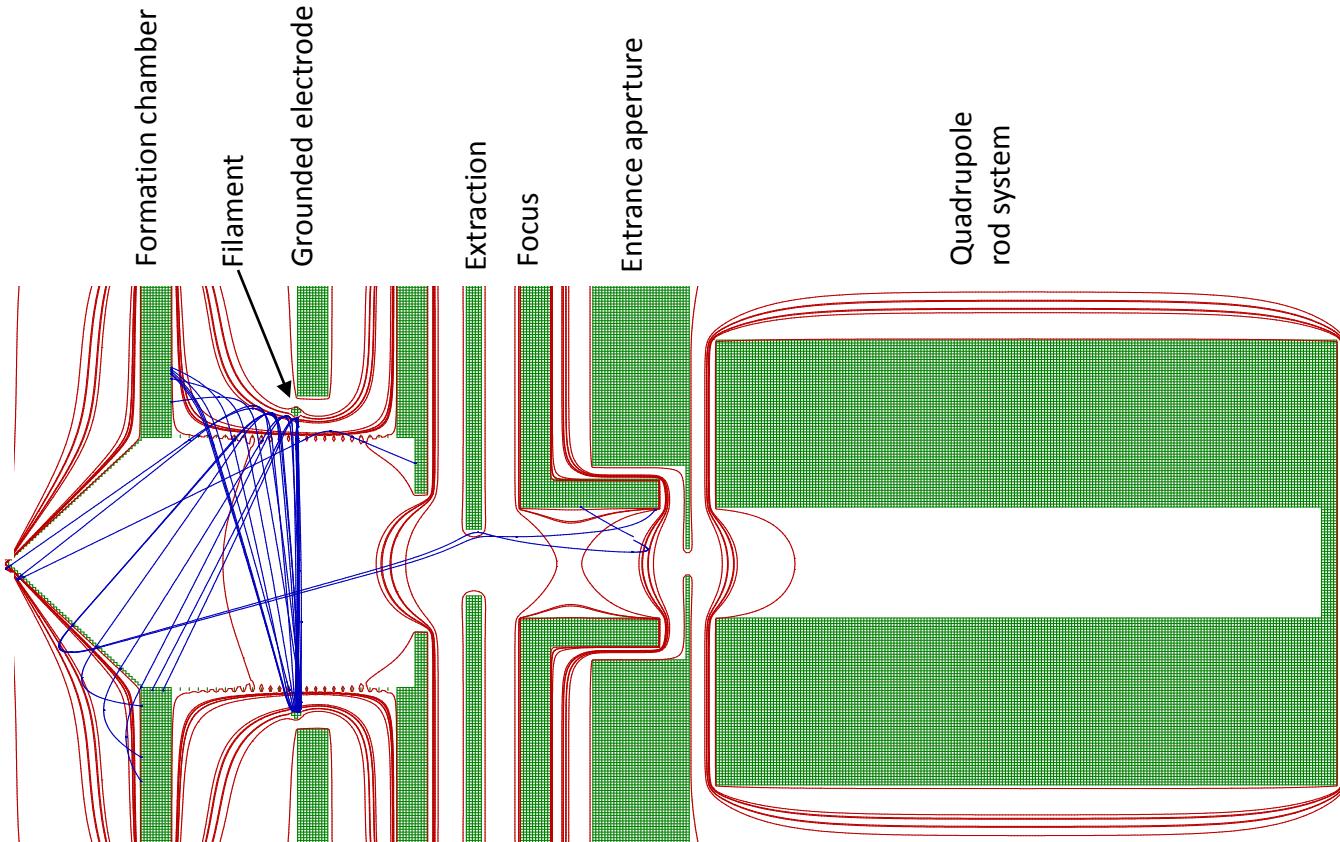
$M_1/z\ 200\ ^{200}\text{Hg}^+$, $M_2/z\ 201,\ ^{201}\text{Hg}^+$, $\Delta M=1$, Unit Resolution 200

Mass separation Quadrupole analyzer

Axial ion source

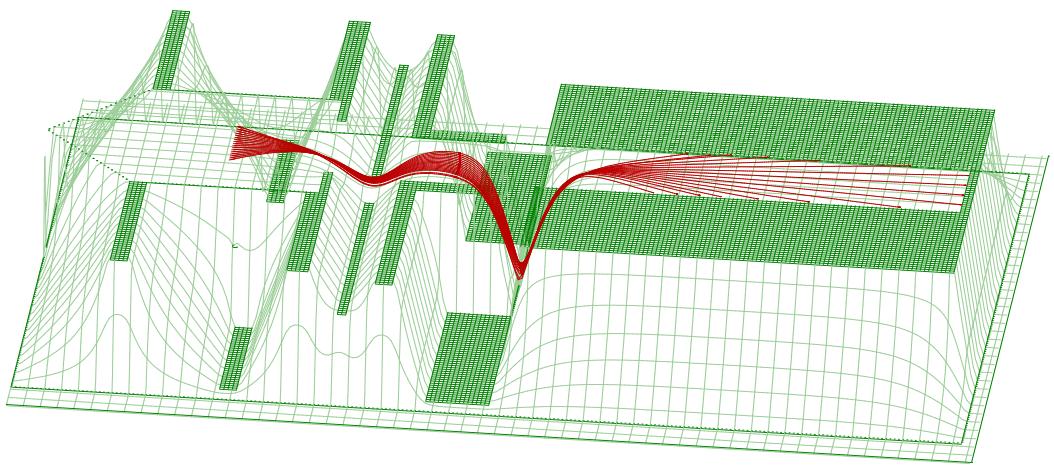
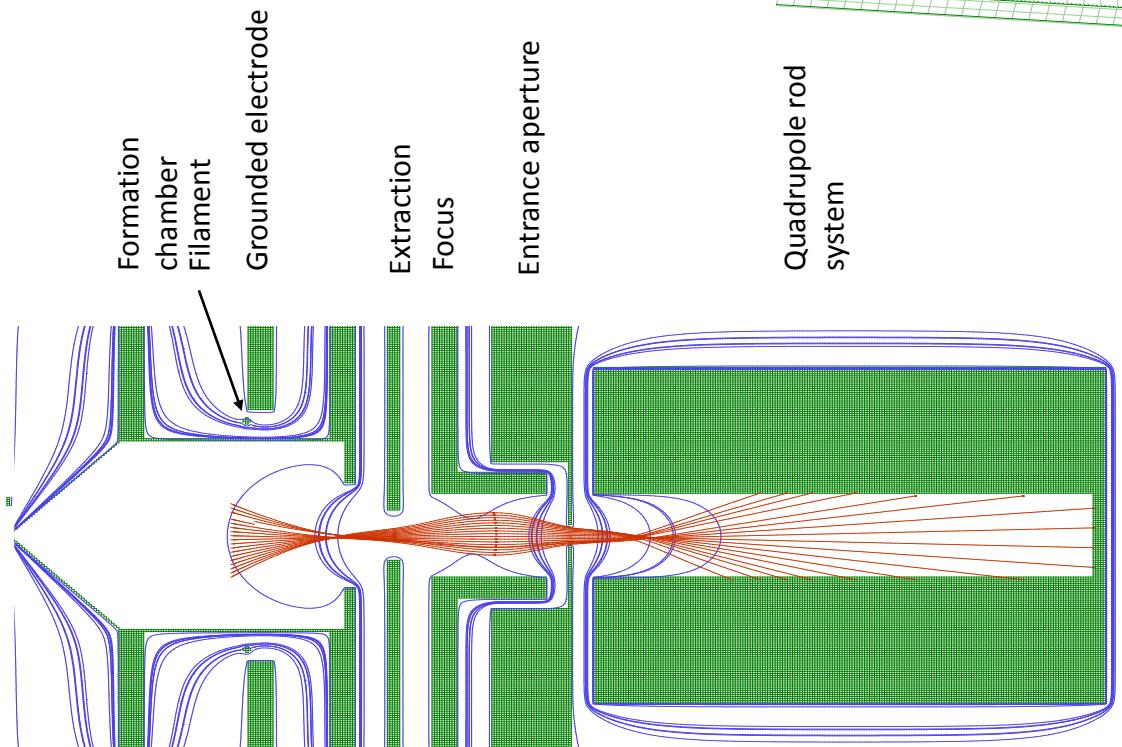


