

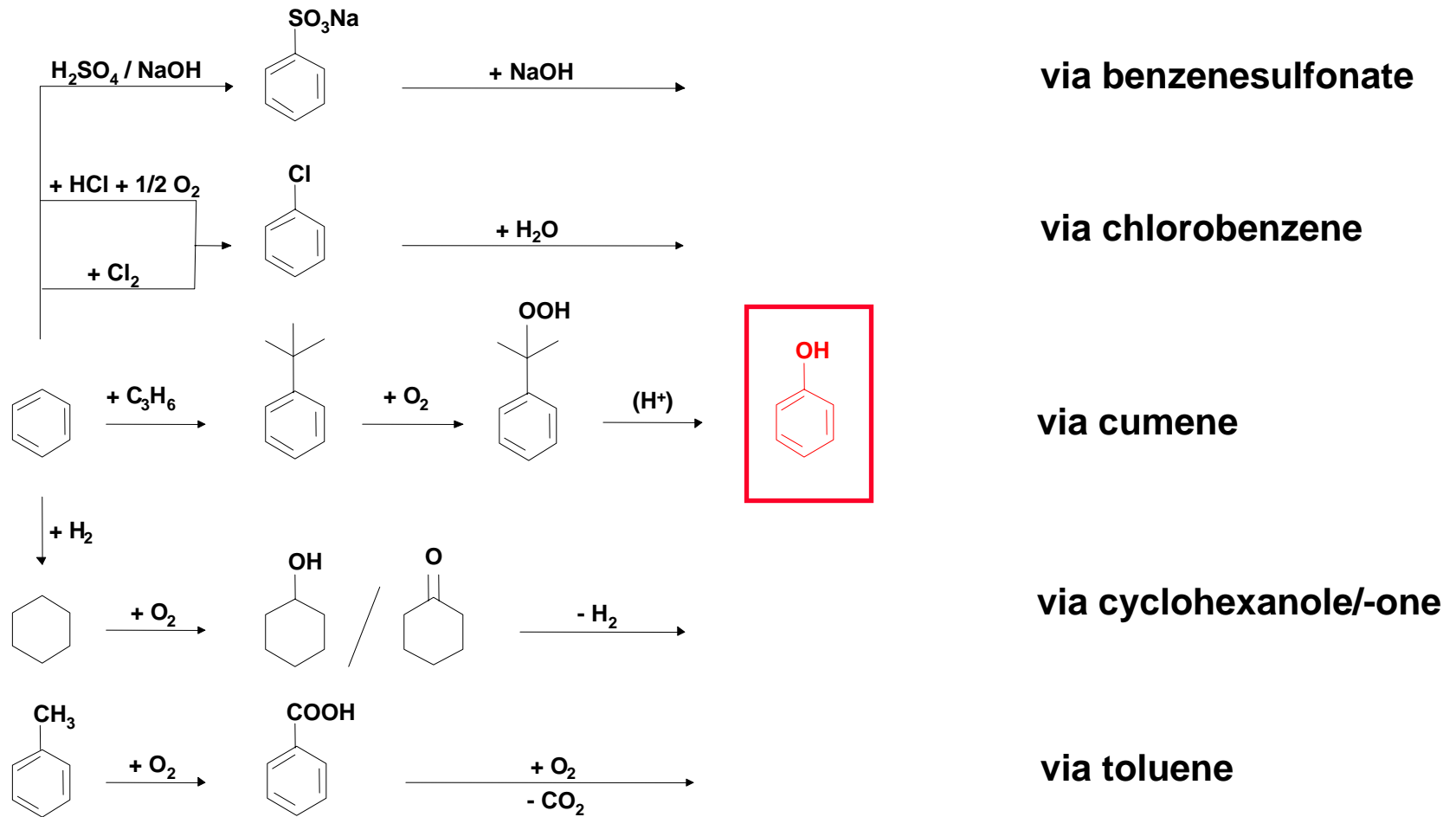
Chemical Reaction Engineering Basics Required to Understand Membrane Reactors

Andreas Seidel-Morgenstern

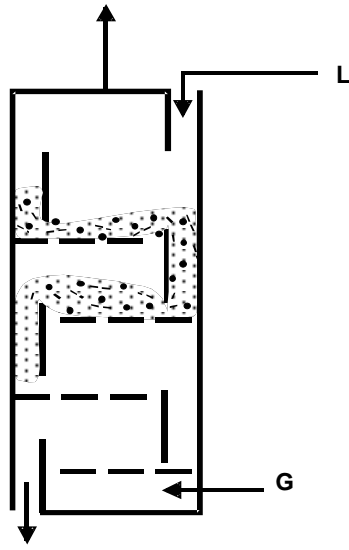


- **Classical reactor concepts**
- **Reaction rates and reactor models**
- **Mass transfer through membranes**
- **Product removal with membranes**
- **Reactant dosing with membranes**

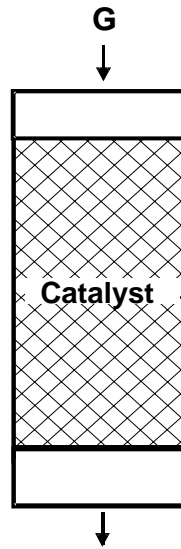
Abundance of chemistry - possible pathways to produce phenol



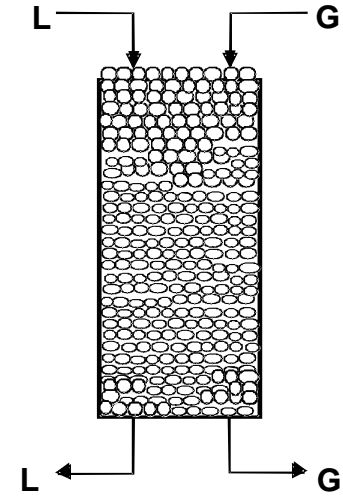
View in the spectrum of reactors



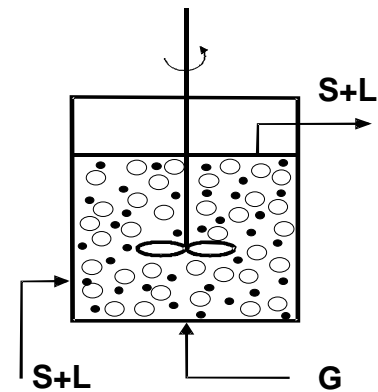
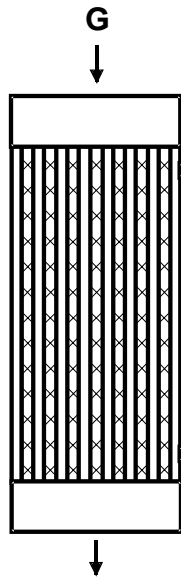
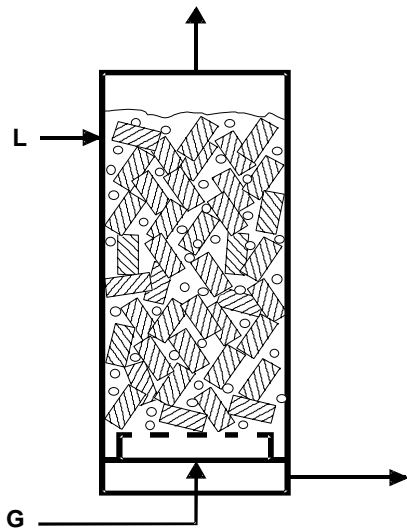
Gas / Liquid



Gas / Solid



Gas / Liquid / Solid



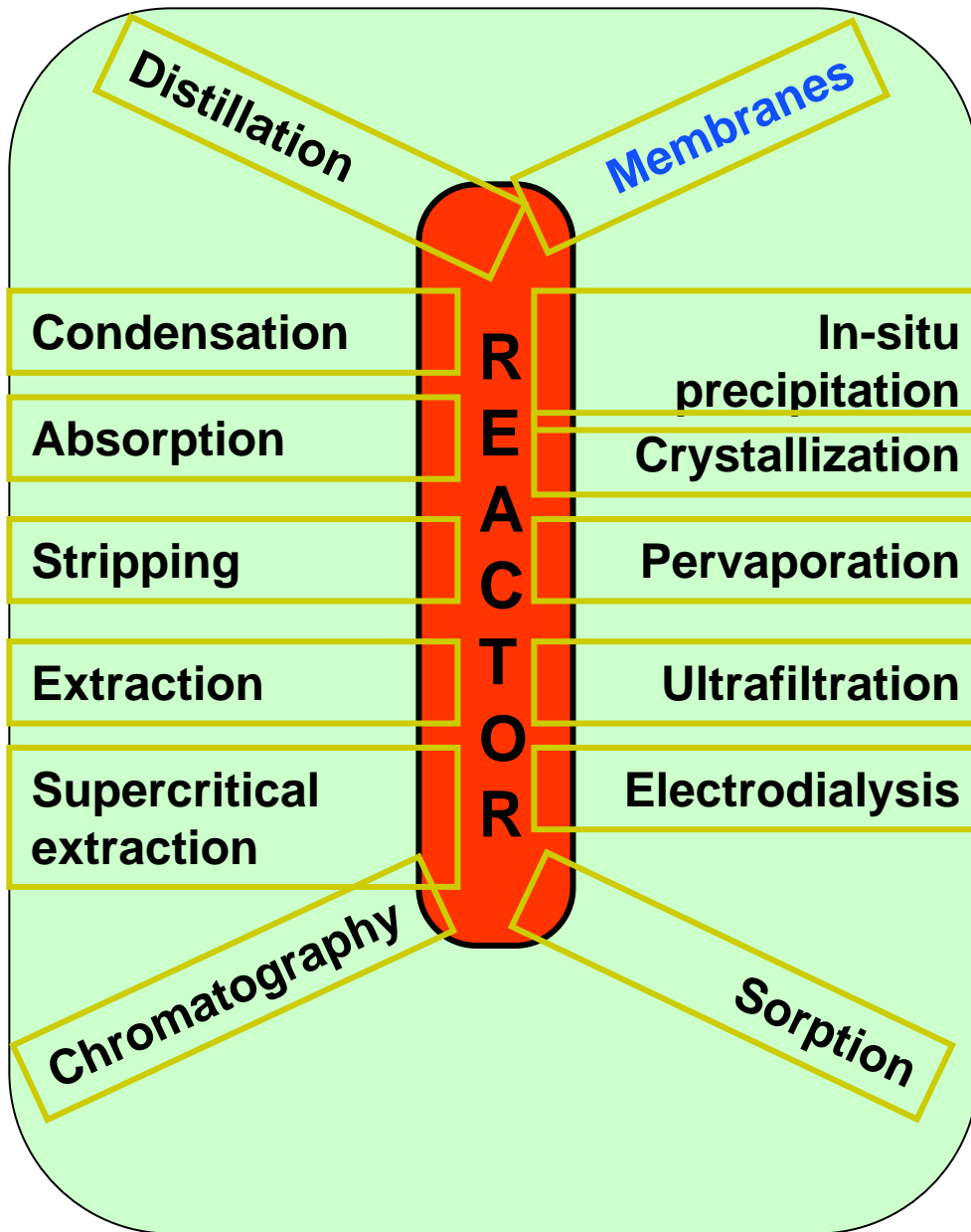
Classical problems

- reactions too slow
- reversibility
- selectivity restricted, yields low
- preheating for exothermic reactions required
- severe thermal effects

Attempts to solve these problems

- search for “alternative chemistry”
- development of selective catalysts
- optimization of operating parameters
- search for new reactor concepts
 - autothermal reactors
 - ...
 - **coupling of reaction and separation**

Possibilities of coupling reaction and separation

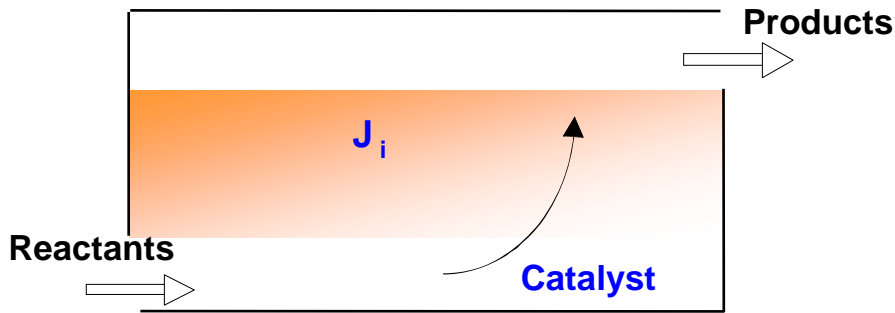


References

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- ↪ Agar, D. W., W. Ruppel 'Multifunktionale Reaktoren für die heterogene Katalyse' Chem.-Ing.-Tech. 60(10): 731-741 (1988)
- ↪ Westerterp, K.R. 'Multifunctional reactors' Chem. Engng. Sci. 47(9-10):2195-2206 (1992)
- ↪ Krishna, R. 'A systems approach to multiphase reactor selection' Adv. Chem. Engng 19:201-249 (1994)
- ↪ Lerou, J.J., K.M. Ng 'Chemical Reaction Engineering: A multiscale approach to a multiobjective task' Chem. Engng. Sci. 51(10): 1595-1614 (1996)
- ↪ Hoffmann, U., K. Sundmacher 'Multifunktionale Reaktoren' Chem.-Ing.-Tech. 69(5):613-622 (1997)
- ↪ Agar, D.W. 'Multifunctional Reactors - old preconceptions and new dimensions' Chem. Engng. Sci. 54(10):1299-1305 (1999)
- ↪ Sundmacher K., Kienle A., Seidel-Morgenstern A. 'Integrated chemical processes' Wiley-VCH, (2005)

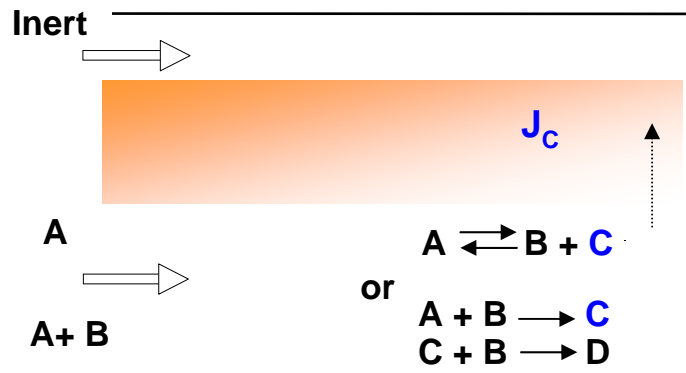
Concepts for membrane reactors

Retaining catalysts



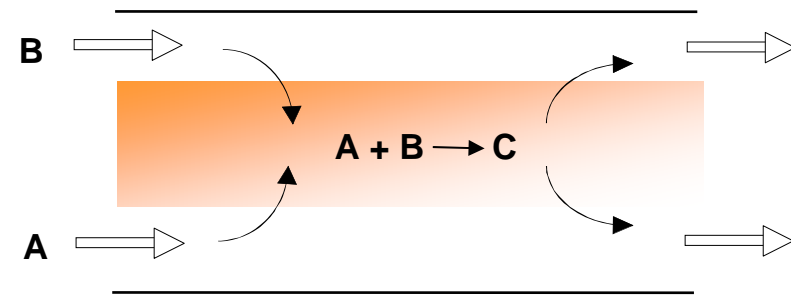
- homogeneous and enzyme catalysis

Selective product removal („Extractor“)



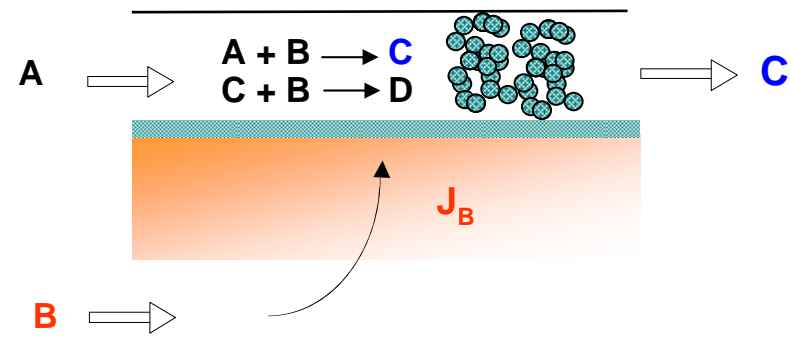
- conversion enhancement for reversible reactions
- selectivity improvements

Membranes as active „contactor“



- conversion front in membrane
- total conversion of toxic components

Controlled reactant dosing („Distributor“)



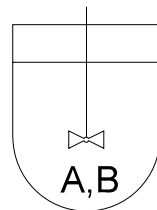
- dosing of critical components
- selectivity improvements

Classification criteria

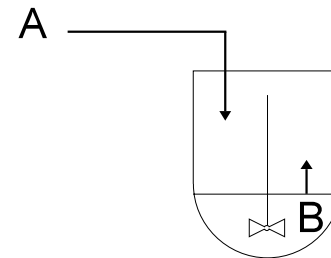
- A) Phases involved (G/L, G/S, G/L/S, ...)
- B) Isothermal, adiabatic, polytropic
- C) One reaction vs. reaction network
- D) Single path operation vs. recycling concepts
- E) Degree of **mixing** (perfect, intermediate, no mixing)
- F) Mode of operation (**discontinuous vs. continuous**)
- G) ...

Discontinuous operation

BR (Batch Reactor, perfect mixing)

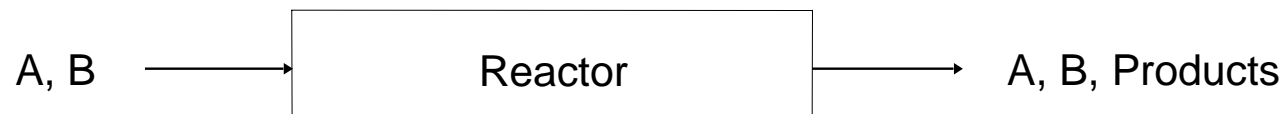


SBR (Semi Batch Reactor)



Continuous operation

CSTR (perfect mixing), **PFTR** (no mixing)



Reaction rates



key information required for quantitative analysis,
couple mass and energy balances

$$\text{Definition: } r = \frac{1}{\text{Scale}} \frac{1}{v_i} \frac{dn_i}{dt} \quad i = 1, N$$

uses stoichiometric balance:
(thus r does not depend on component)

$$\frac{dn_i}{v_i} = \frac{dn_j}{v_j}$$

$$\text{Scale: often } V_R \quad (\text{if } V_R = \text{const.} : r = \frac{1}{v_i} \frac{dc_i}{dt})$$

How can the rate be influenced?

$$r = r(T, c_1, c_2, \dots, c_N, \text{catalyst})$$

$$r = f(T) \cdot g(c_1, c_2, \dots, c_N) \quad f(T) = ? \quad g(\bar{c}) = ?$$

Concentration and temperature dependencies of reaction rates

Collision theory (kinetic theory of gases and statistical mechanics)

$$N_{coll} = \left[8\pi RT \left(\frac{1}{M_A} + \frac{1}{M_B} \right) \right]^{1/2} \delta_{AB}^2 c_A c_B$$

N_{coll} - (big) number of collisions between molecules

δ_{AB} - mean molecule diameter

$$N_{coll}^{eff} = N_{coll} e^{-\frac{E}{RT}}$$

N_{coll}^{eff} - (smaller) number of effective collisions

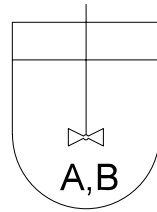
E - activation energy

$$f(T) = k_{\infty}^* T^{1/2} e^{-\frac{E}{RT}} \quad g(c_A, c_B) = c_A c_B$$

Formal generalisation and application:

$$f(T) = k_{\infty} \exp\left(-\frac{E}{RT}\right) \quad g(\bar{c}) = \prod_{i=1}^{Reactants} c_i^{|v_i|}$$

Reaction orders and concentration time curves in a BR



$$\frac{dn_i}{dt} = V_R v_i r$$

$$i = 1, N$$

if $V_R = \text{const.}$:

$$\frac{dc_i}{dt} = v_i r$$

$$i = 1, N$$

Example: $A \rightarrow B$

$$v_A = -1$$

$$r = kc_A^n$$

n : order of reaction

First order:

$$\frac{dc_A}{dt} = -k \cdot c_A$$

$$c_A = c_A^0 e^{-kt}$$

Second order:

$$\frac{dc_A}{dt} = -k \cdot c_A^2$$

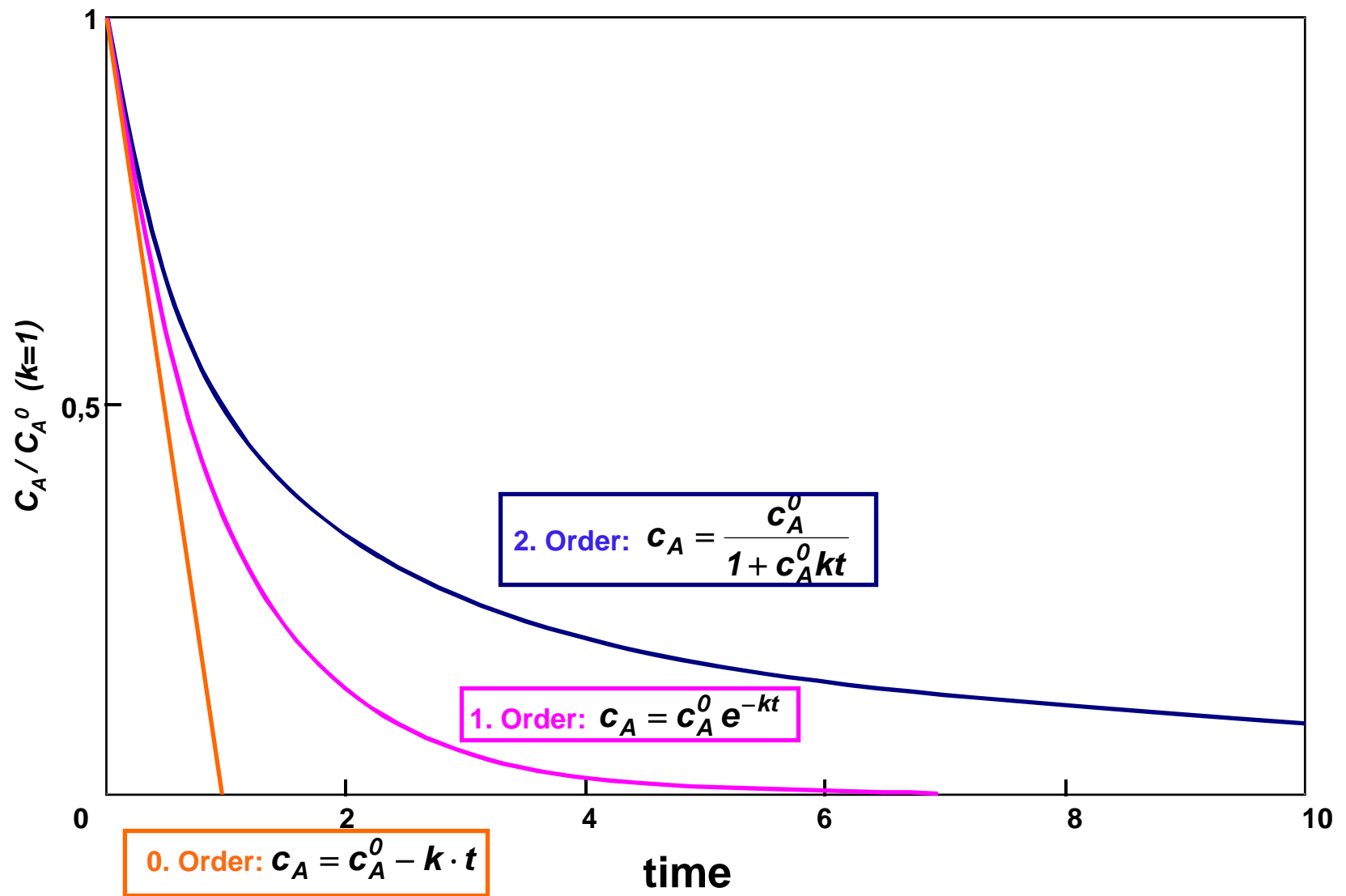
$$c_A = \frac{c_A^0}{1 + c_A^0 kt}$$

Zero order:

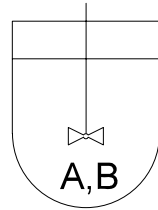
$$\frac{dc_A}{dt} = -k$$

$$c_A = c_A^0 - k \cdot t$$

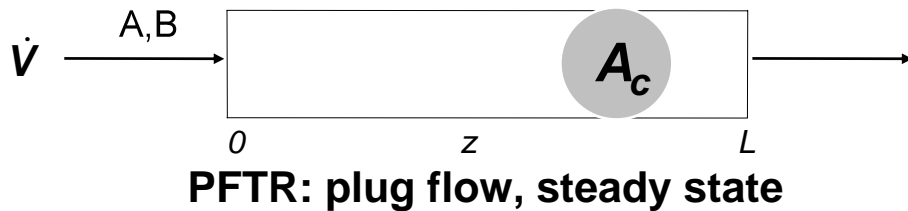
Reaction orders and concentration time curves in a BR



Mass balances BR and PFTR



BR: perfectly mixed



PFTR: plug flow, steady state

$$\frac{dc_i}{dt} = \sum_{j=1}^M \nu_{ij} r_j$$

$$\frac{dc_i}{dz} = \frac{A_c}{\dot{V}} \sum_{j=1}^M \nu_{ij} r_j$$

} similar ODE systems
(initial value problems)

Dimensionless mass balances

$$\tau \frac{\partial \mathbf{X}}{\partial t} = \frac{\tau}{c_A^0} \text{div } \vec{j}_A + Da \cdot \Phi(\mathbf{X})$$

with

$$\mathbf{X} = (c_A^0 - c_A) / c_A^0 \quad r = r_0(T) \Phi(\mathbf{X})$$

(conversion)

$$Da = \frac{(-\nu_A) r_0 \tau}{c_A^0} \quad \left(\tau = t \text{ or } \tau = \frac{L A_c}{\dot{V}} \right)$$

(Damköhler number)

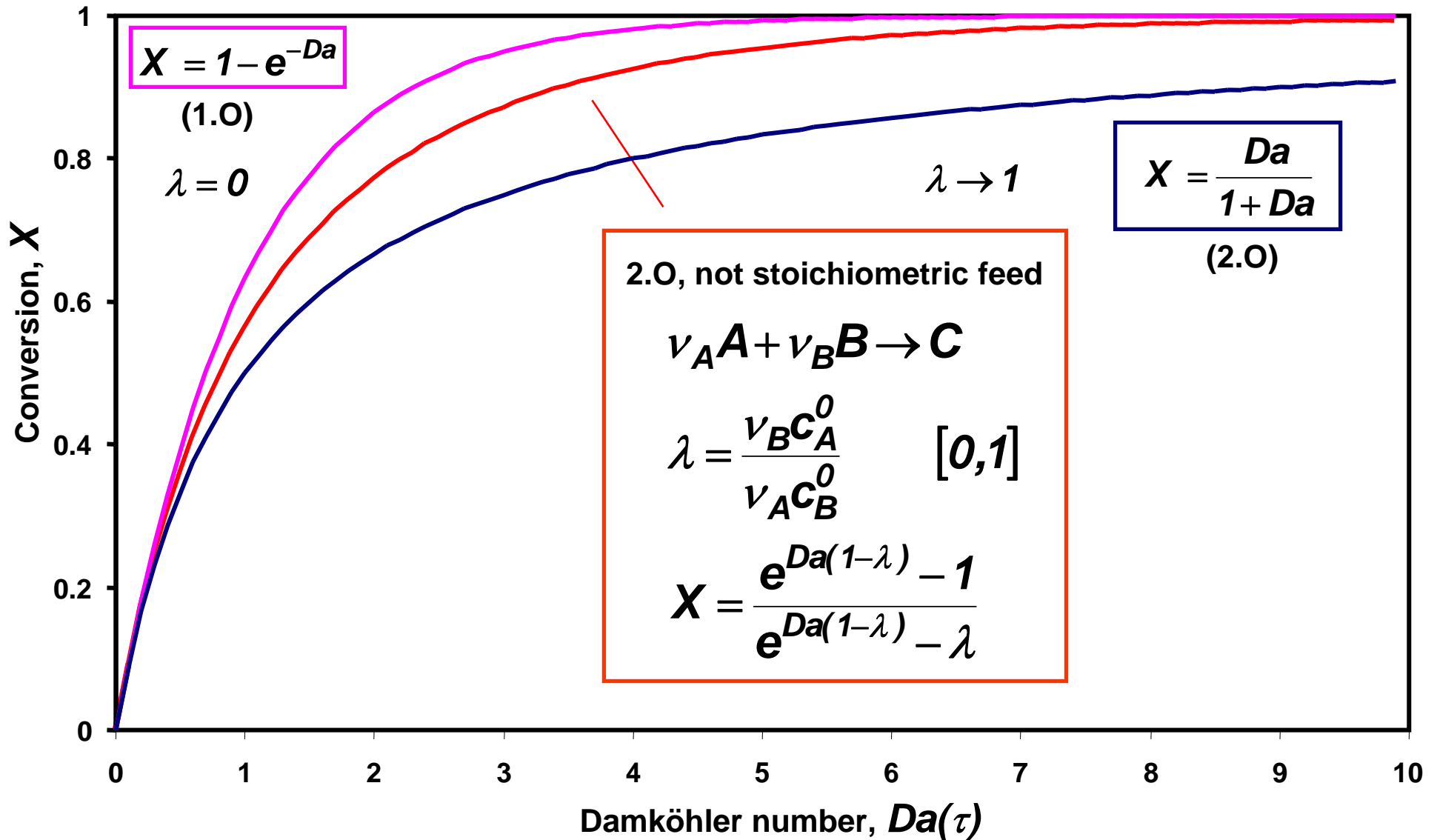
BR and PFTR:

$$Da(\tau) = \int_0^{X(\tau)} \frac{dX}{\Phi(X)}$$

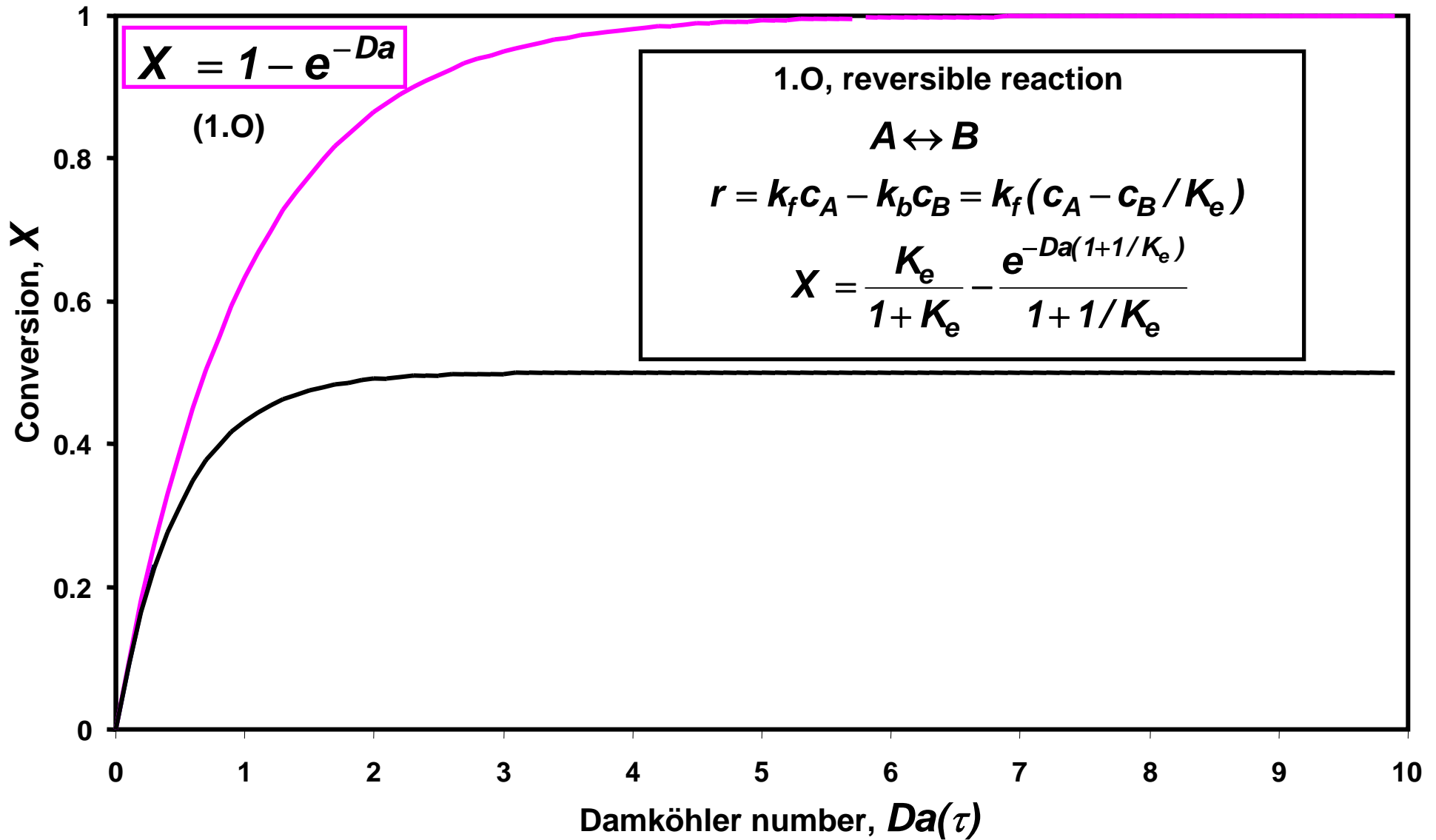
Example: $r = kc_A$, $r_0 = kc_A^0$, $\Phi(X) = 1 - X$
(1. Order)

$$X(\tau) = 1 - e^{-Da(\tau)}$$

Conversion vs. Damköhler number (BR and PFTR)