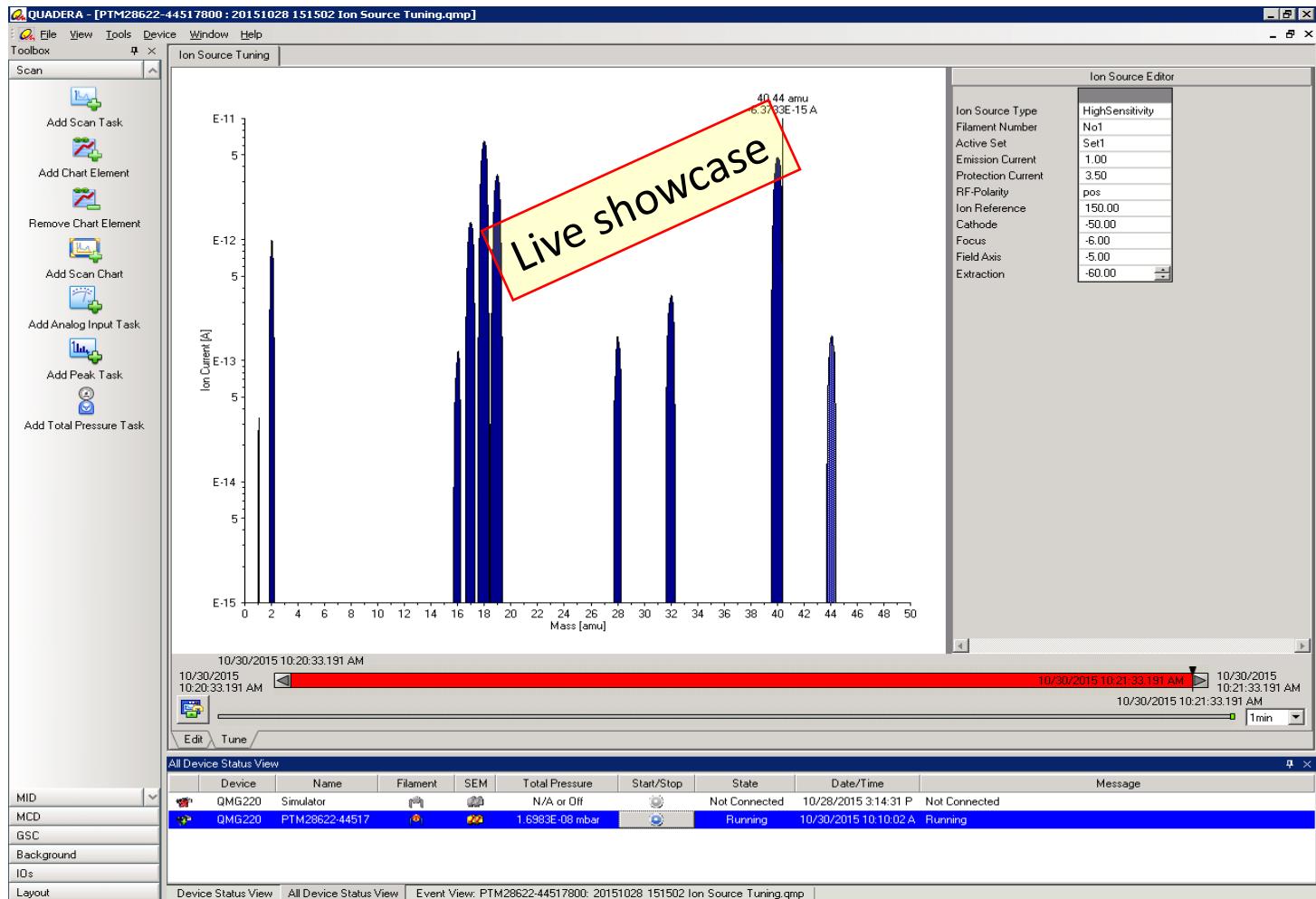
Flight paths of electrons





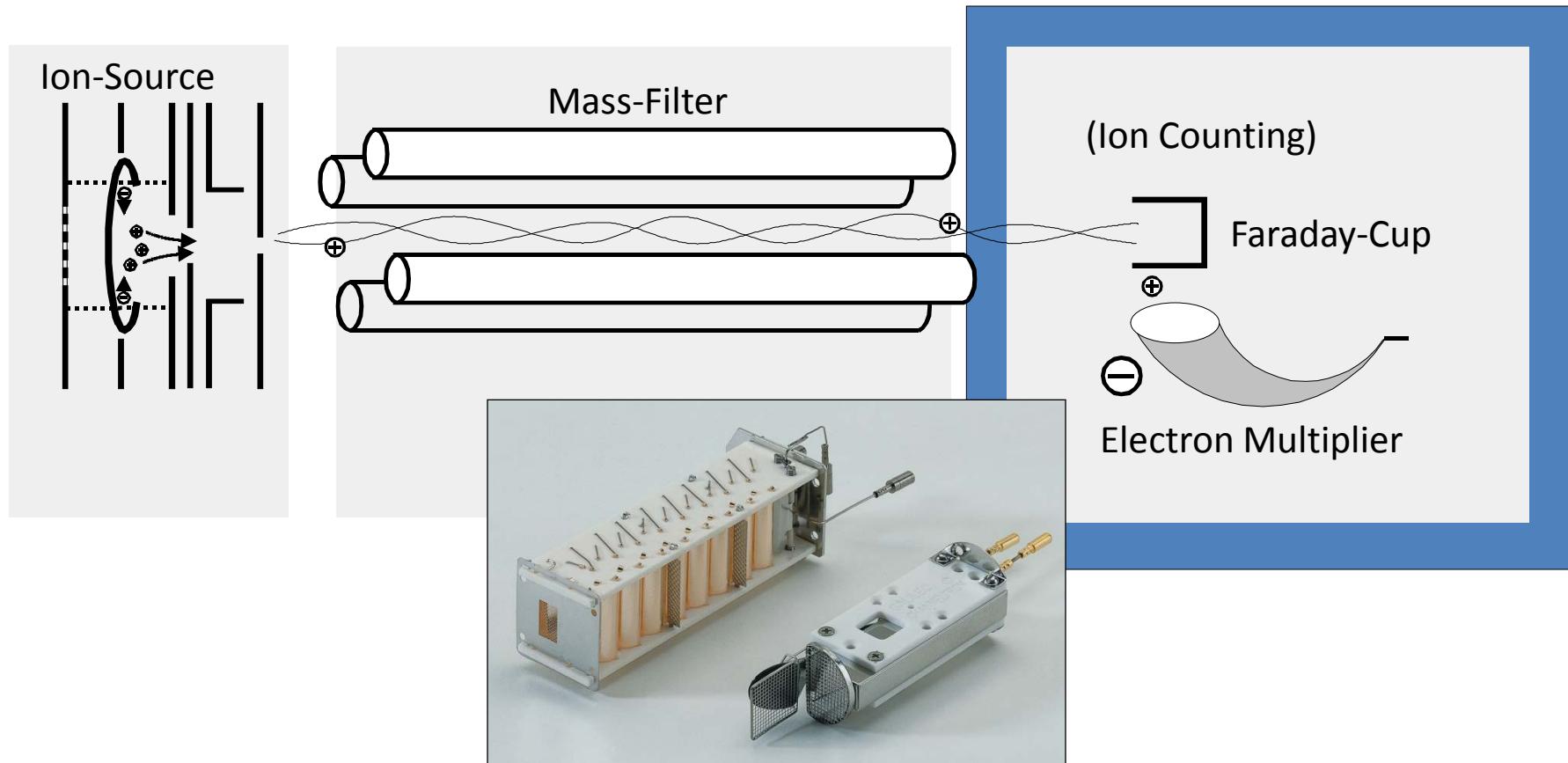
Flight paths of positive ions





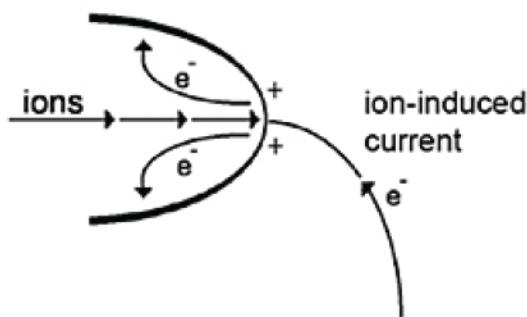


Detectors

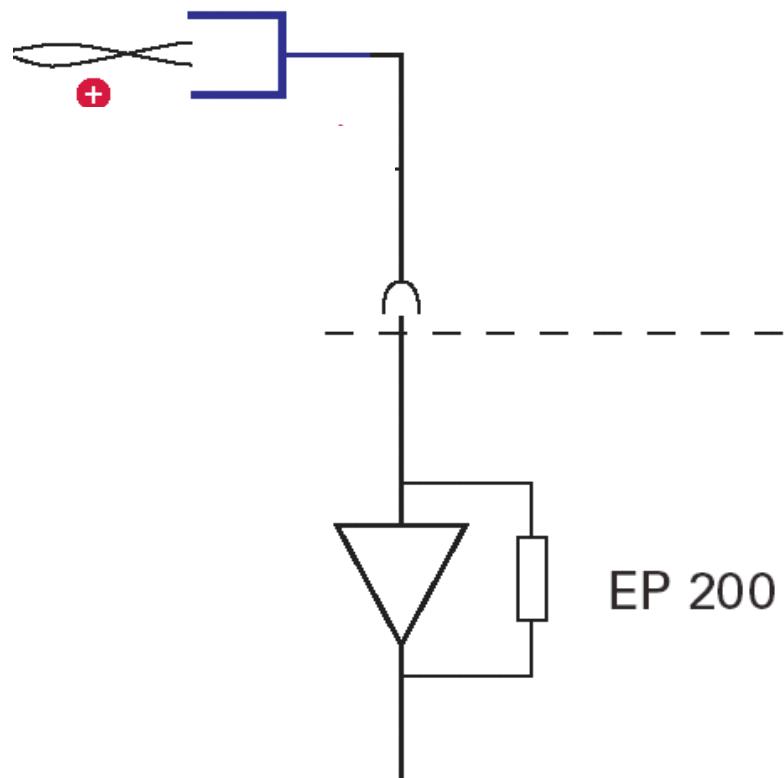


Detectors

- Ions leaving the quadrupole mass spectrometer fly into a *Faraday-Cup* and give off their charge
- Charge is measured as a current by an electrometer amplifier
- Measurable currents are from 1E-15 to 1E-8 A



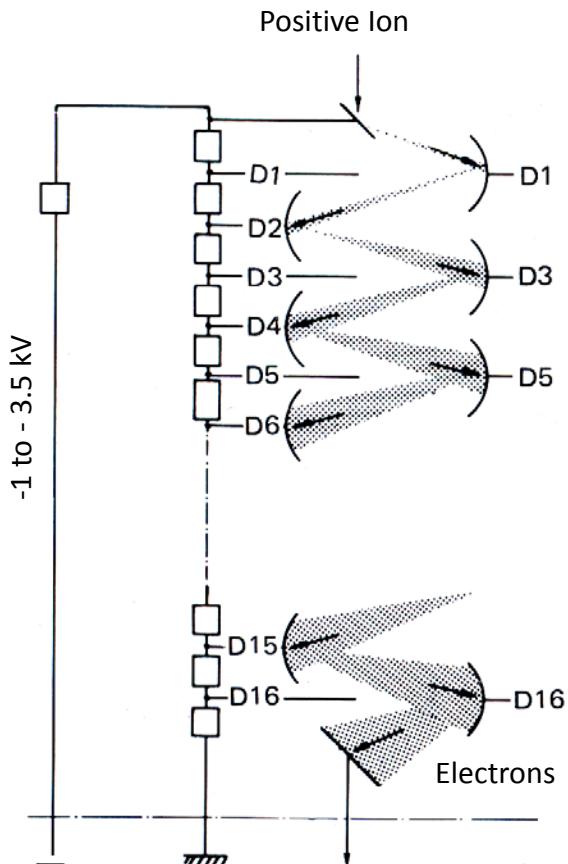
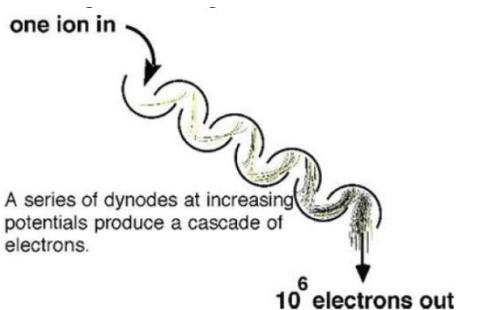
Faraday-Cup



Detectors

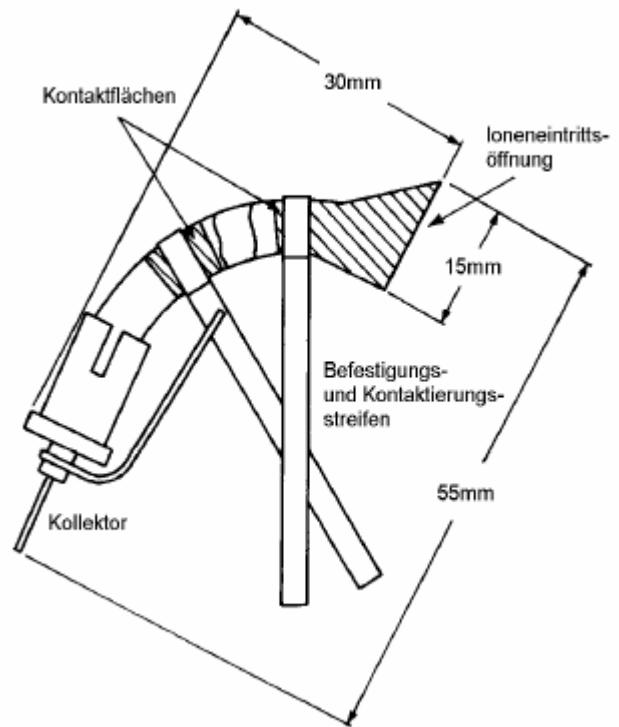
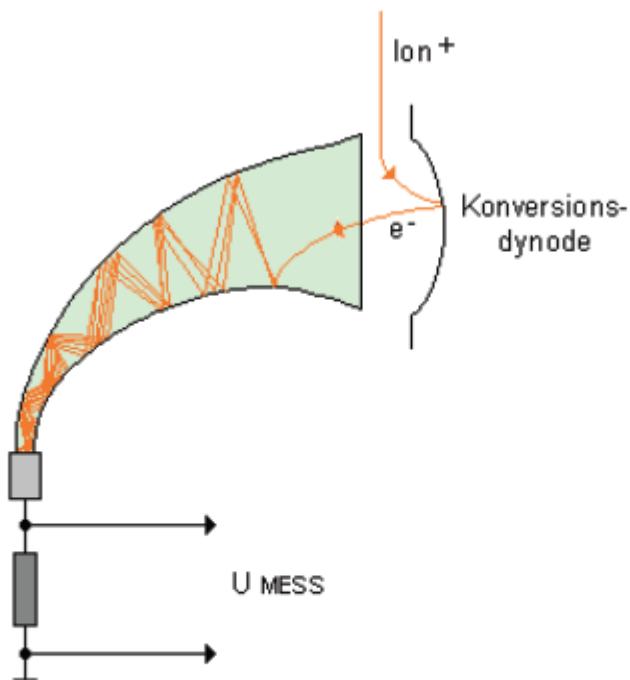
Secondary Electron Multiplier, discrete

- Particles (ions, neutrals, electrons, photons) hitting a surface with high energy release several „secondary electrons“
- Use of several dynodes allows for an amplification of up to 10^8
- Low ion currents can be detected easily
- Measuring range $1E-15$ to $1E-5A$
- „Counting“ of individual electron bursts allows for the detection of single ions ($1E-19A$)



Detectors

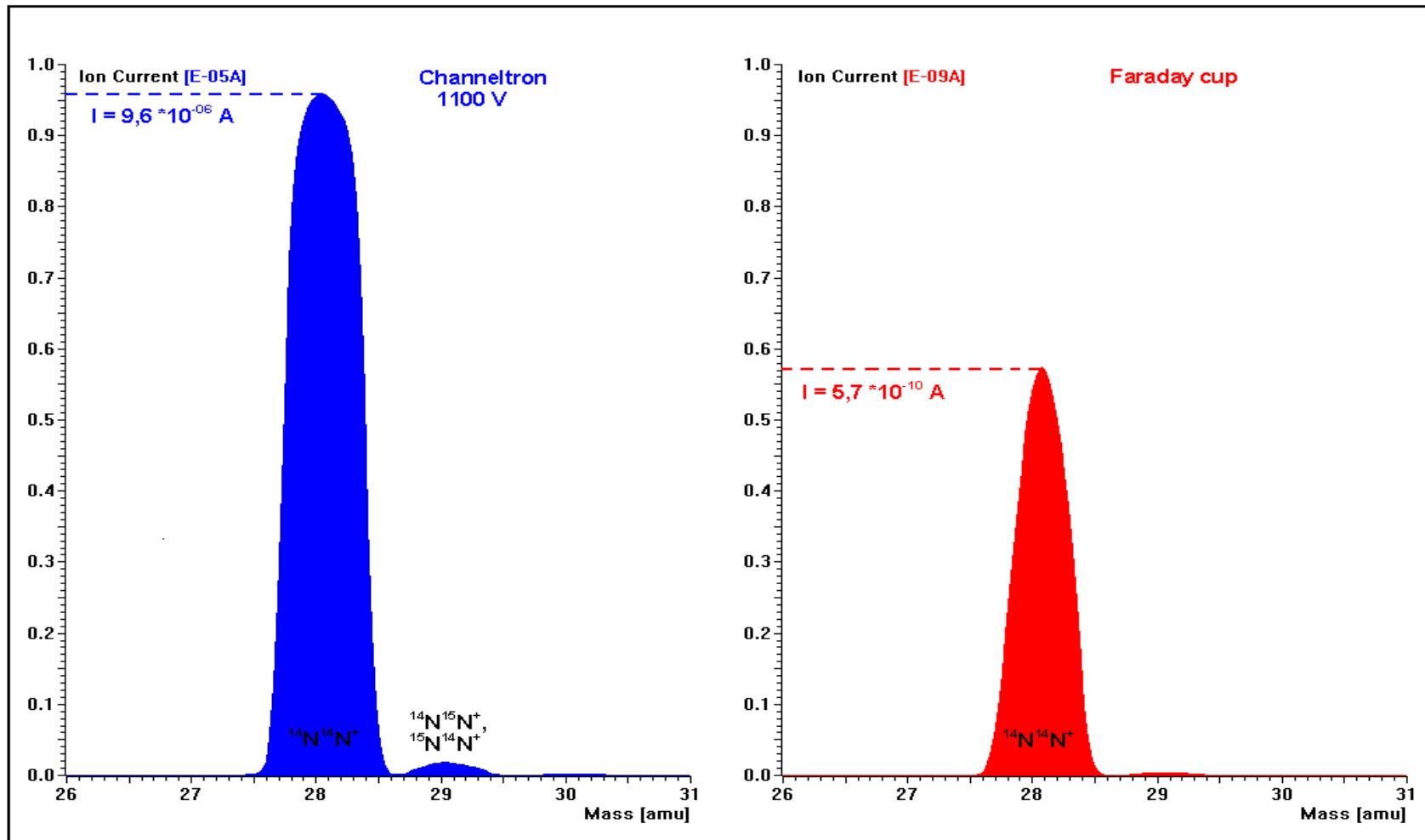
Channeltron



- Glastube covered with antimony based material
- Gain $10^5 - 10^6$
- for voltage amplification high resistance is used