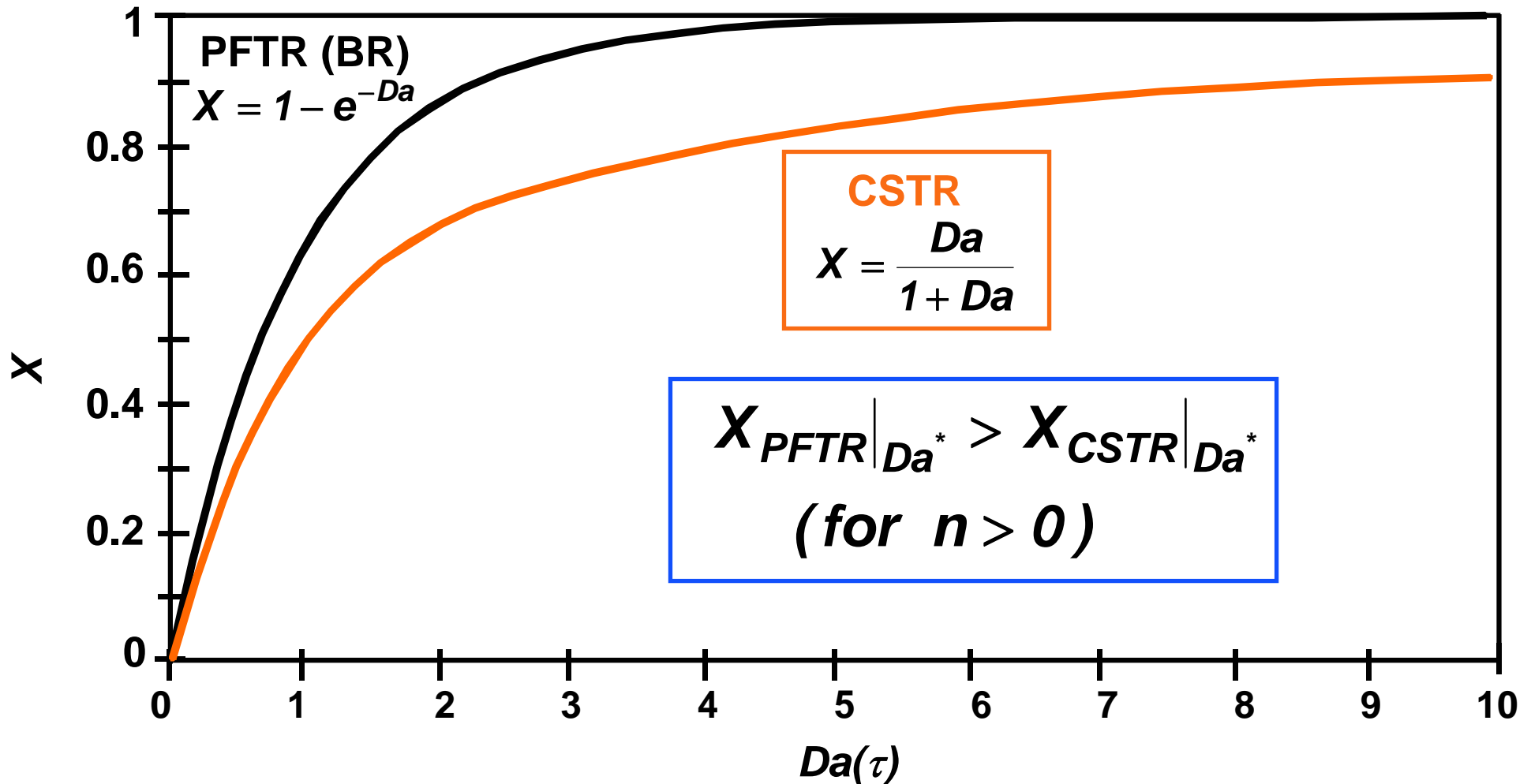
Reversibility



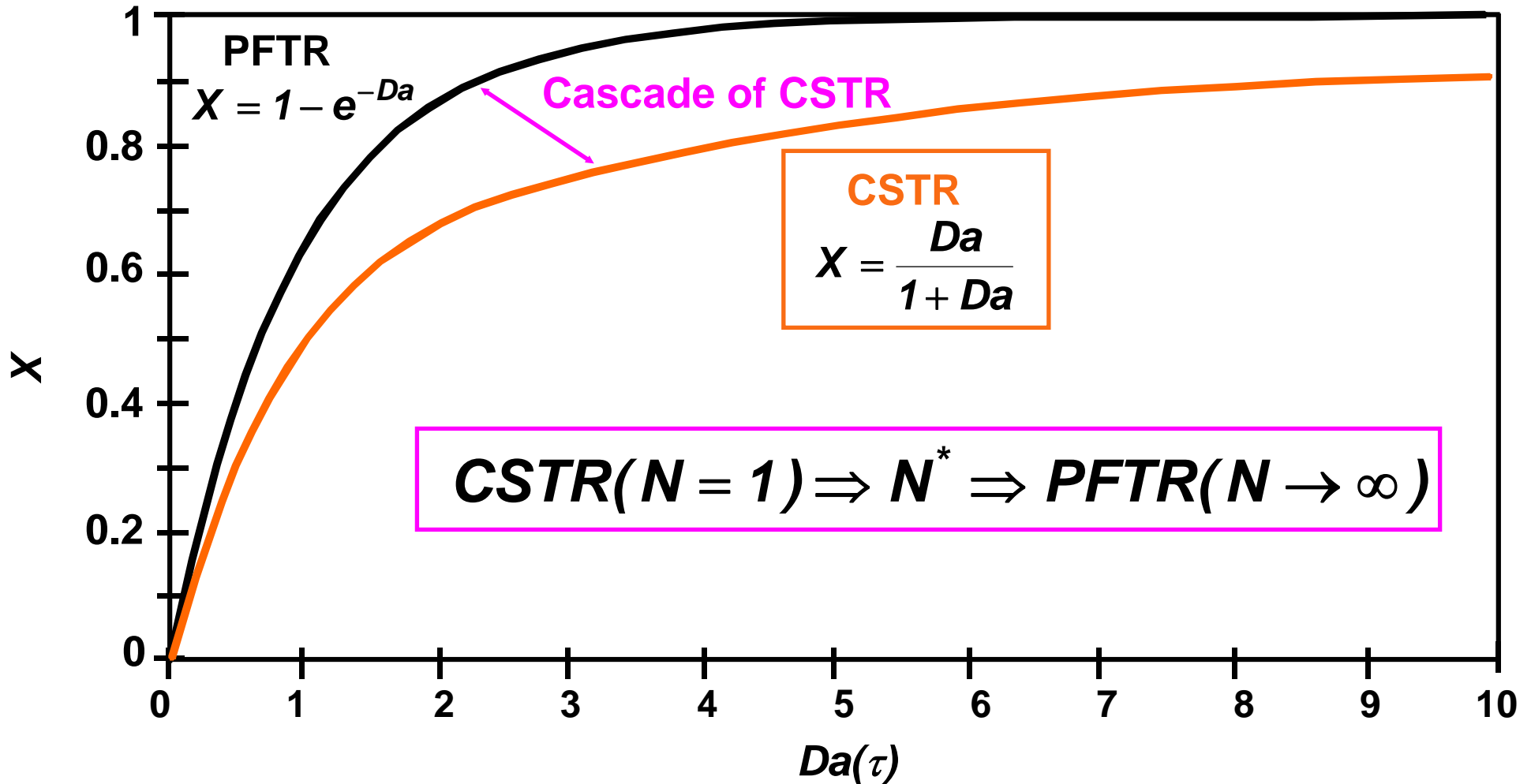
PFTR vs. CSTR (1. Order, $\Phi(X)=1-X$)

$$\text{PFTR (BR): } Da = \int_0^X \frac{dX}{\Phi(X)}$$

$$\text{CSTR: } Da = \frac{X}{\Phi(X)}$$

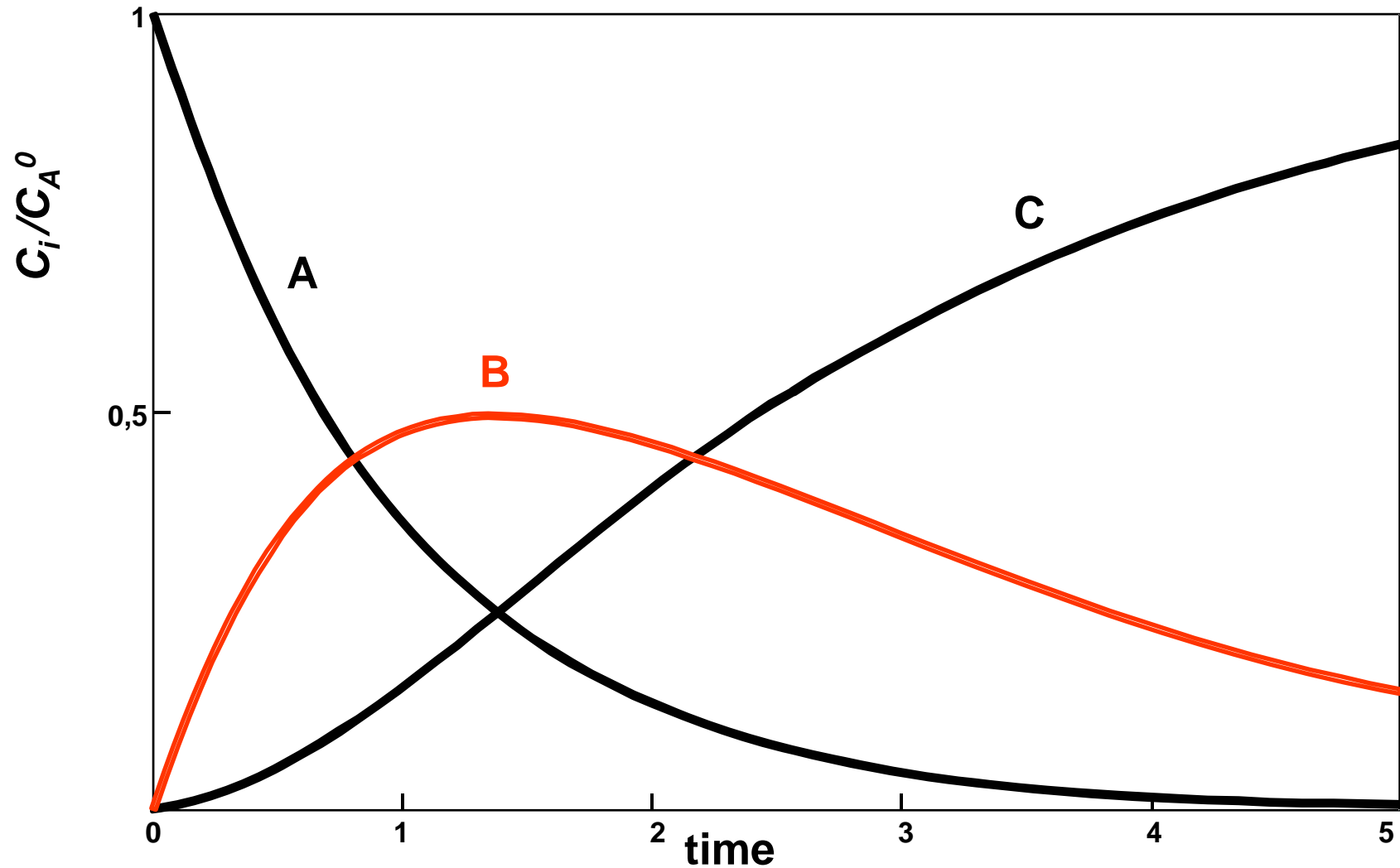


CSTR – Cascade of CSTR - PFTR

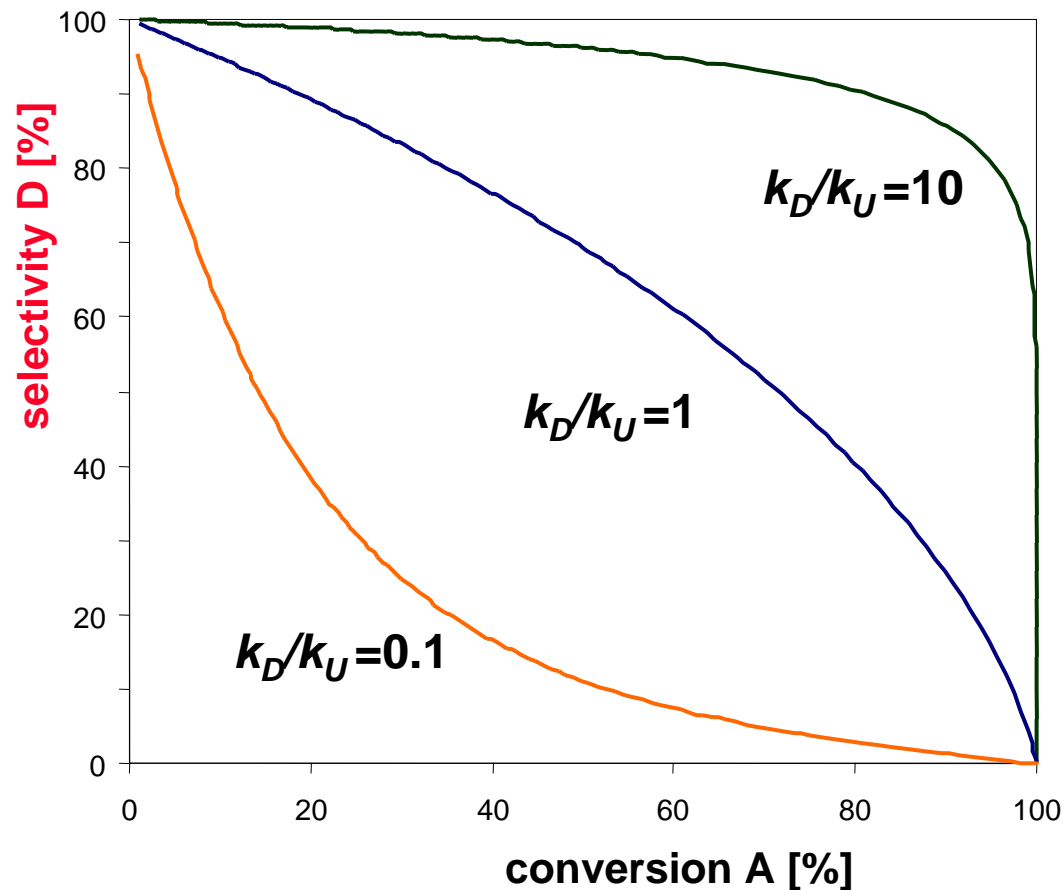


Consecutive reactions in a BR (PFTR)

$$\frac{dc_i}{dt} = \sum_{j=1}^M v_{ij} r_j$$



Selectivity vs. Conversion (PFTR)



Heterogeneous catalysis

(adsorption and Langmuir-Hinshelwood kinetics)

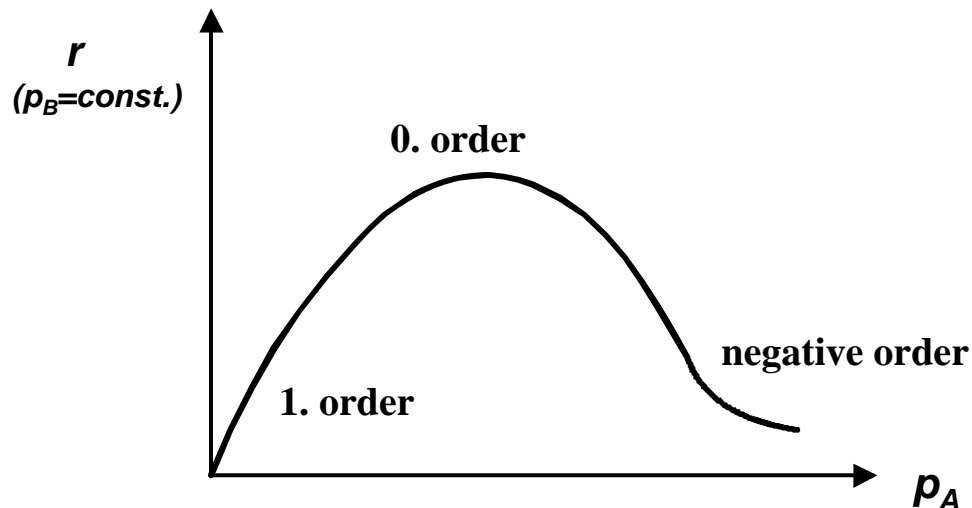


Problem: Θ_i not directly accessible

Assumption: established adsorption equilibrium

$$r = k \Theta_A \Theta_B = k \frac{K_A p_A K_B p_B}{(1 + K_A p_A + K_B p_B)^2}$$

with $\Theta_i = \frac{K_i p_i}{(1 + K_A p_A + K_B p_B)}$

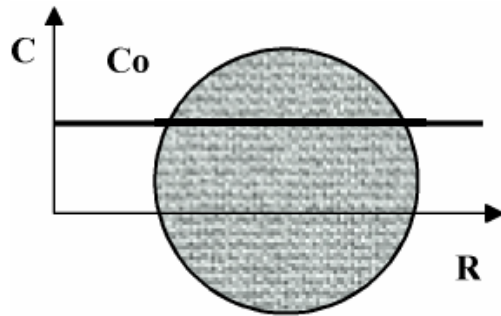


$$r \approx \frac{1}{\text{Scale}}$$

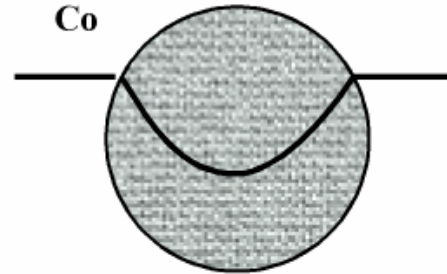
often: $\text{Scale} = m_{\text{Cat}}$

Heterogeneous catalysis

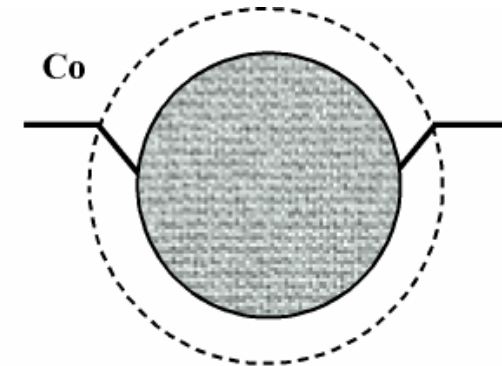
(mass transfer limitations)



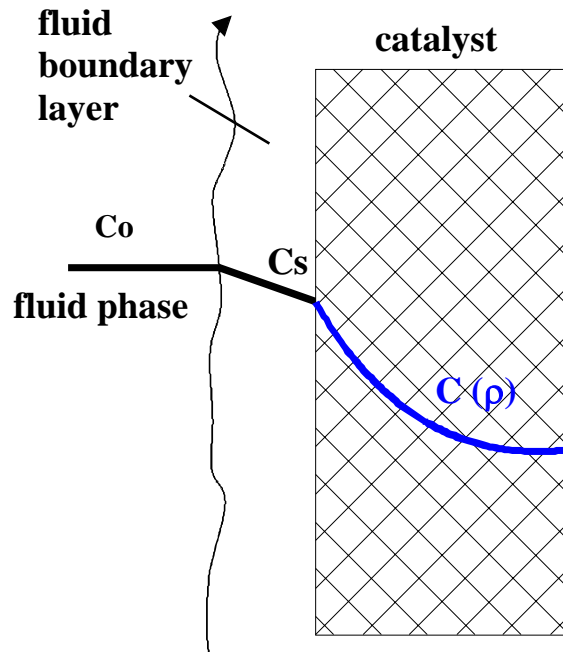
kinetically controlled



pore diffusion



film diffusion



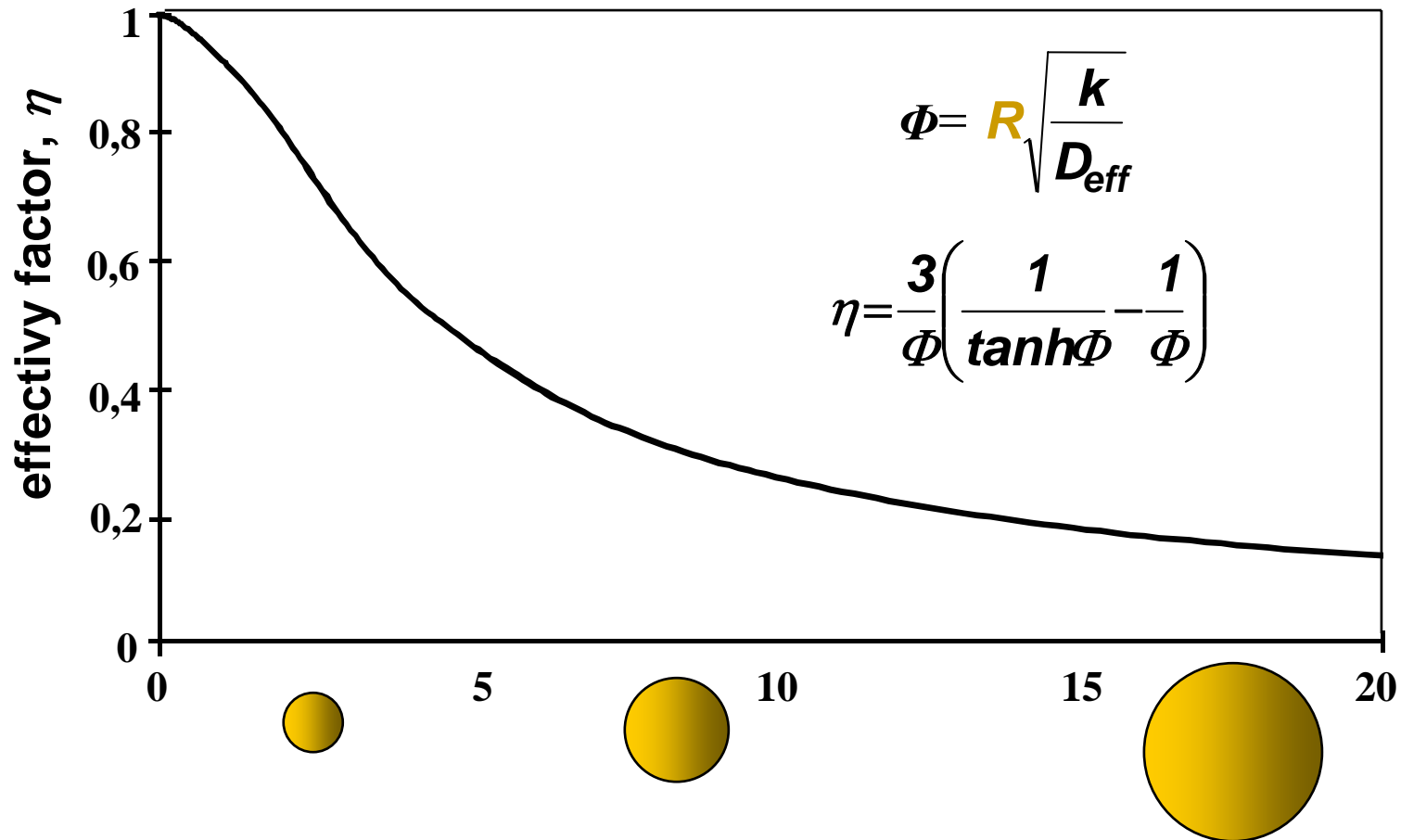
$$D_{eff} \left(\frac{d^2c}{d\rho^2} + \frac{2}{R} \frac{dc}{d\rho} \right) = k c$$

$$\text{BC: } c(\rho = R) = c_0, \quad \left. \frac{dc}{d\rho} \right|_{\rho=0}$$

$$c(\rho) = c_0 \frac{R}{\rho} \frac{\sinh(\Phi \frac{\rho}{R})}{\sinh(\Phi)} \quad \text{with} \quad \Phi = R \sqrt{\frac{k}{D_{eff}}}$$