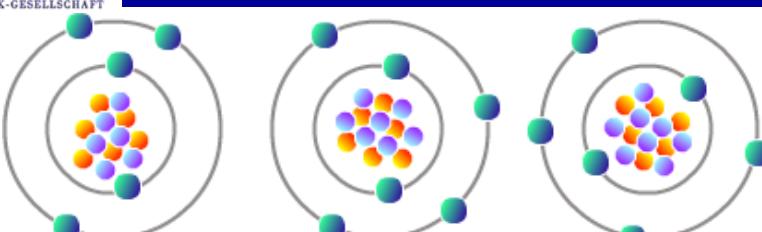


Detectors





MAX-PLANCK-GESSELLSCHAFT



Carbon

6 Protons
6 Neutrons

Carbon-13

6 Protons
7 Neutrons

Carbon-14

6 Protons
8 Neutrons

Nuclear number

$$= 6 + 6 \\ = 12$$

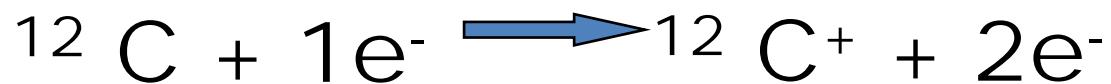
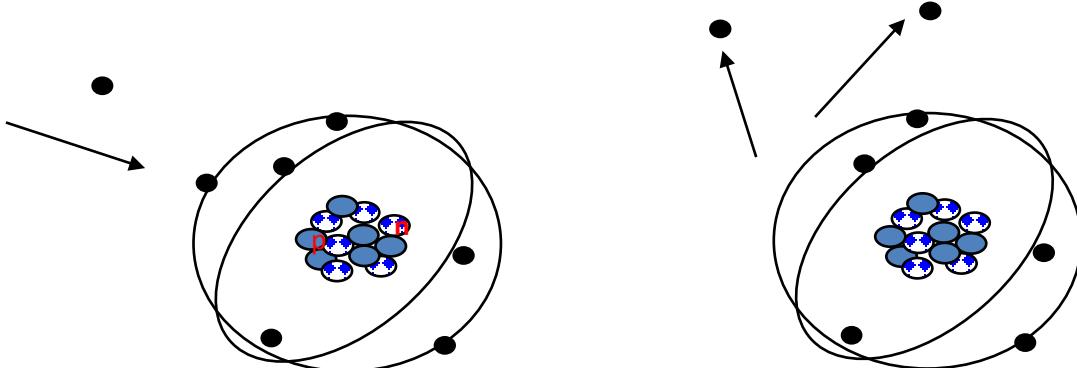
Nuclear number

$$= 6 + 7 \\ = 13$$

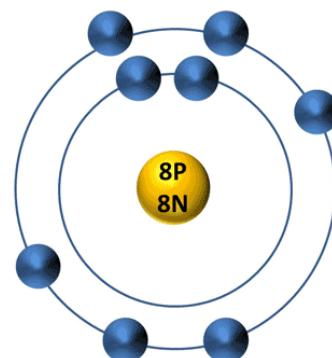
Nuclear number

$$= 6 + 8 \\ = 14$$

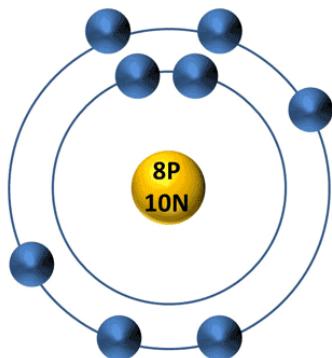
➤ Ionization



Oxygen Isotopes

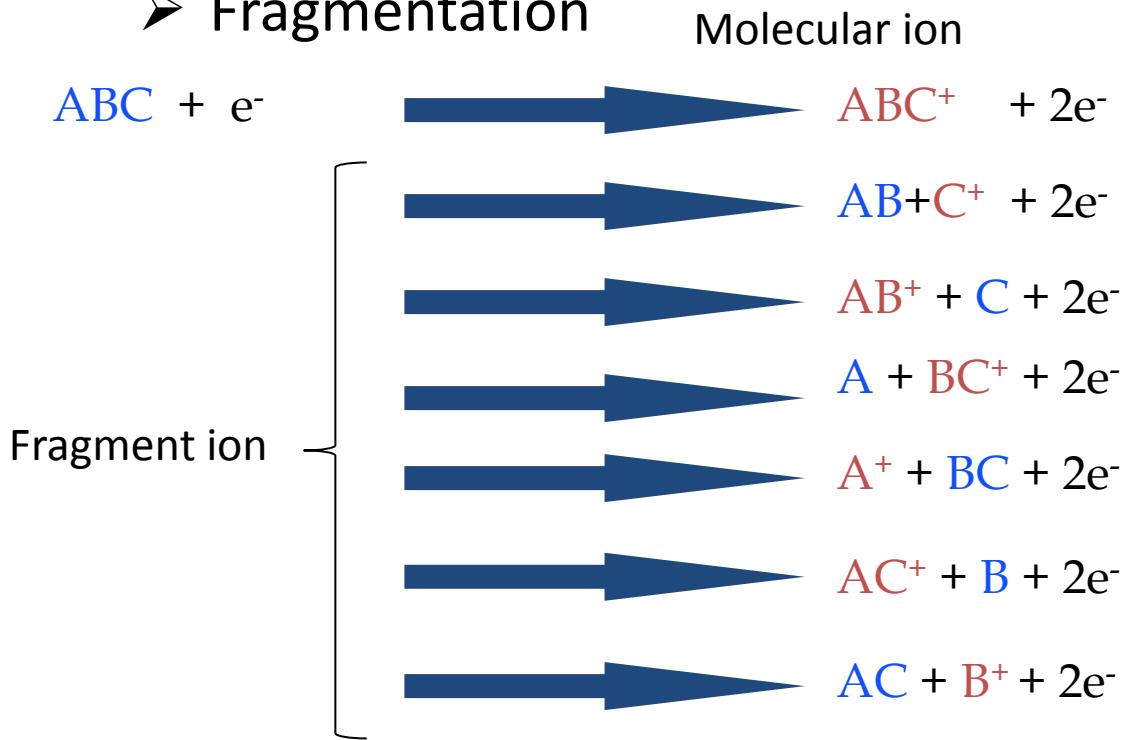


^{16}O Isotope

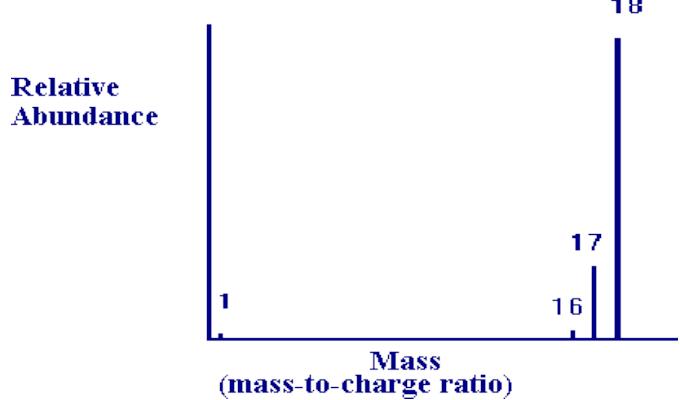


^{18}O Isotope

➤ Fragmentation



Mass Spectrum of Water

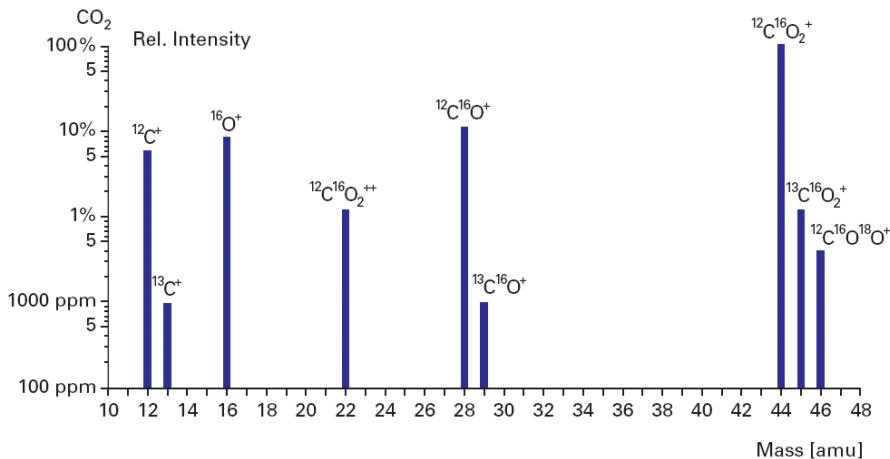


Fragmentation

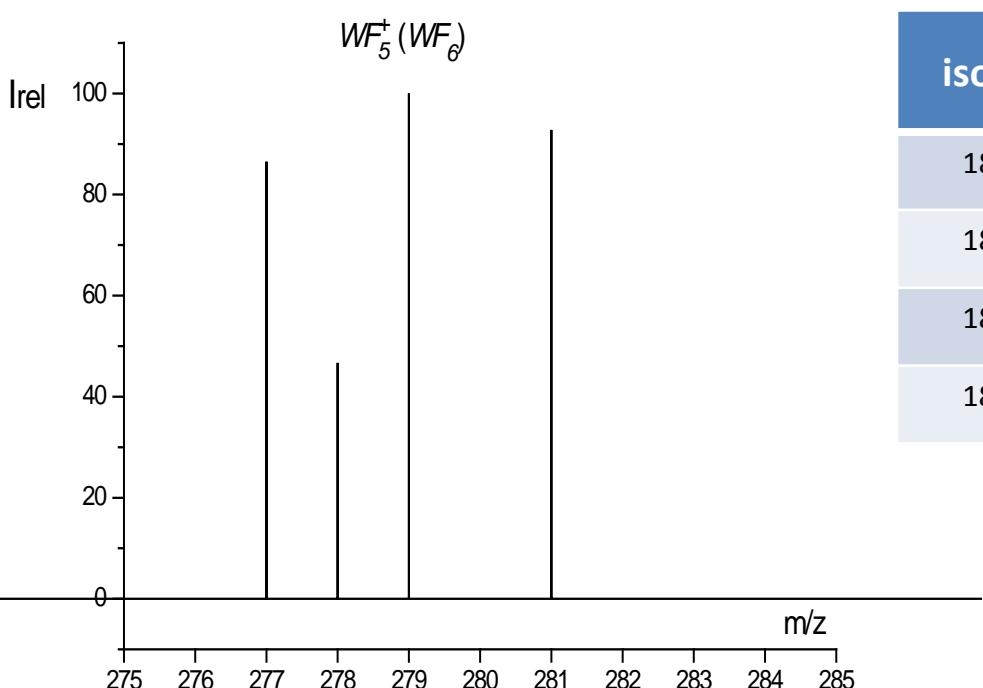
Molecule + electron

- Ionization of molecule
- Fragmentation of molecule
- Multiple charged ions
- Isotopic effects are shown

Example:



- $\text{CO}_2^+ + 2 e^-$ 44 m/e
- $\text{C}^+ + 2 e^-$ 12 m/e
- $\text{O}^+ + 2 e^-$ 16 m/e
- $\text{CO}^+ + 2 e^-$ 28 m/e
- $\text{CO}_2^{++} + 3 e^-$ 22 m/e
- $^{12}\text{C}^{16}\text{O}^{18}\text{O}^+ + 2 e^-$ 46 m/e

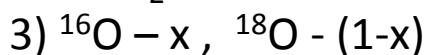
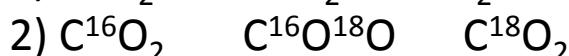


isotope	Natural Abundance, %	I _{rel} , %
^{182}W	26.5	83.7
^{183}W	14.3	45.2
^{184}W	31.6	100.0
^{186}W	28.4	89.8

- Isotopic pattern is a fingerprint of all chemical elements and compounds

Isotope scrambling

Theoretical pattern of CO_2^+

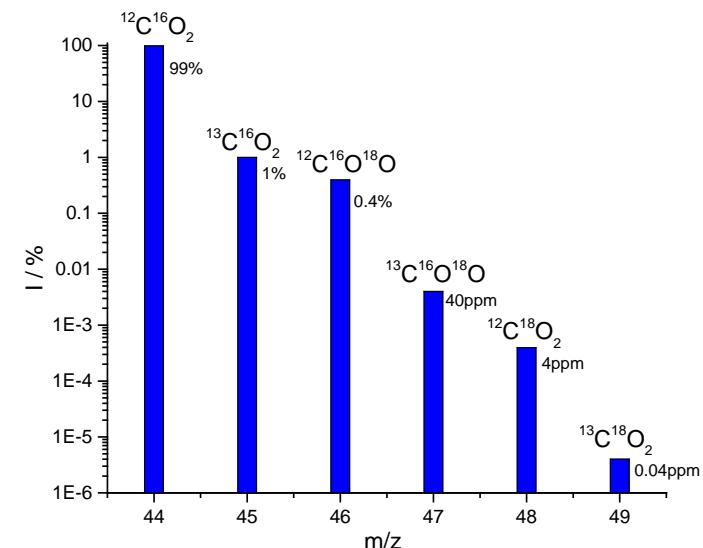
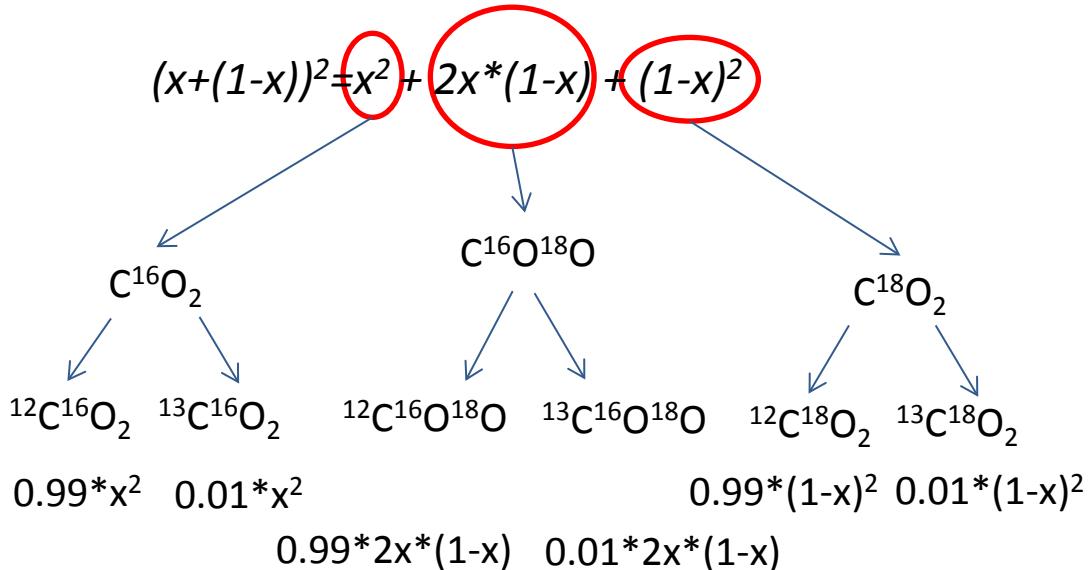


$${}^{16}\text{O} = 0.998, {}^{18}\text{O} = 0.002$$

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

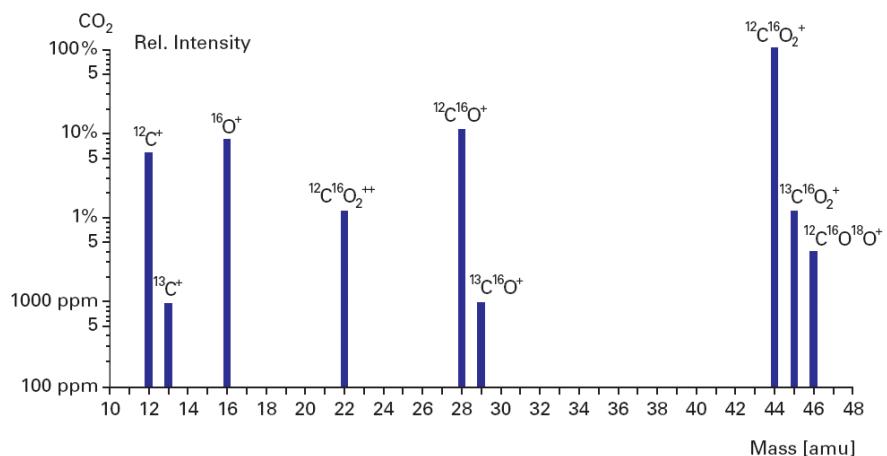
n – number of equivalent positions, row number, 2

a, b , abundance of isotopes, $x, 1-x$.