Thiele modulus

Pore diffusion and effectivity factor η



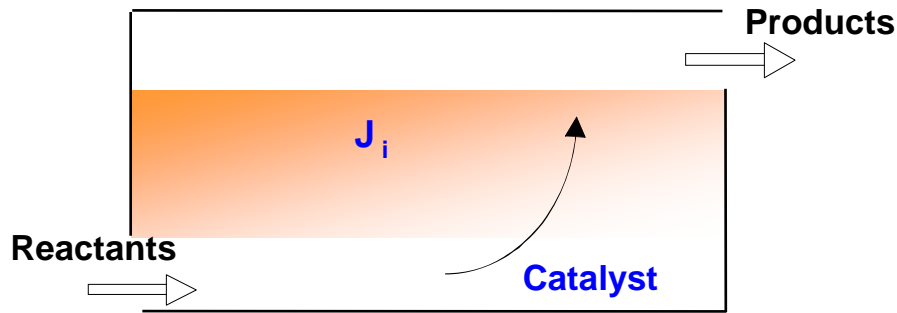
Thiele modulus Φ (\sim reaction rate / diffusion rate)

$$r_{eff} = \eta \cdot r$$

(instead of solving diffusion equation)

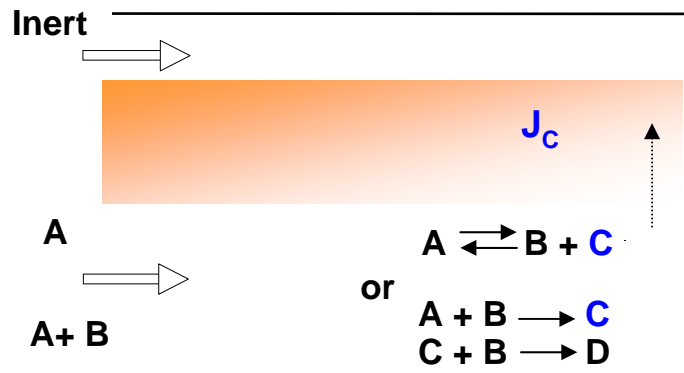
Concepts for membrane reactors

Retaining catalysts



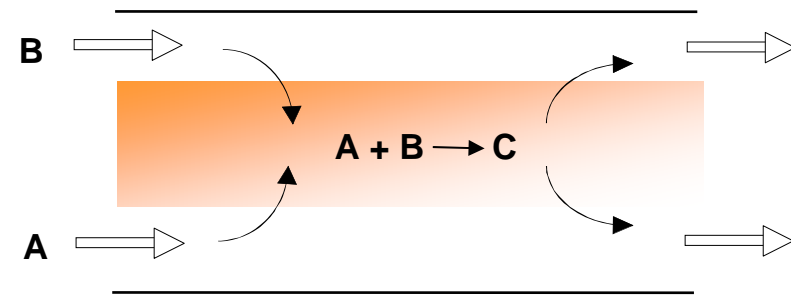
- homogeneous and enzyme catalysis

Selective product removal („Extractor“)



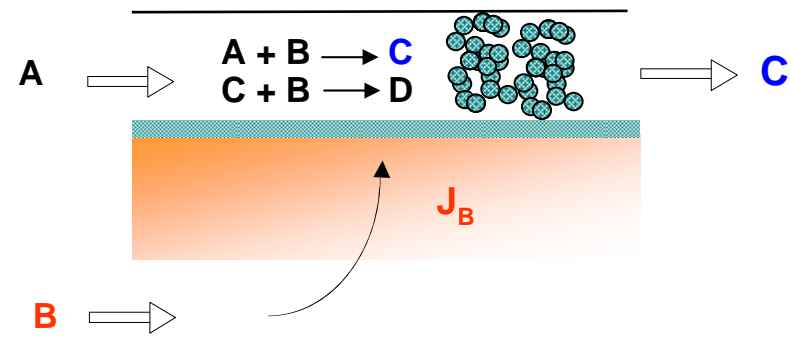
- conversion enhancement for reversible reactions
- selectivity improvements

Membranes as active „contactor“



- conversion front in membrane
- total conversion of toxic components

Controlled reactant dosing („Distributor“)



- dosing of critical components
- selectivity improvements

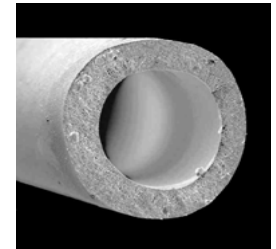
Temperature resistant inorganic membranes

Dense membranes:

- ★ **metals** (Pd, Pd-Ru, Pd-Ag)
- ★ **solid ion conducting electrolytes** ($ZrO_2 - Y_2O_3$, $ThO_2 - Y_2O_3$, Perovskites ABO_3)



selective, limited permeability



IGB-Stuttgart
Perovskite hollow fibre
(\varnothing 1 mm)

Porouse membranes: (symmetric / asymmetric)

- ★ **metals** (sintered metall powder)
- ★ **porous glasses** (acidic leaching)
- ★ **ceramic membranes** (Al_2O_3 , TiO_2) (sol-gel processes)
- ★ **zeolites** (crystallisation)



less selective, more permeabel



Ceramic module
HITK, Hermsdorf

Transport mechanisms in membranes

(previous workshop)

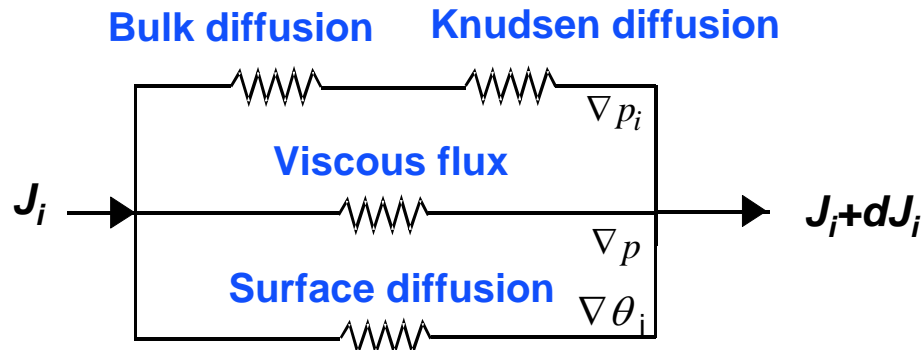
- convection
- molecular diffusion
- Knudsen diffusion
- surface diffusion
- configurational diffusion
- solution and diffusion
- ion and electron conduction
- . . .

essential parameters:

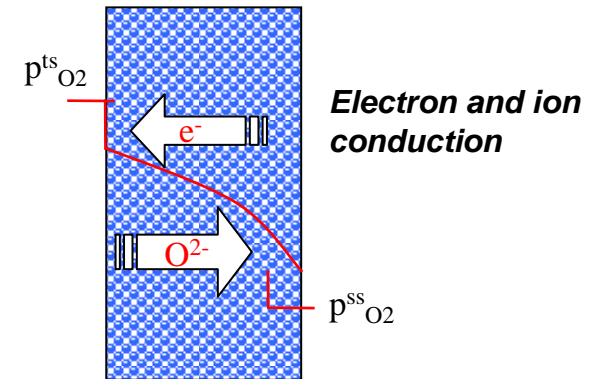
- properties of membrane
- properties of fluid
- operating conditions (driving forces)

Mass transfer mechanisms and models

Porous membrane



Dense membrane



Stefan-Maxwell theory (Dusty Gas Model, DGM)

$$\sum_{\substack{j=1 \\ j \neq i}}^N \frac{x_j J_i^P - x_i J_j^P}{\frac{\varepsilon}{\tau} D_{ij}^O} + \frac{J_i^P}{D_{K,i}^{eff}} = -\frac{p}{RT} \nabla x_i - \frac{x_i}{RT} \left(1 + \frac{B_o}{\eta D_{K,i}^{eff}} p \right) \nabla p$$

(Wesselingh and Krishna, *Mass transfer in multicomponent mixtures*, Delft University Press, 2000)

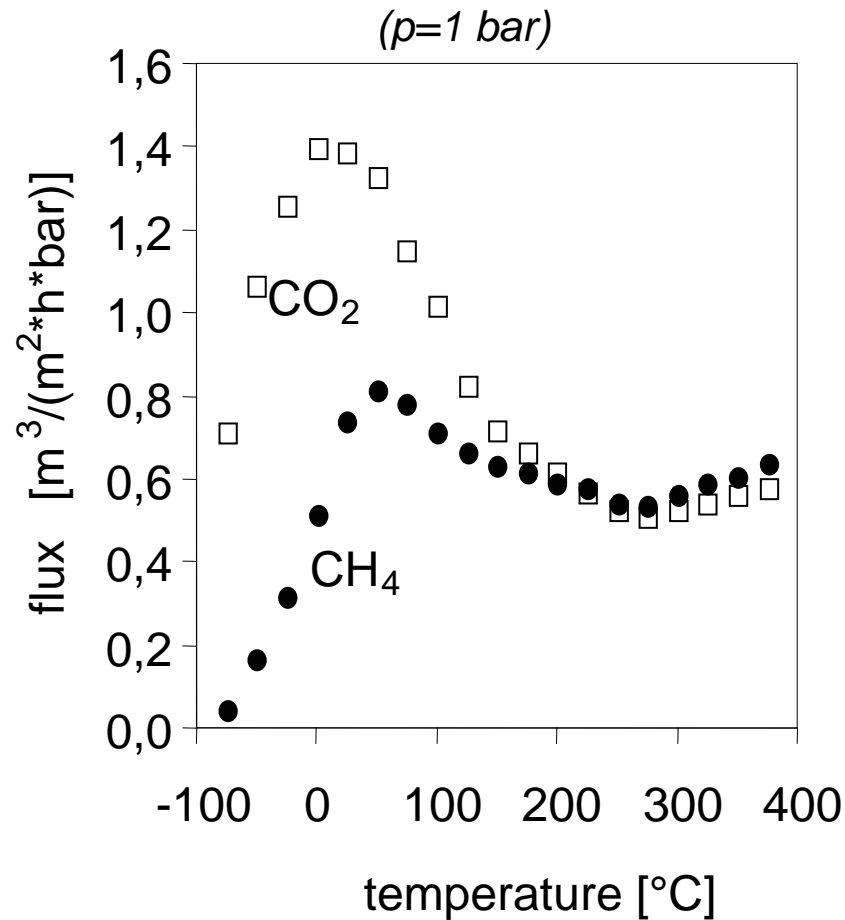
Wagner theory

$$J_{O_2} = -\frac{RT}{4^2 F^2 L_M} \int_{\ln p_{O_2}^{ss}}^{\ln p_{O_2}^{ts}} \frac{\sigma_{el} \cdot \sigma_{ion}}{\sigma_{el} + \sigma_{ion}} d \ln p_{O_2}$$

(Bouwmeester, *Catalysis Today*, 82, 2003, 141)

Illustration of complex influence of temperature on fluxes

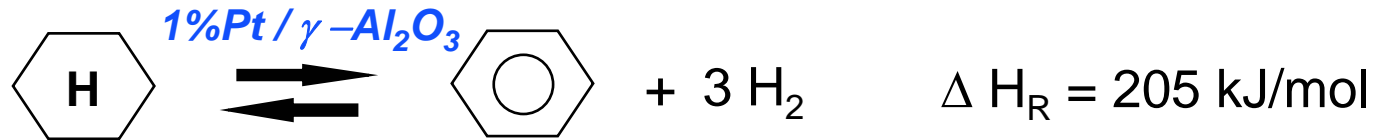
Silicalite layer ($\sim 10 \mu\text{m}$) on a stainless steel support