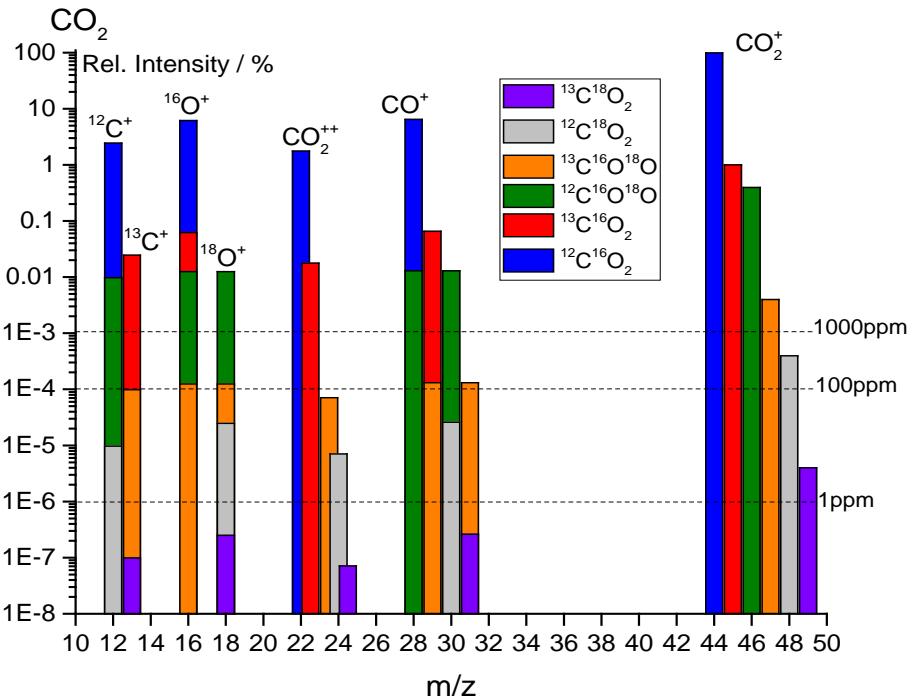


Fragmentation

Cracking pattern of CO_2 (experimental)
 Fragmentation distribution at 70 eV



Cracking pattern of CO_2 (calculated)

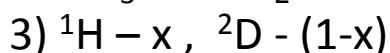
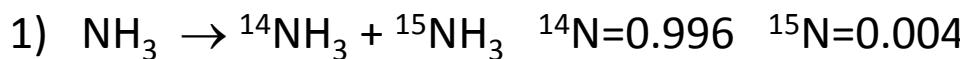


Deviation of experimental and theoretical spectra caused by:

- Proximity to equilibrium (equilibrium is not reached)
- Kinetic hindrance
- Thermodynamically favorable configuration
- Isotopic composition

Isotope scrambling

Theoretical pattern of NH_3^+



$${}^1\text{H} = 0.9999, {}^2\text{D} = 0.0001$$

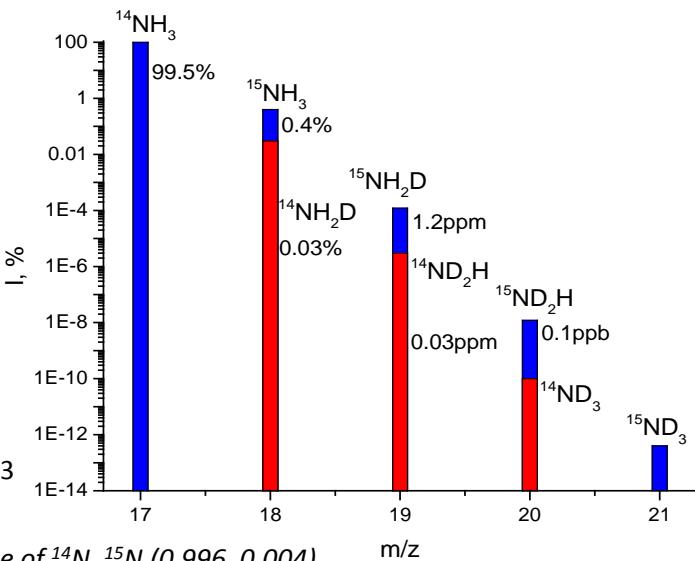
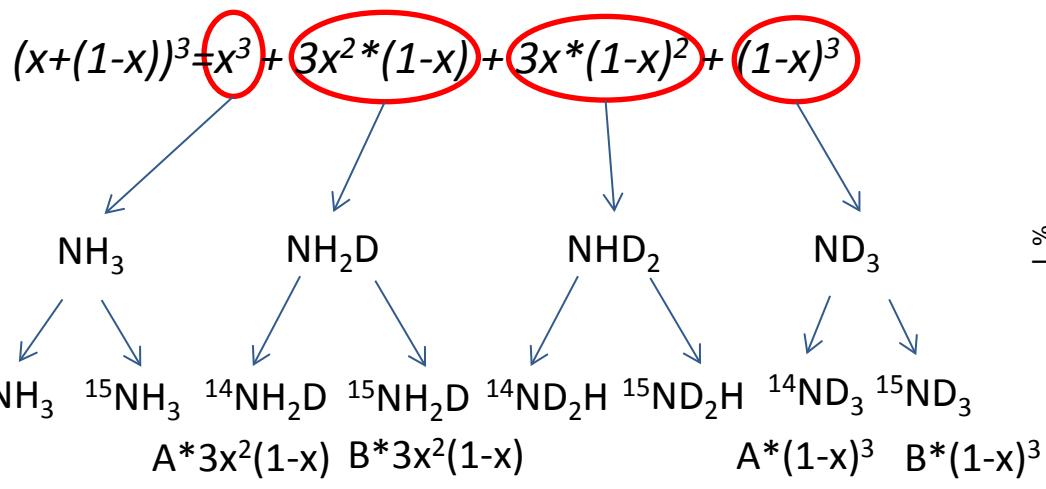
$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

n – number of equivalent positions, row number, 3

a, b , natural abundance of isotopes, $x, 1-x$.

- Distribution is determined by binomial coefficients

n	k							
0	1							Row 0
1	1 1							Row 1
2	1 2 1							Row 2
3	1 3 3 1							Row 3
4	1 4 6 4 1							Row 4
5	1 5 10 10 5 1							Row 5
6	1 6 15 20 15 6 1							Row 6
7	1 7 21 35 35 21 7 1							Row 7



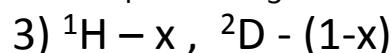
A, B , abundance of ${}^{14}\text{N}, {}^{15}\text{N}$ (0.996, 0.004)



Isotope scrambling

MAX-PLANCK-GESELLSCHAFT

Theoretical pattern of CH_4^+



$${}^1\text{H} = 0.9999, {}^2\text{D} = 0.0001$$

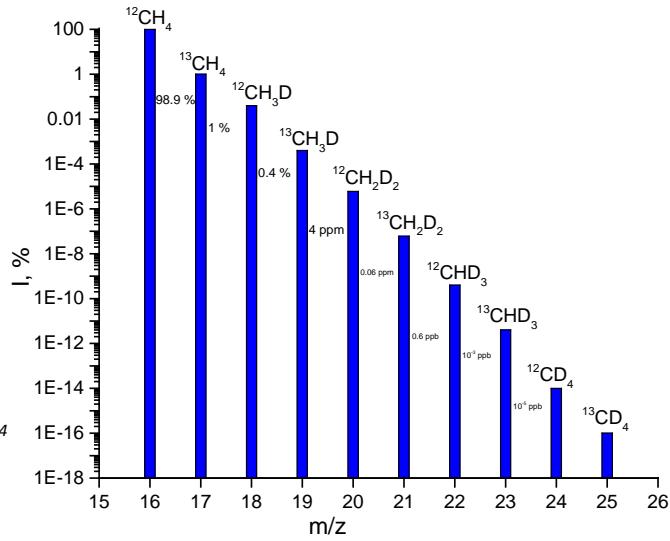
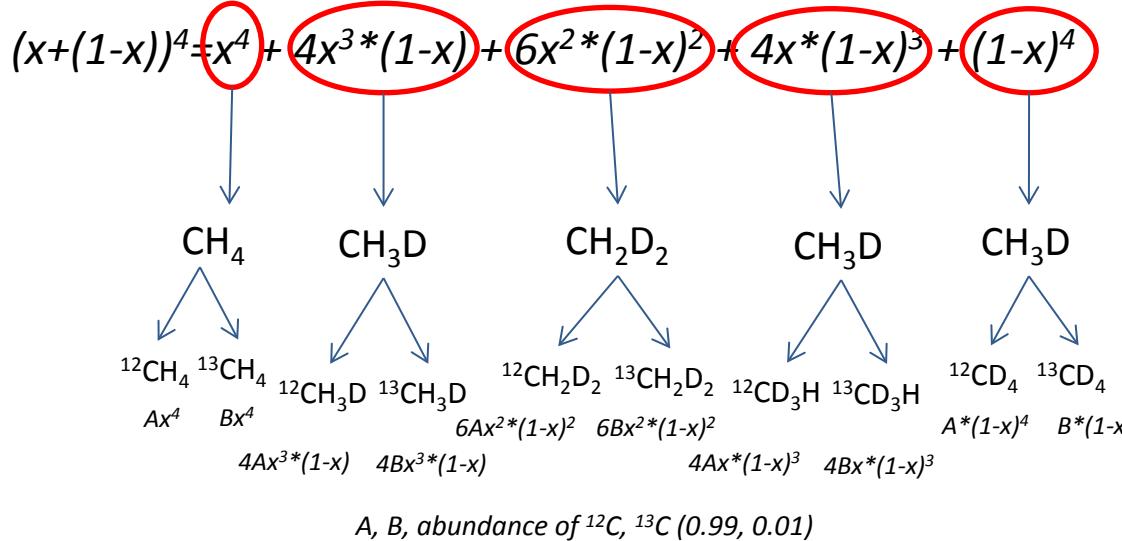
$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

n – number of equivalent positions, row number, 4

a, b , abundance of isotopes, $x, 1-x$.

- Distribution is determined by binomial coefficients

n	k								
0									Row 0
1					1		1		Row 1
2			1		2		1		Row 2
3		1	3		3		1		Row 3
4	1	4	6	4	1				Row 4
5	1	5	10	10	5				Row 5
6	1	6	15	20	15	6	1		Row 6
7	1	7	21	35	35	21	7	1	Row 7





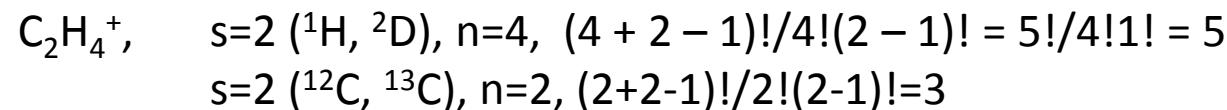
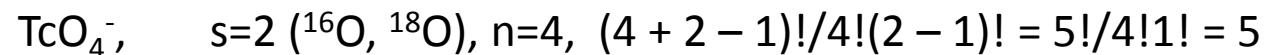
Isotope scrambling

s - type of particles (number of isotopes of one element)

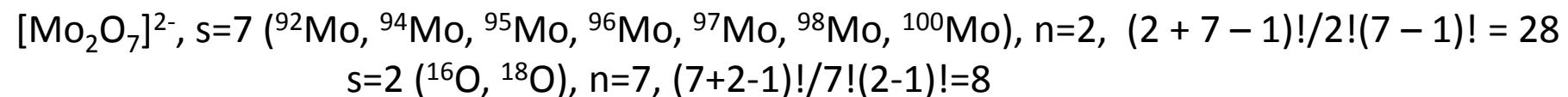
n – number of particles

C_s^s – number of combinations of n particles with s isotopes (number of lines in spectra)

$$C_s^s = \frac{(n+s-1)!}{n!(s-1)!}$$



$\text{C}_4^2 * \text{C}_2^2 = 15$ (15 possible lines in spectra for ethylene (C_2H_4^+))

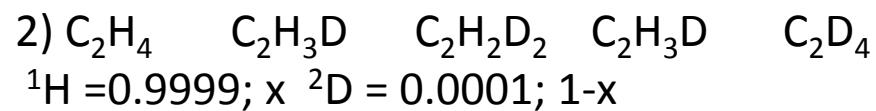
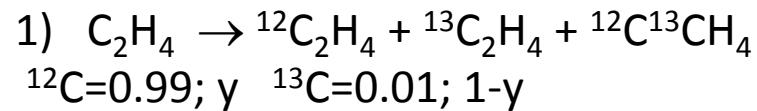


$\text{C}_2^7 * \text{C}_2^2 = 224$ (224 possible lines in spectra of $[\text{Mo}_2\text{O}_7]^{2-}$)



Isotope scrambling

Theoretical pattern of $C_2H_4^+$



$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

- Distribution is determined by binomial coefficients

n	k								
0	1								Row 0
1	1 1								Row 1
2	1 2 1								Row 2
3	1 3 3 1								Row 3
4	1 4 6 4 1								Row 4
5	1 5 10 10 5 1								Row 5
6	1 6 15 20 15 6 1								Row 6
7	1 7 21 35 35 21 7 1								Row 7

n – number of equivalent positions, row number, 2 (C) and 4 (H)

a, b , abundance of isotopes, $y, 1-y$ and $x, 1-x$,

$$(y+(1-y))^2 = y^2 + 2y*(1-y) + (1-y)^2 \text{ for carbon}$$

$$(x+(1-x))^4 = x^4 + 4x^3*(1-x) + 6x^2*(1-x)^2 + 4x*(1-x)^3 + (1-x)^4 \text{ for hydrogen}$$

$$(y+(1-y))^2 \cdot (x+(1-x))^4 \longrightarrow \text{Intensity distribution for 15 possible lines of } (C_2H_4^+)$$

Theoretical pattern of $[Mo_2O_7]^{2-}$

1) Mo

^{92}Mo , ^{94}Mo , ^{95}Mo , ^{96}Mo , ^{97}Mo , ^{98}Mo , ^{100}Mo
a, b, c, d, e, f, g

^{92}Mo 61.5%

^{94}Mo 38.3%

^{95}Mo 66.0%

^{96}Mo 69.1%

^{97}Mo 39.6%

^{98}Mo 100%

^{100}Mo 39.9%

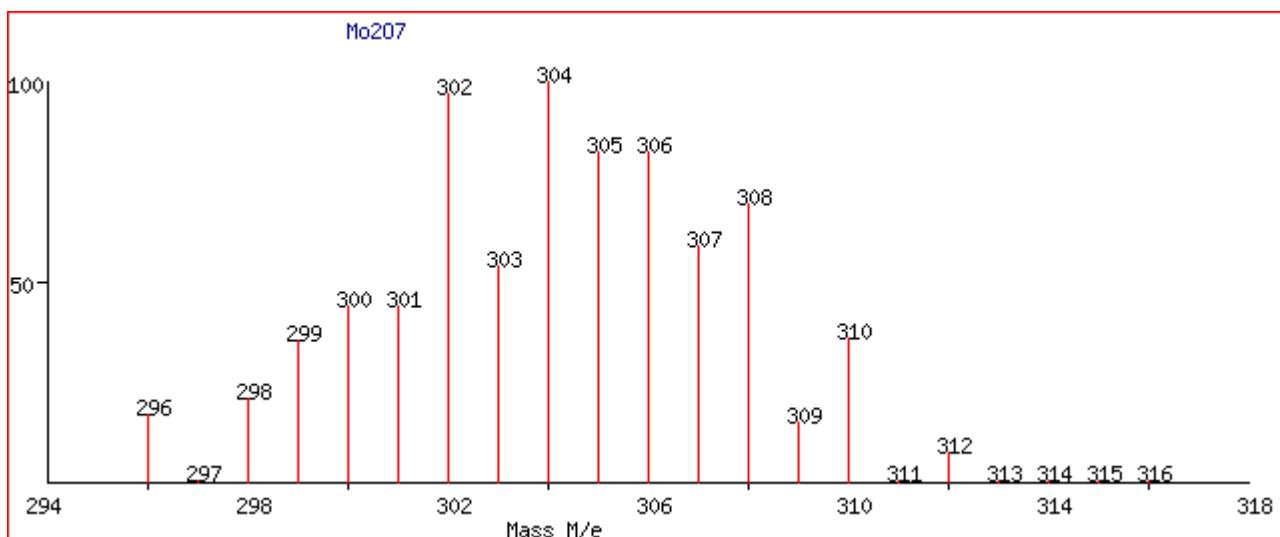
2) $^{16}\text{O} - x$, $^{18}\text{O} - (1-x)$

$^{16}\text{O} = 0.998$, $^{18}\text{O} = 0.002$

n – number of equivalent positions, row number, 2 (Mo) and 7 (O)

a, b, abundance of isotopes, a, b, c, d, e, f, g (Mo); x, 1-x, (O)

$(x+(1-x))^7 \cdot (a+b+c+d+e+f+g)^2 \Rightarrow$ Intensity distribution for 224 possible lines of (Mo_2O_7)





Quantitative gas analysis

For Isolated spectra

$$P(x) = \frac{\text{total ion current } (x)}{GK(x)}$$

$K(x)$ – sensitivity to the specific component*

For overlapping spectra

$$i_{28} = SG[a_{11}P(N_2) + a_{12}P(CO)]$$

$$i_{14} = SG[a_{21}P(N_2) + a_{22}P(CO)]$$

K - instrument sensitivity

G - multiplier gain

$$I_M = KG * \sum \sigma_{mk} * n_k \sim KG * \sum a_{mk} * P_k$$

i_m – ion current of the mass M

σ_{mk} – ionization cross section of component (molecule) k with formation of ion with mass M

$a_{mk} \sim \sigma_{mk}/RT$ – component of cracking pattern, calibration factor

P_k – pressure of the k component.

$$\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_m \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1k} \\ a_{12} & \dots & a_{2k} \\ \vdots & \vdots & \vdots \\ a_{m1} & \dots & a_{mk} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_k \end{bmatrix}$$

$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_k \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1k} \\ a_{12} & \dots & a_{2k} \\ \vdots & \vdots & \vdots \\ a_{m1} & \dots & a_{mk} \end{bmatrix}^{-1} \cdot \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_m \end{bmatrix}$$

* For the rough estimation the sensitivity provided by the manufacturer for nitrogen could be used



Quantitative gas analysis

Internal standard calibration

$$\gamma_i^k = \frac{I_i}{x_i} \Bigg/ \frac{I_{Ar}}{x_{ar}}$$

γ_i^k – response coefficient of species i of component k with respect to Ar

I_{Mi} – MS intensities (ion currents of mass M_i)

x_i – the mole fractions.

For mixture of CO and CO₂ with Ar as internal standart.

$$I_{Mi} = \sum \gamma_{ik} * C_k$$

$$\begin{aligned} i_{28} &= \gamma_{28}^{CO} [CO] + \gamma_{28}^{CO_2} [CO_2] \\ i_{44} &= \gamma_{44}^{CO_2} [CO_2] \end{aligned}$$



Quantitative gas analysis

MCD Parameter < mcdf.mcp >

File Edit Options

0 [] ▶

#	0	1	2	3	4	5	6	7	8	9		
#	Comp.	Mass	2	4	14	16	18	28	32	40	44	55
0	Ar	0	0	0	0	0	0	1	0	0	0	0
1	CO2	0	0	0	0	0	0.1364	0	0	0.9	0	0
2	H2	0.8	0	0	0	0	0	0	0	0	0	0
3	H2O	0	0	0	0	0.75	0	0	0	0	0	0
4	CxHy	0	0	0	0	0	0.03	0	0	0	0.25	0
5	N2 / CO	0	0	?	0	0	0.8747	0	0	0	0	0
6	O2	0	0	0	?	0	0	0.676	0	0	0	0
7	He	0	0.3	0	0	0	0	0	0	0	0	0
8												



MAX-PLANCK-GESELLSCHAFT

Quantitative gas analysis

$$\begin{bmatrix} a_{11} & \dots & a_{1k} \\ a_{12} & \dots & a_{2k} \\ \vdots & \vdots & \vdots \\ a_{m1} & \dots & a_{mk} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_k \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_m \end{bmatrix}$$

$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_k \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1k} \\ a_{12} & \dots & a_{2k} \\ \vdots & \vdots & \vdots \\ a_{m1} & \dots & a_{mk} \end{bmatrix}^{-1} \cdot \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_m \end{bmatrix}$$

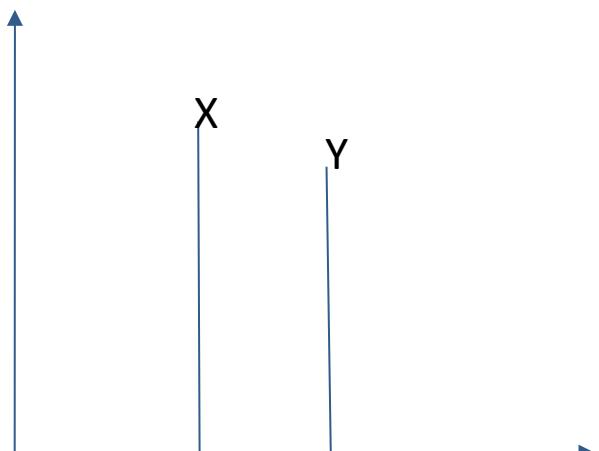
$$\left. \begin{array}{l} ak + bq = X \\ ar + bs = Y \end{array} \right\}$$

$$\begin{bmatrix} k & q \\ r & s \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

Calibration factors

Unknown concentrations

Experimental ion currents



$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} k & q \\ r & s \end{bmatrix}^{-1} \cdot \begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{ks - qr} \cdot \begin{bmatrix} s & -q \\ -r & k \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \end{bmatrix}$$

Inverse matrix

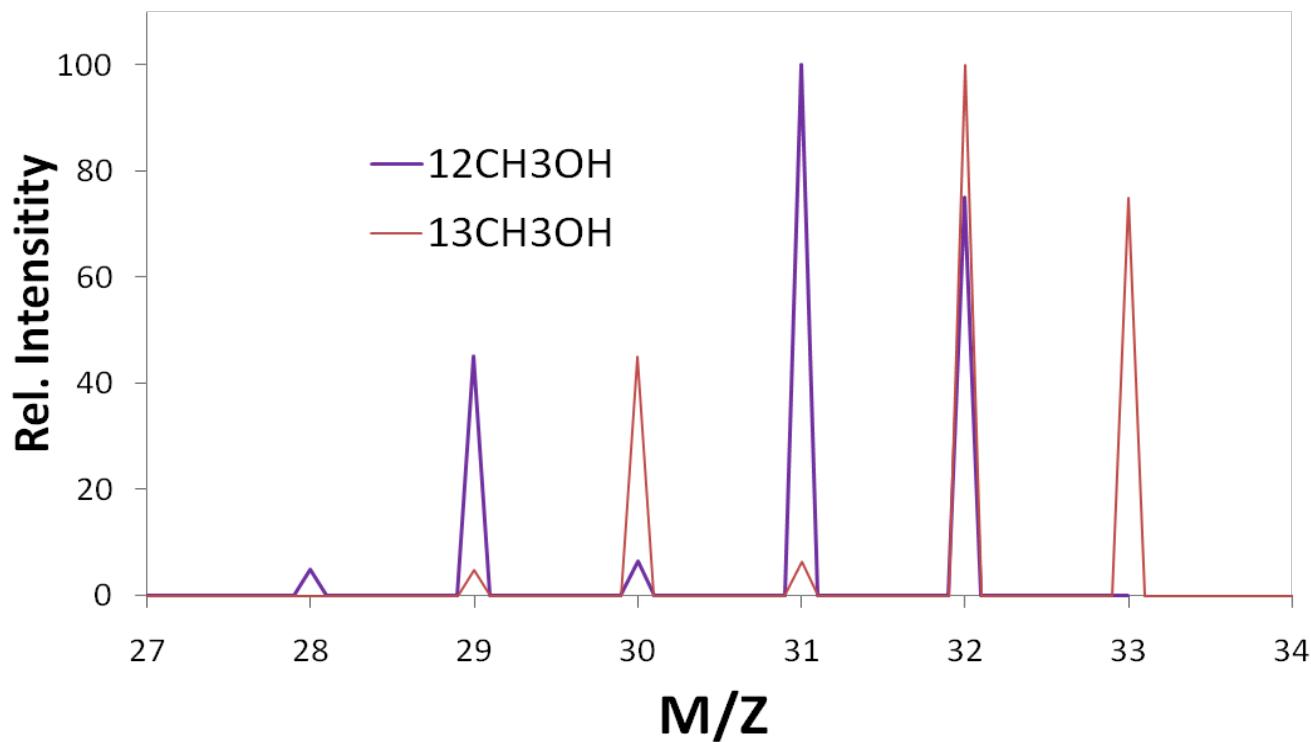
Adjugate matrix

$$ks - qr = D, \text{matrix determinant}$$

$$a = \frac{sx - qy}{ks - qr} \quad b = \frac{ky - rx}{ks - qr}$$

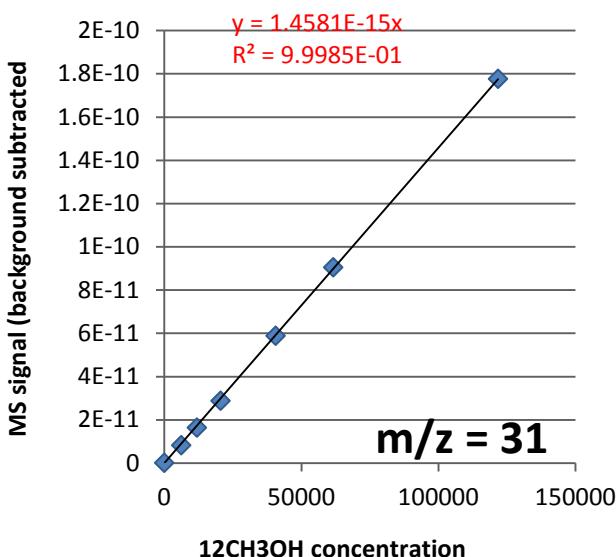
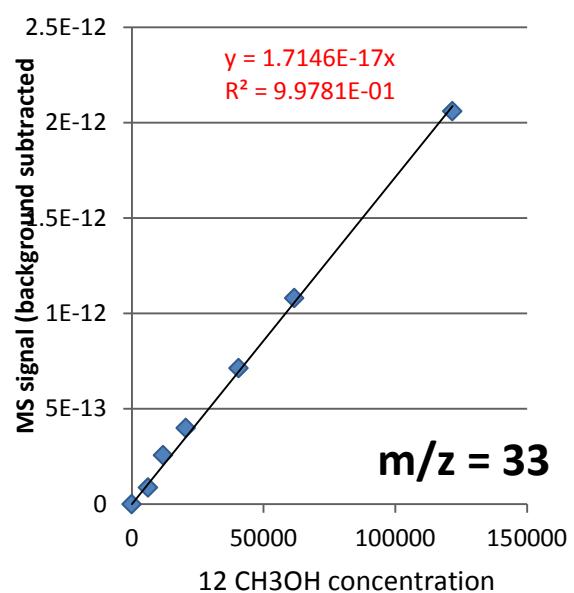
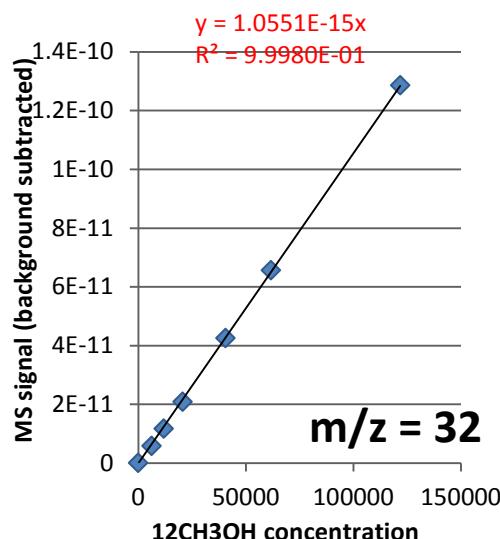
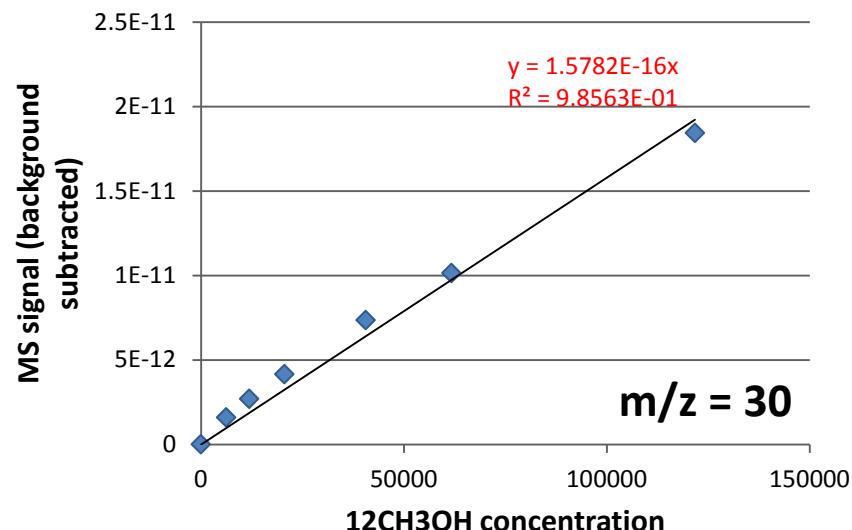
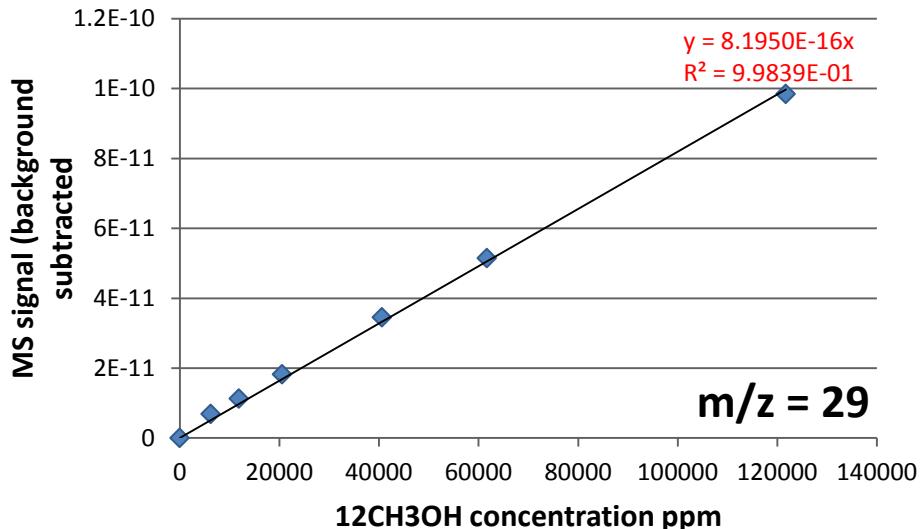
$$b/a = \frac{ky - rx}{sx - qy}$$