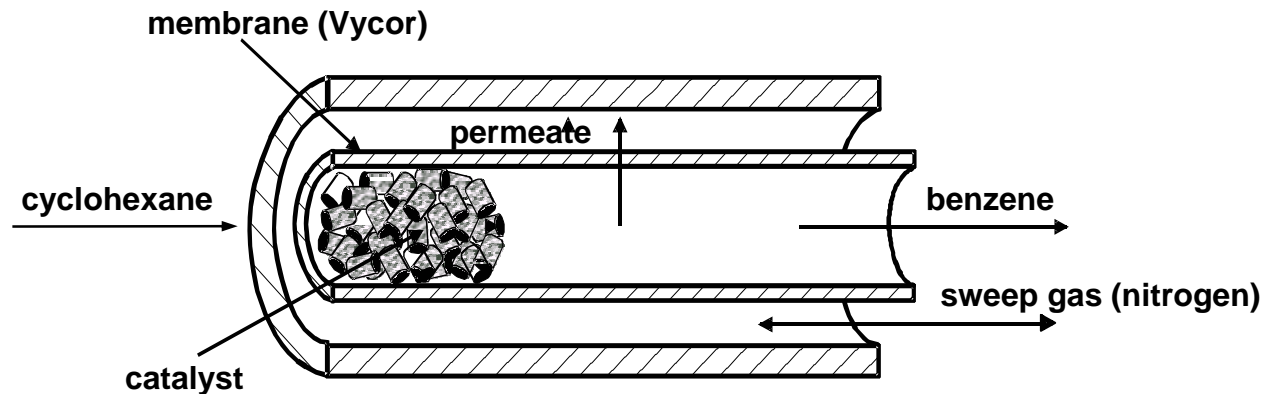
*van den Broeke, Bakker, Kapteijn, Moulijn
AIChE Journal 45 (1999) 976-985*

Example 1: Product removal in a membrane reactor („Extractor“)

Model reaction



- ★ at low temperatures possible, high selectivities
- ★ quantification of the reaction rates in a Berty reactor



Membrane: Vycor glass (d_p appr. 4 nm)

- ★ quantification of mass transfer of reactants with transient diffusion experiments

Determination of reaction rates in a Berty-reactor

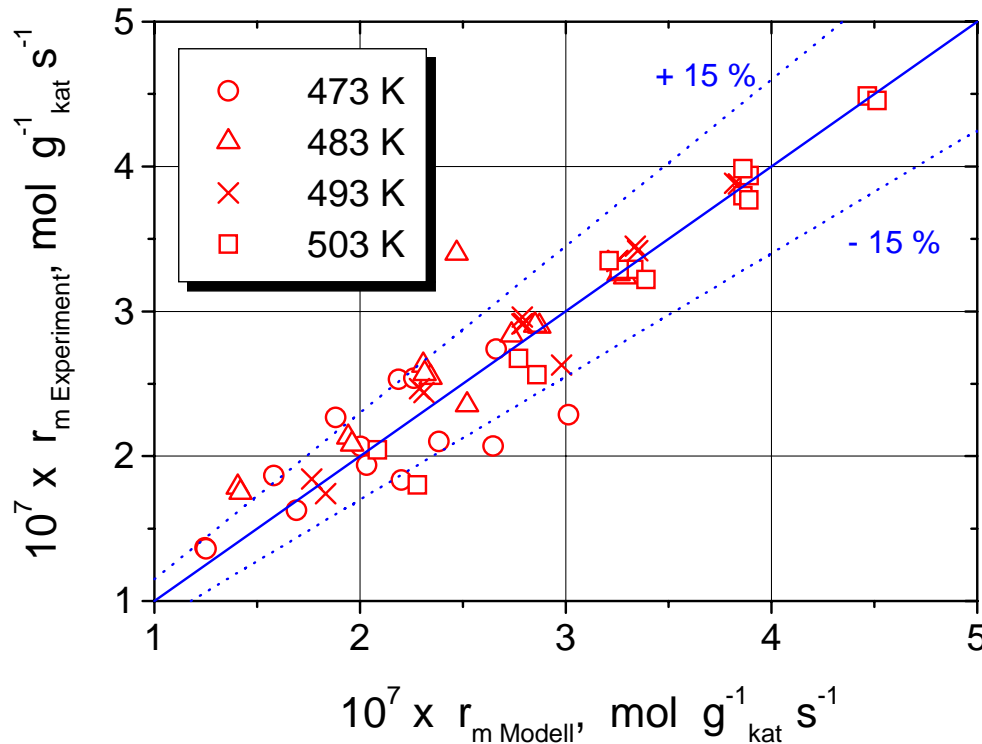
Experimental conditions

T = 473 ... 503 K
 p = 10^5 Pa
 x_{cyc} = 2 ... 6 Vol - %
 space velocity: 400 ... 1000 $[\text{h}^{-1}]$

„Best“ reaction rate equation

$$r = k \frac{P_{\text{cyc}} - P_{\text{ben}} P_{\text{hyd}}^3 / K_p}{1 + k_{\text{ads}} P_{\text{ben}}}$$

benzene adsorbed on catalyst



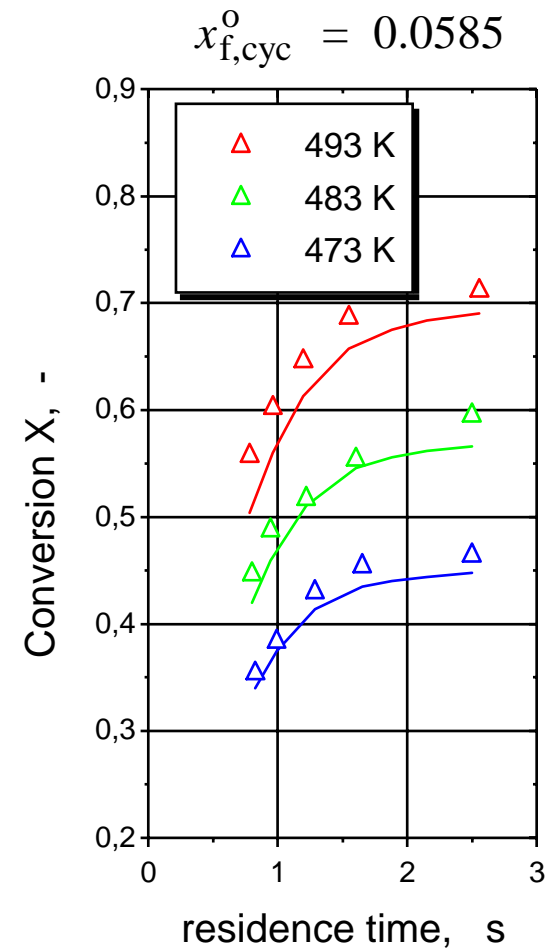
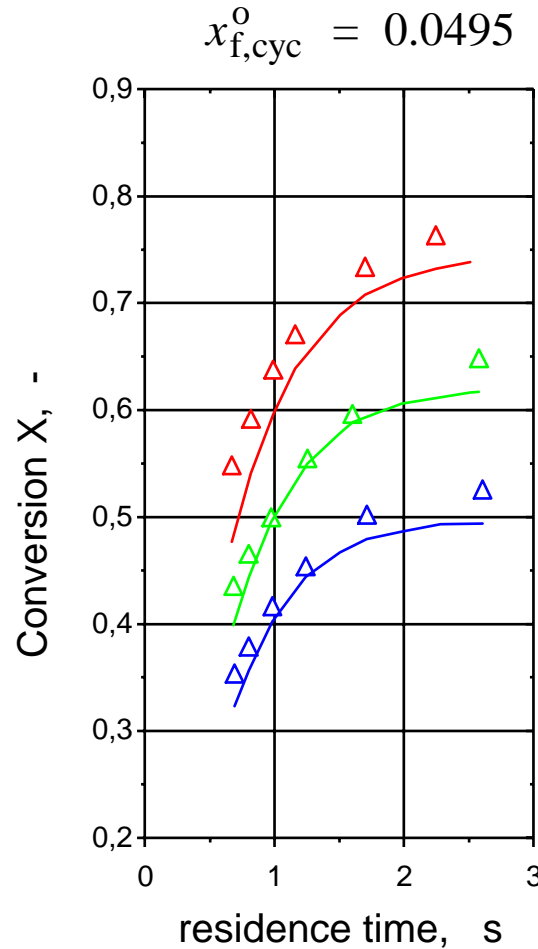
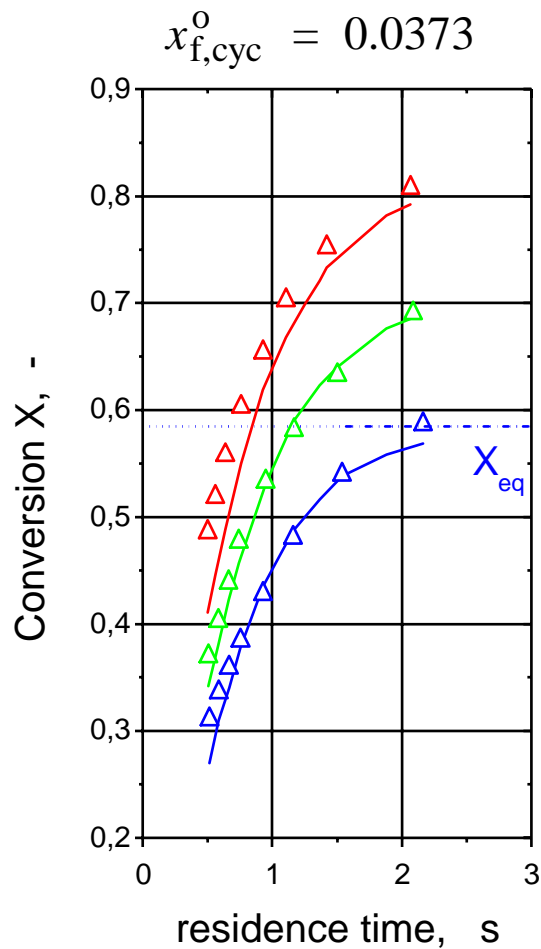
Results (reaction kinetics)

$$k = 7 \cdot 10^{-7} \cdot \exp(-44600 / RT) \left[\frac{\text{mol}}{\text{g s Pa}} \right]$$

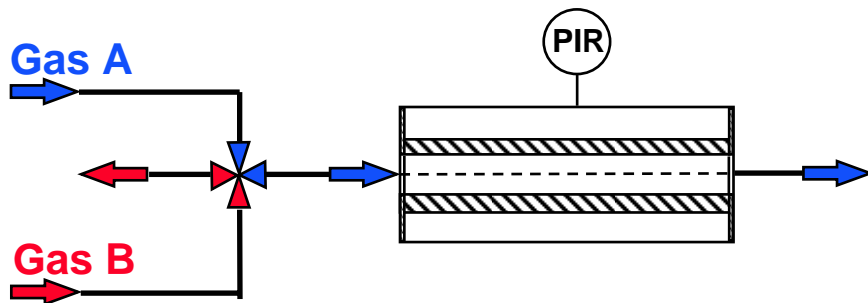
$$k_{\text{ads}} = 1,5 \cdot 10^{-8} \cdot \exp(39930 / RT) \left[\frac{1}{\text{Pa}} \right]$$

Conventional fixed-bed reactor (FBR)

Reactor model:
$$0 = -\frac{p_f}{RT} \frac{1}{q_f} \frac{\partial(\dot{V}_f x_{f,i})}{\partial z} + D_{ax} \frac{p_f}{RT} \frac{\partial^2 x_{f,i}}{\partial z^2} + R_i$$