Quantitative gas analysis



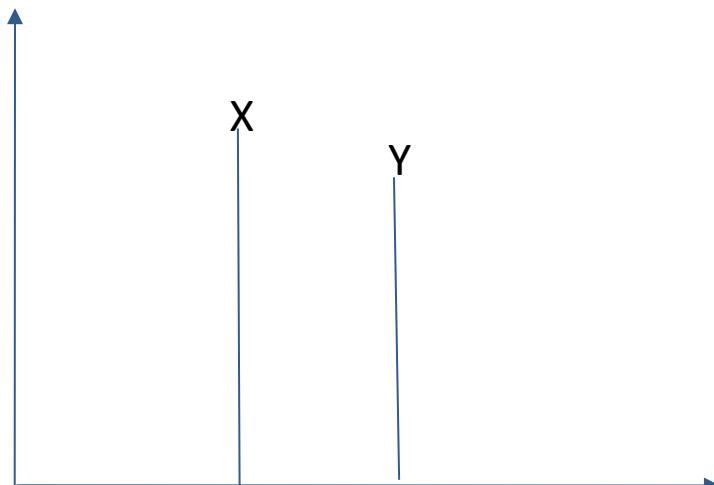
Direct calibration, mass matrix of $^{13}\text{CH}_3\text{OH}/^{12}\text{CH}_3\text{OH}$ mixtures

Quantitative gas analysis





Quantitative gas analysis



Mass Matrix Calculation

We have 2 species a and b. a = C 12 methanol b = C 13 methanol

F_{ax} = Contribution of species a(C12 methanol) to the peak X }

F_{by} = Contribution of species b(C13 methanol) to the peak Y }

Sensitivity factors

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} F_{ax} & F_{bx} \\ F_{ay} & F_{by} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \quad X = a * F_{ax} + b * F_{bx} \\ Y = a * F_{ay} + b * F_{by}$$

$$fraction\ C13\ methanol = \frac{b}{a+b} = \frac{F_{ax} \cdot y - F_{ay} \cdot x}{y \cdot (F_{ax} - F_{bx}) + x \cdot (F_{by} - F_{ax})}$$

Done By E. Kunkes and N. Thomas in ChemCatChem 2015, 7, 1105-1111

Quantitative gas analysis

Sample	Amount of catalyst Loaded	% of C-13 Methanol		
		GC and MS	MS alone 31 and 33 signals	MS alone 32 and 33 signals
Cu/ZnO/Al ₂ O ₃	50mg	93	97	92
Cu/Al ₂ O ₃	1000mg	94	87	87
Cu/MgO	500 mg	91	88	86
Cu/MgO/Zn	50mg	96	94	91

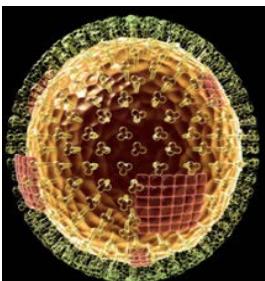
Leaks



„technically“
tight
 $0.1 \mu\text{m}$



„gas“-
tight
 $0.8 \mu\text{m}$



Virus-
tight
 $3 \mu\text{m}$



Bacteria-
tight
 $10 \mu\text{m}$



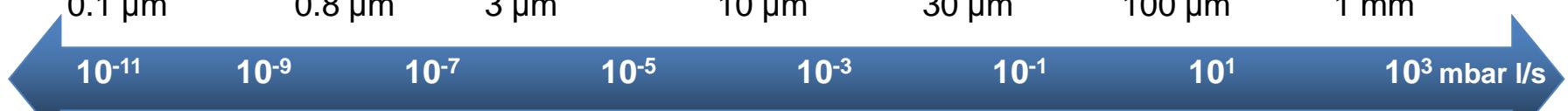
water-
tight
 $30 \mu\text{m}$



water
drops
 $100 \mu\text{m}$



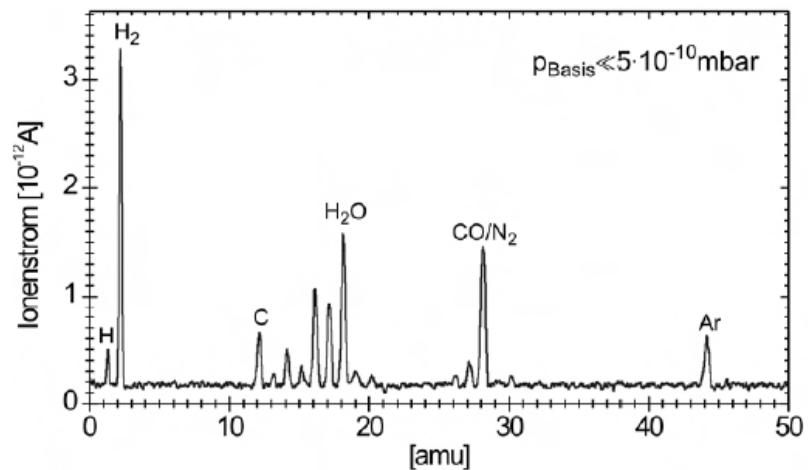
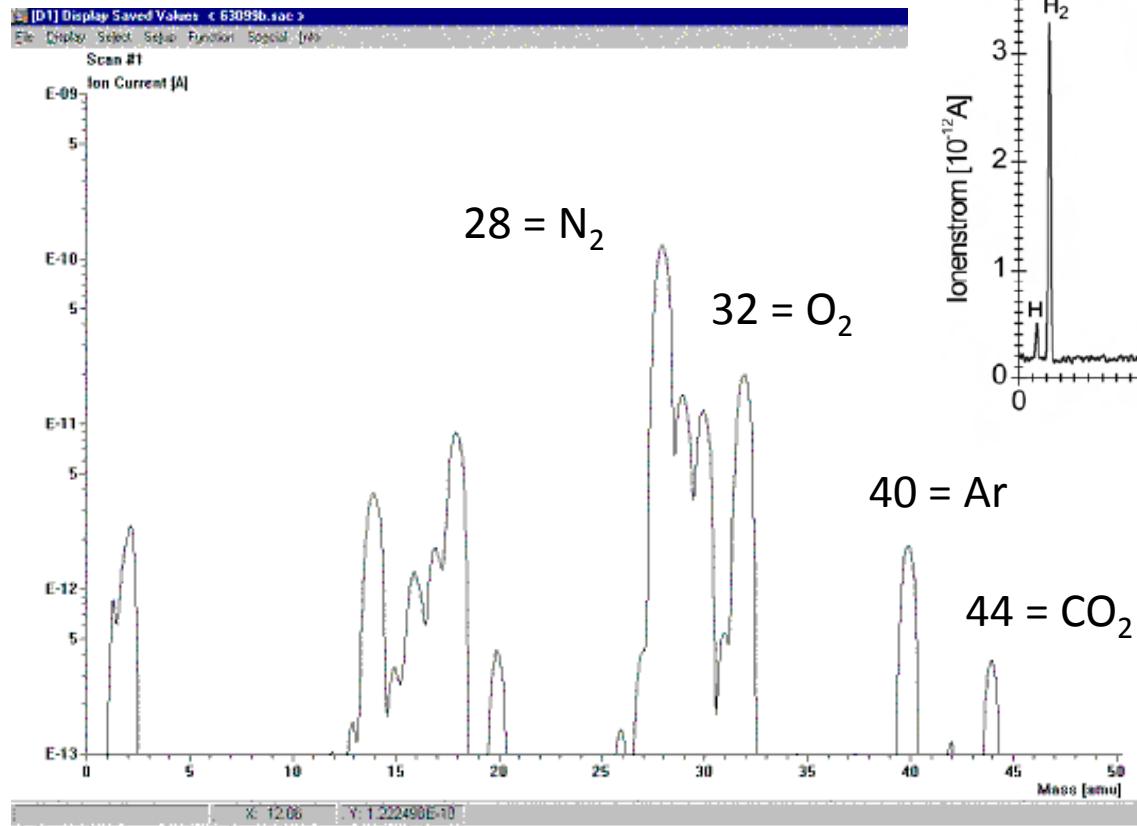
water
flows
 1 mm

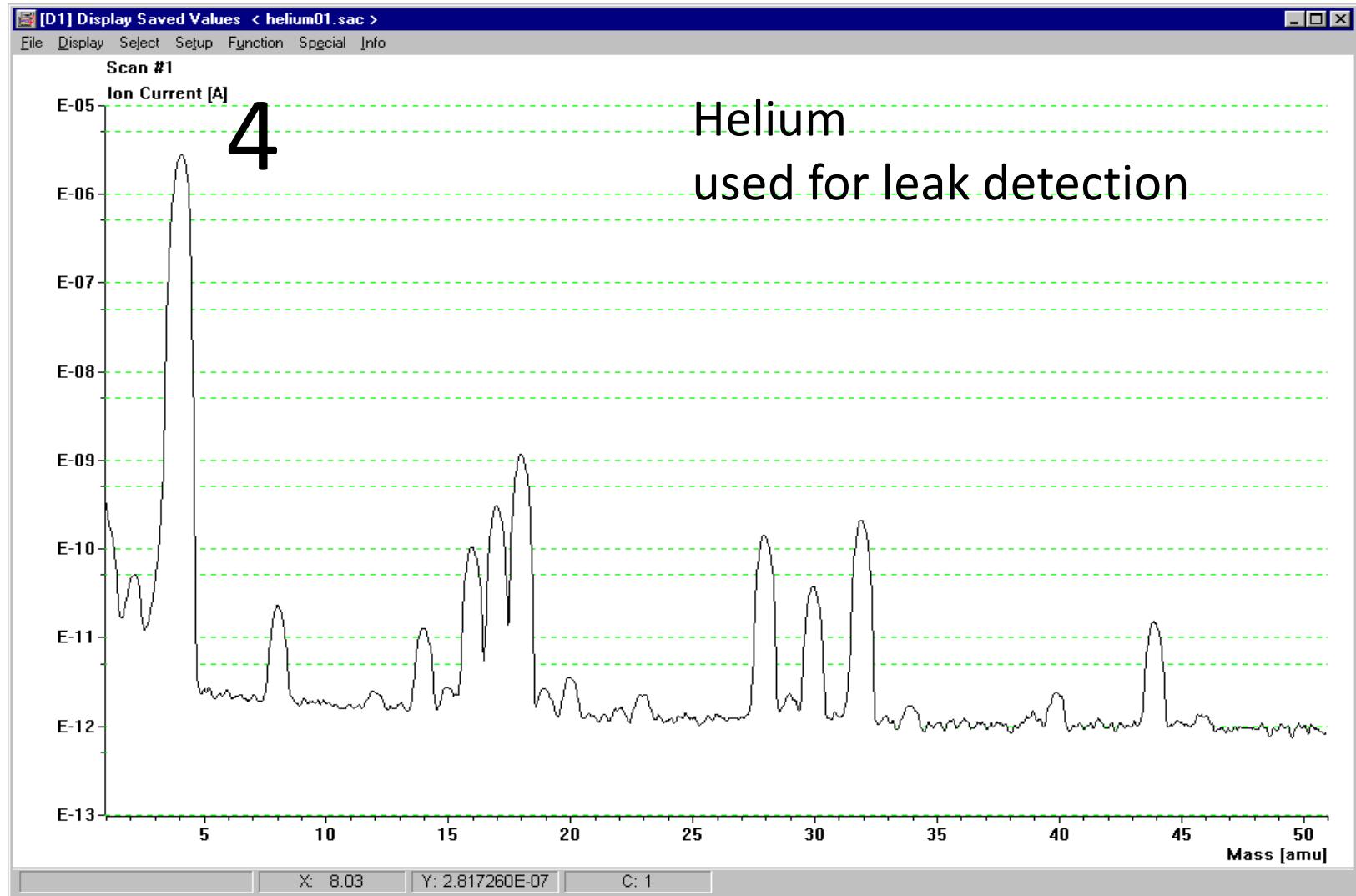


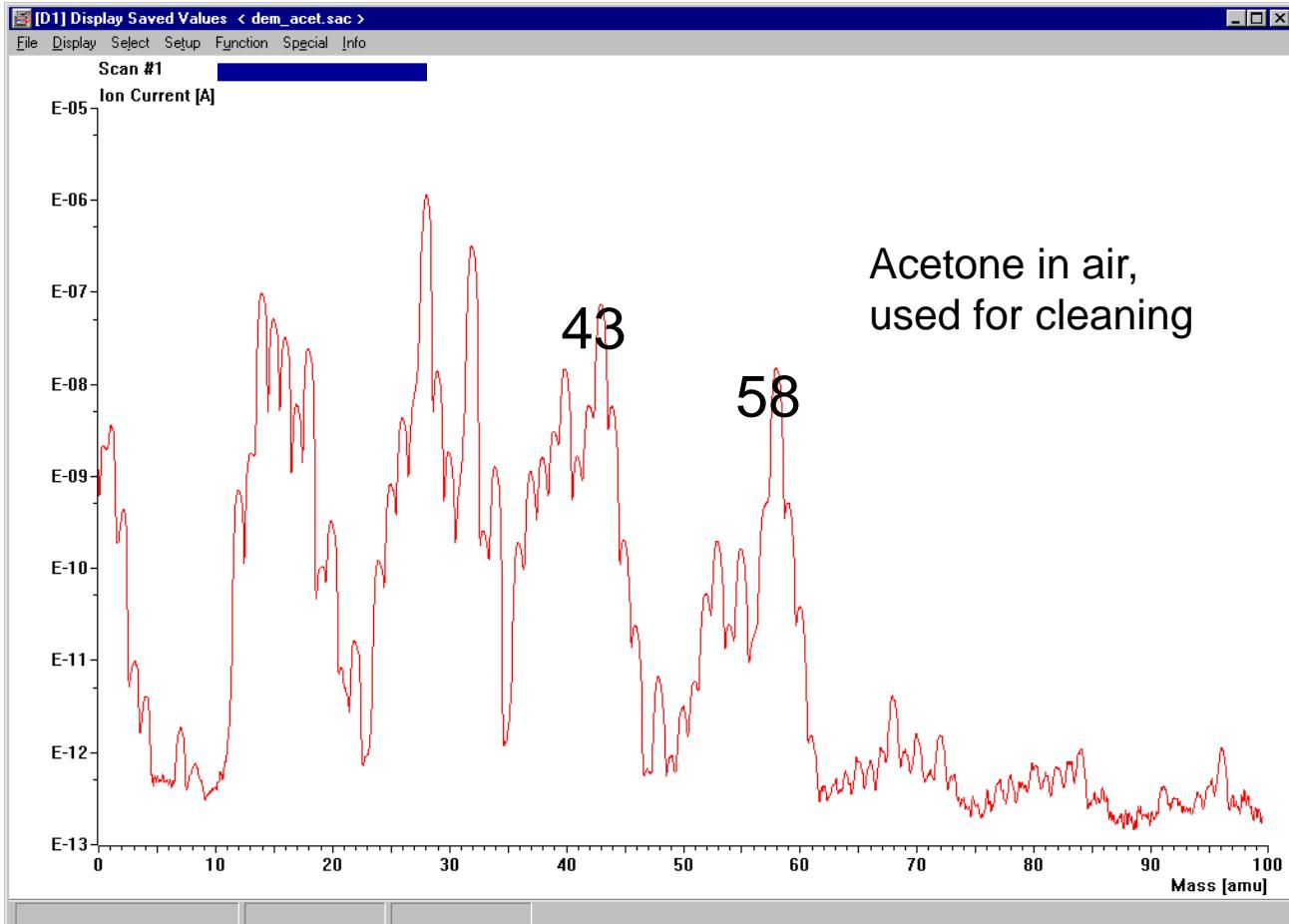
Lower detection limit
of vacuum methods

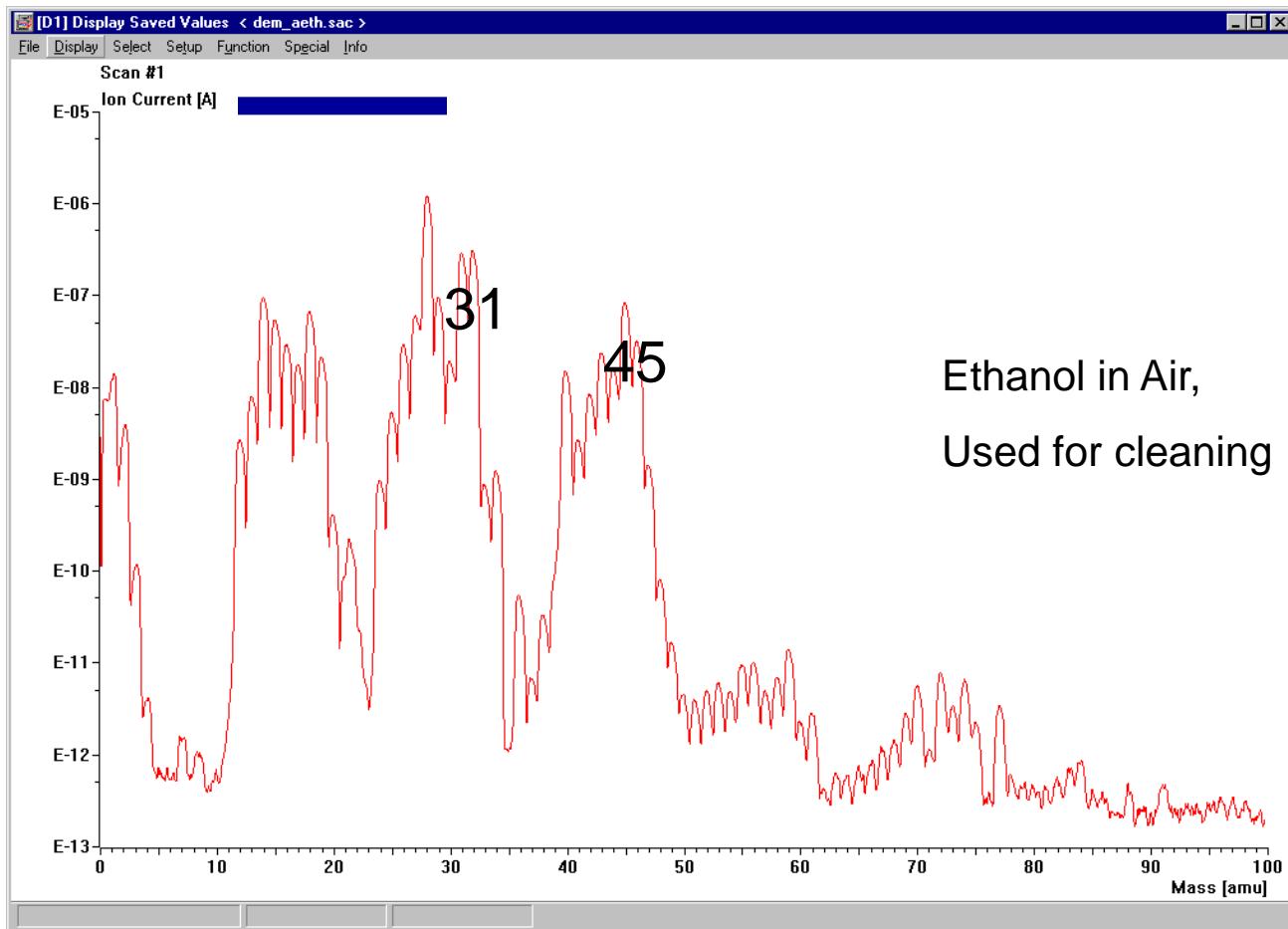
Lower detection limit
of snooping methods

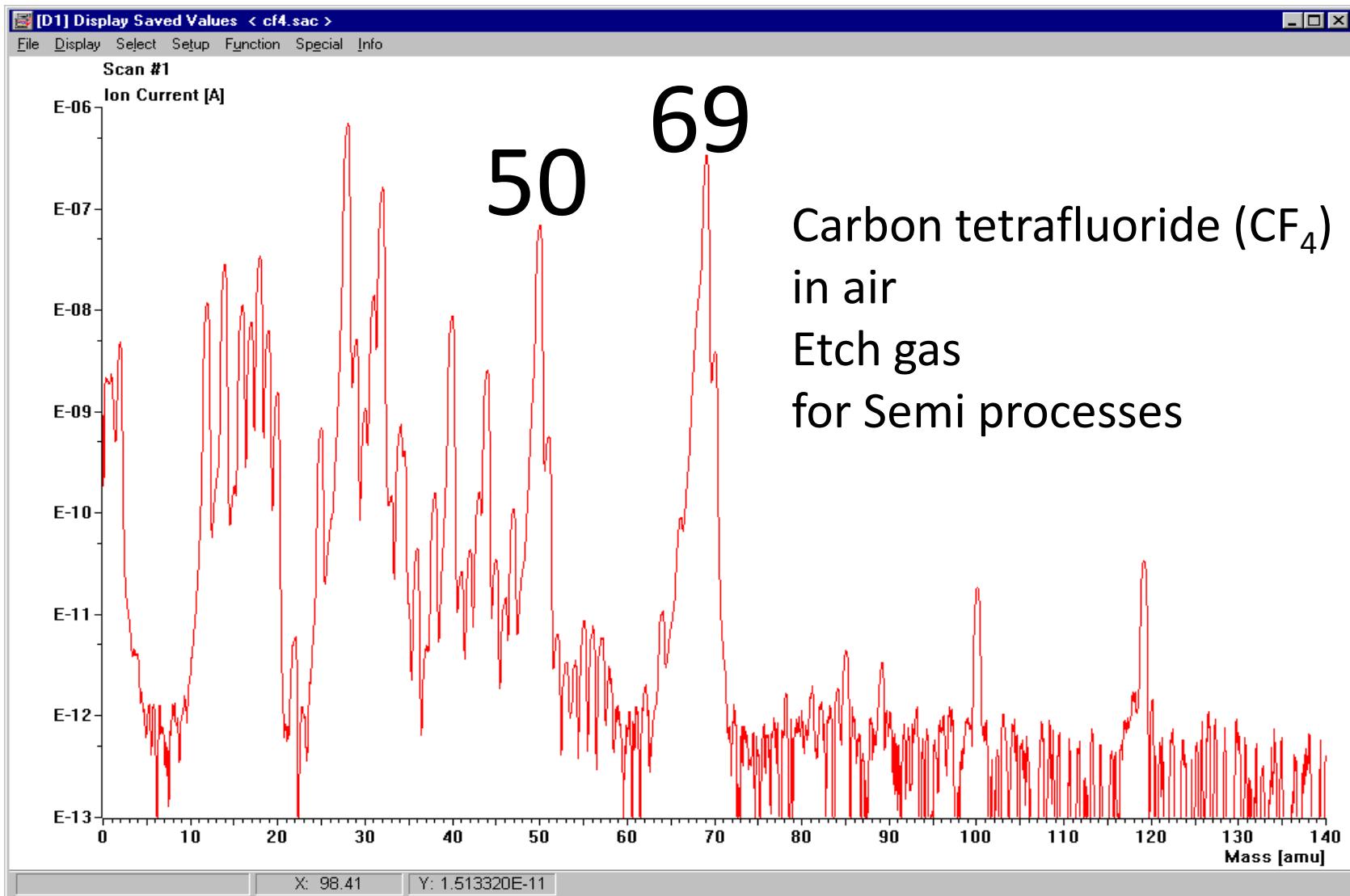
Air, residual gas in the system

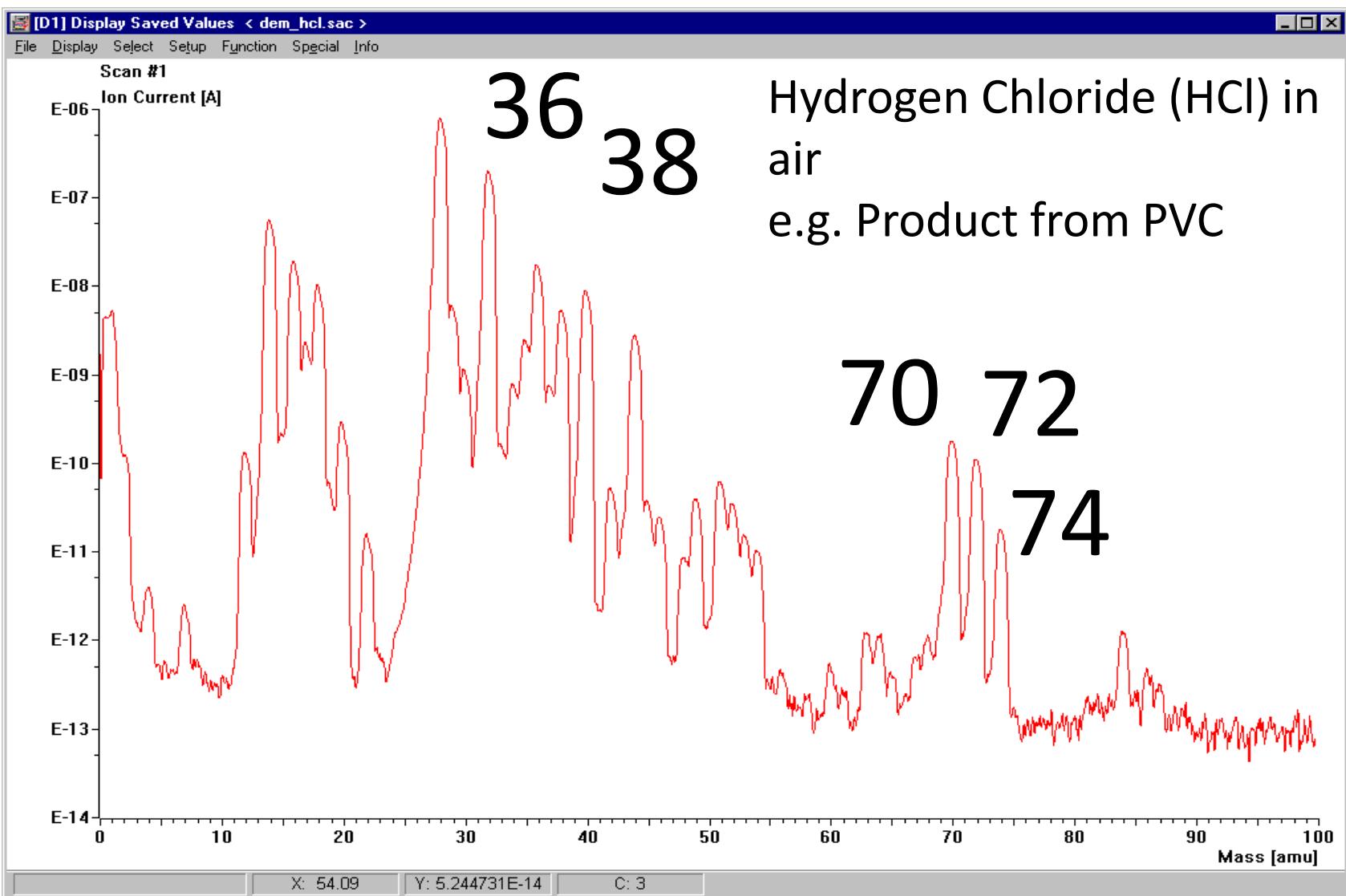


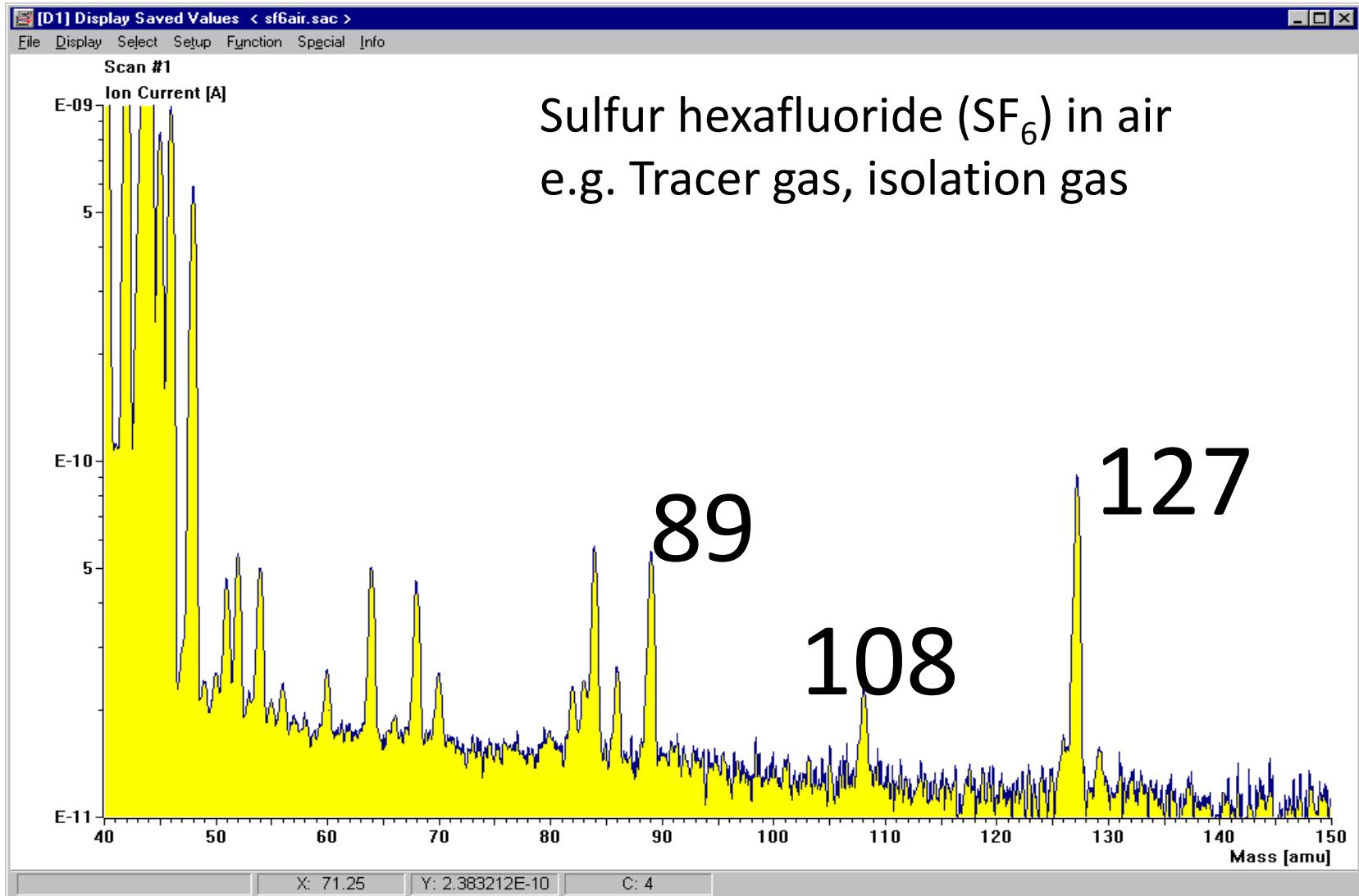


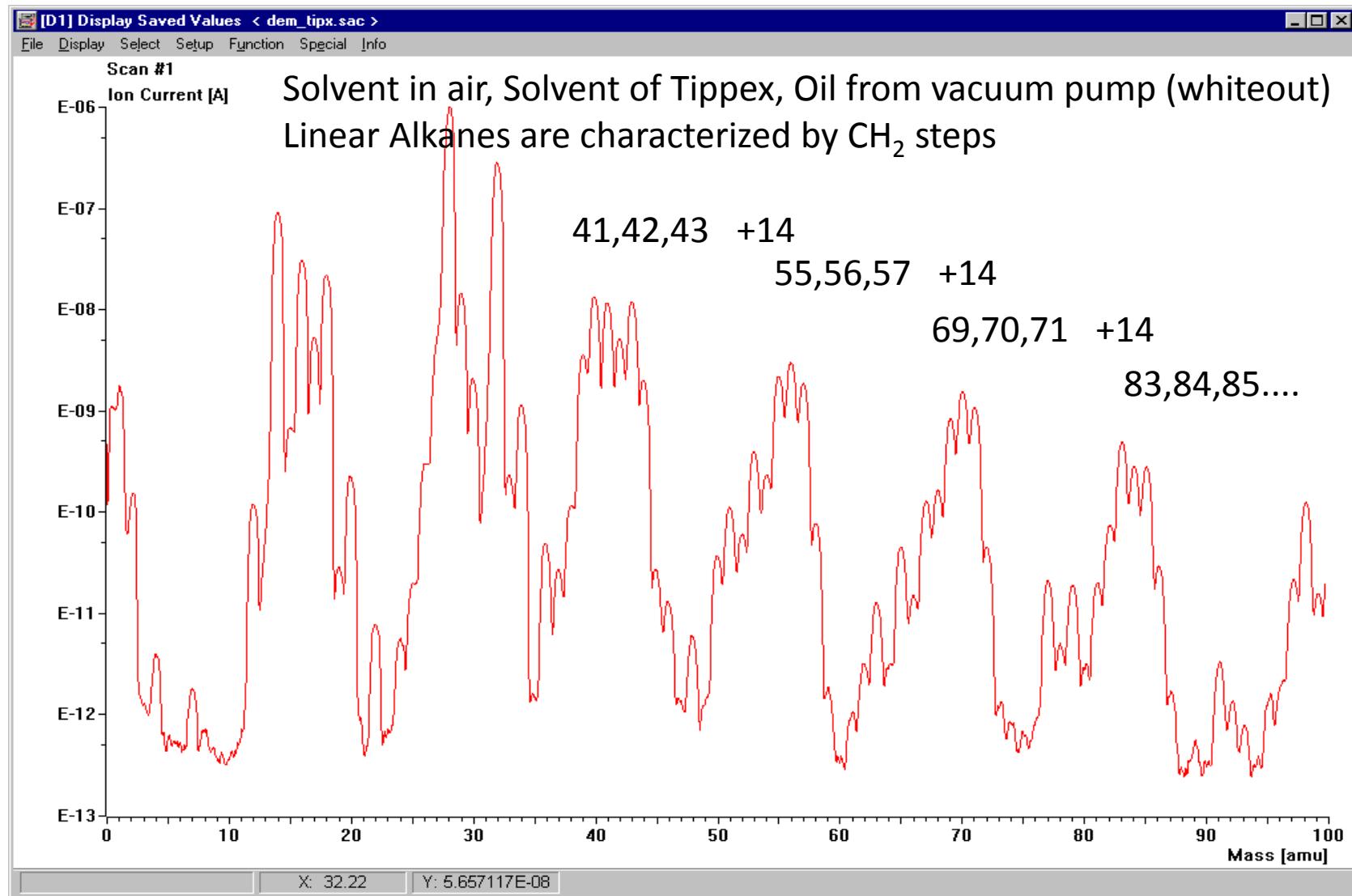


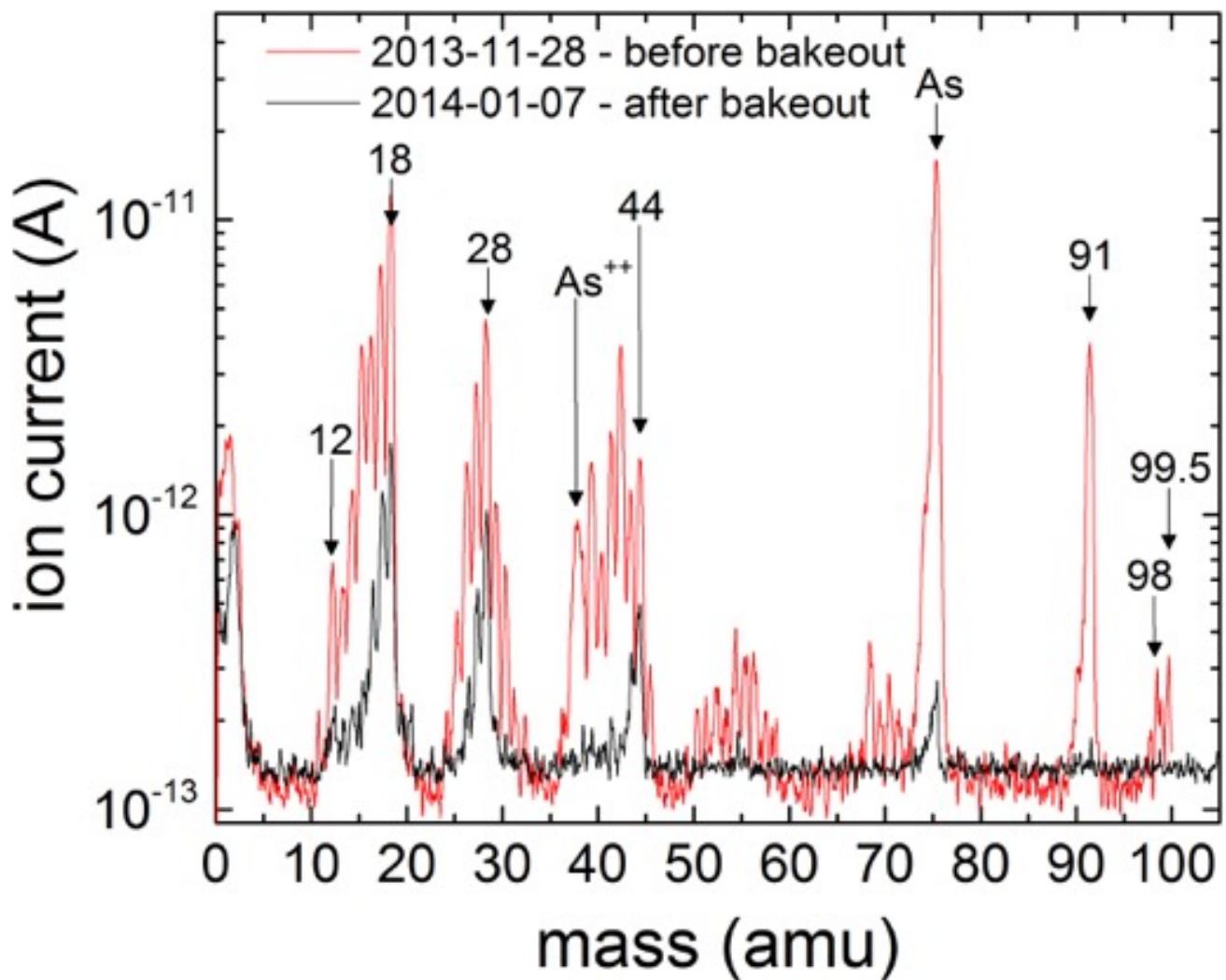












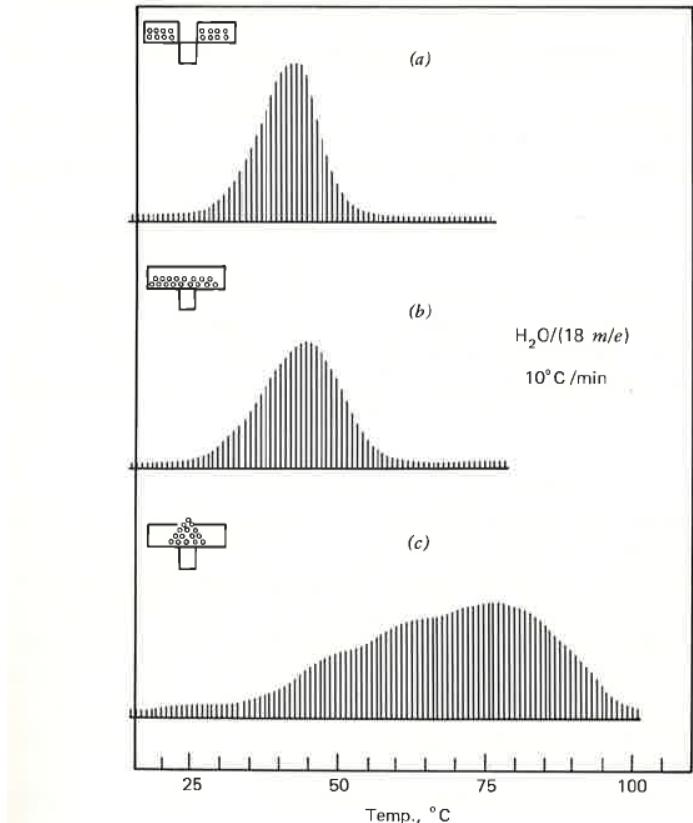


Figure II.13. Effect of sample packing and sample-holder geometry on the evolution of water from $\text{CaC}_2\text{O}_4 \cdot \text{H}_2\text{O}$ (43).

W Wendland Thermal methods of analysis. John Wiley & Sons, 1974.