Transient diffusion experiment



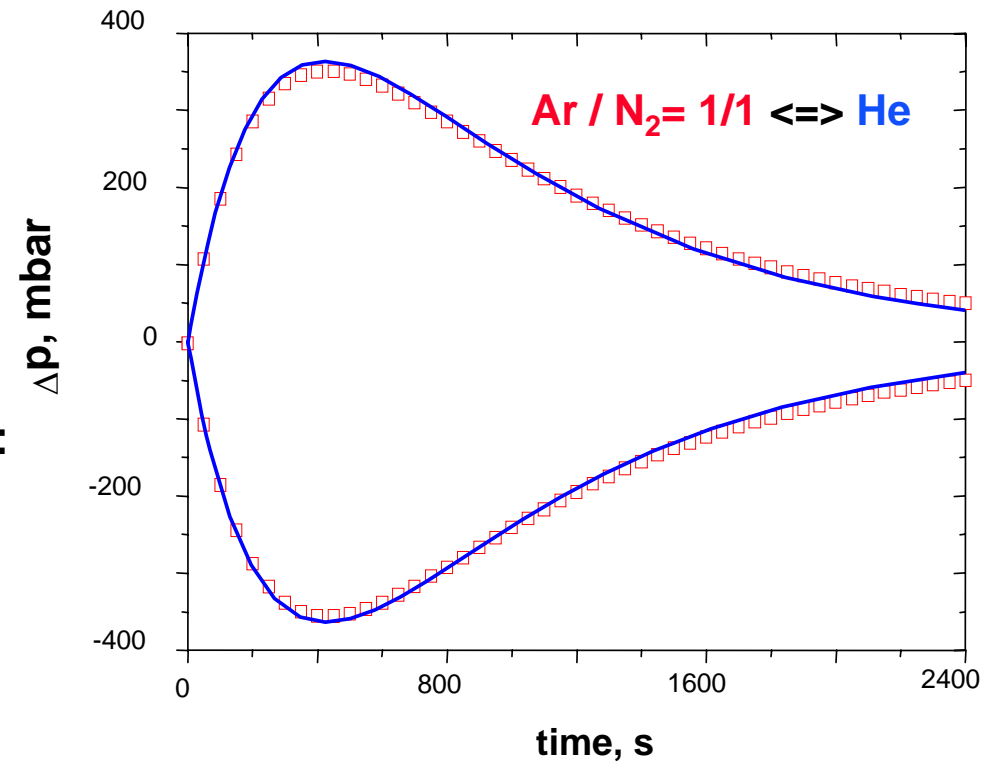
Results (transport through membrane)

Dusty Gas Model parameters of Vycor:

$$\varepsilon / \tau = 0.03$$

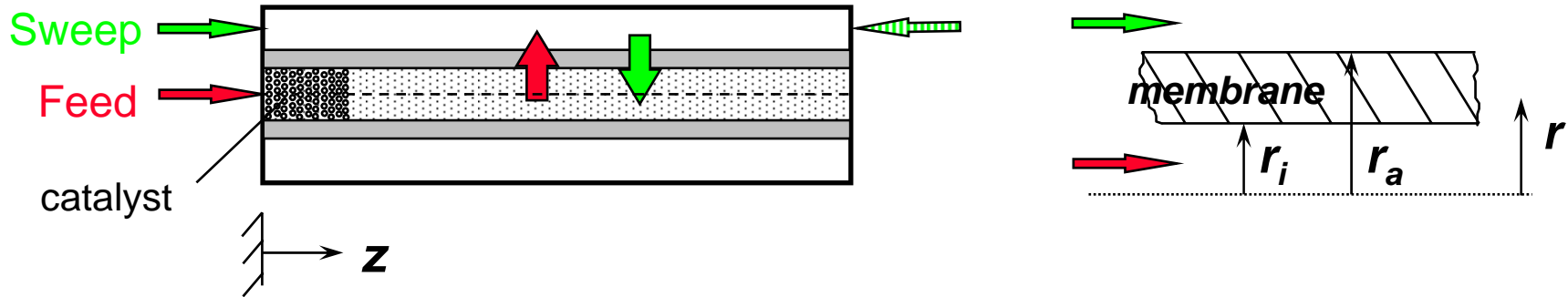
$$K_o = 6.6 \cdot 10^{-11} \text{ m } (\rightarrow D_{K,i})$$

$$B_o = 11.3 \cdot 10^{-20} \text{ m}^2$$



rates of H_2 transport compatible to rates of reaction

Mass balances (tubular membrane reactor)



Mass balance of the feed volume

$$0 = -\frac{p_f}{RT} \frac{1}{q_f} \frac{\partial(\dot{V}_f x_{f,i})}{\partial z} + D_{ax} \frac{p_f}{RT} \frac{\partial^2 x_{f,i}}{\partial z^2} - \frac{2\pi r_i}{q_f} J_i|_{r=r_i} + R_i$$

Mass balance of the membrane

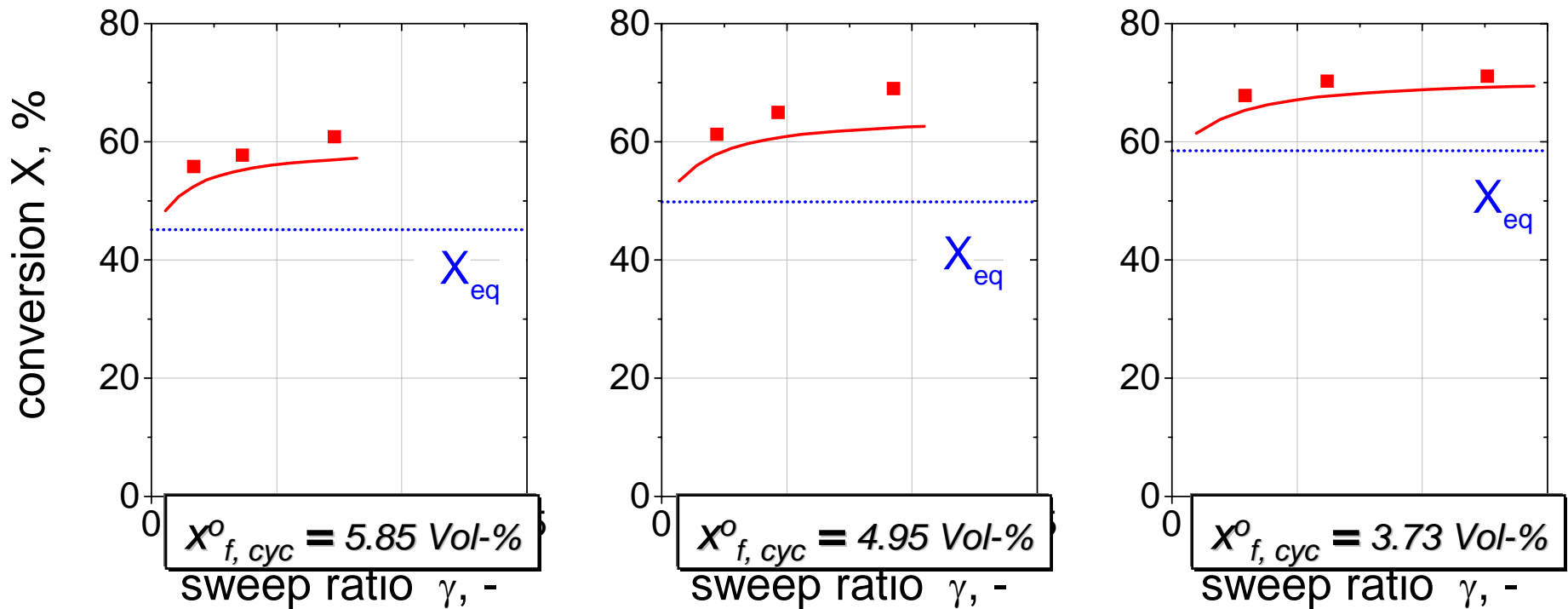
$$0 = \frac{1}{r} \frac{\partial}{\partial r} (r J_i)$$

Mass balance of the sweep volume

$$0 = \pm \frac{p_s}{RT} \frac{1}{q_s} \frac{\partial(\dot{V}_s x_{s,i})}{\partial z} + D_{ax} \frac{p_s}{RT} \frac{\partial^2 x_{s,i}}{\partial z^2} + \frac{2\pi r_a}{q_s} J_i|_{r=r_a}$$

Membrane reactor (MR)

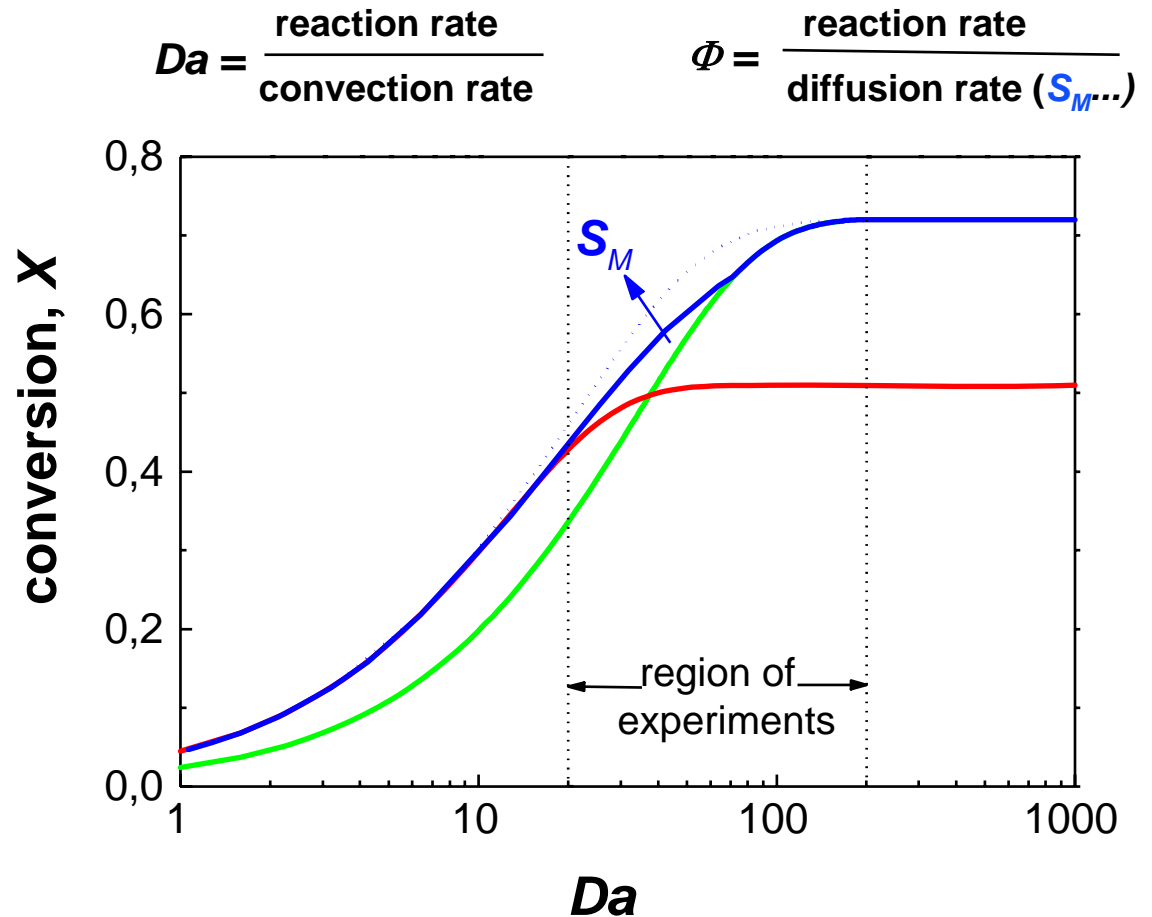
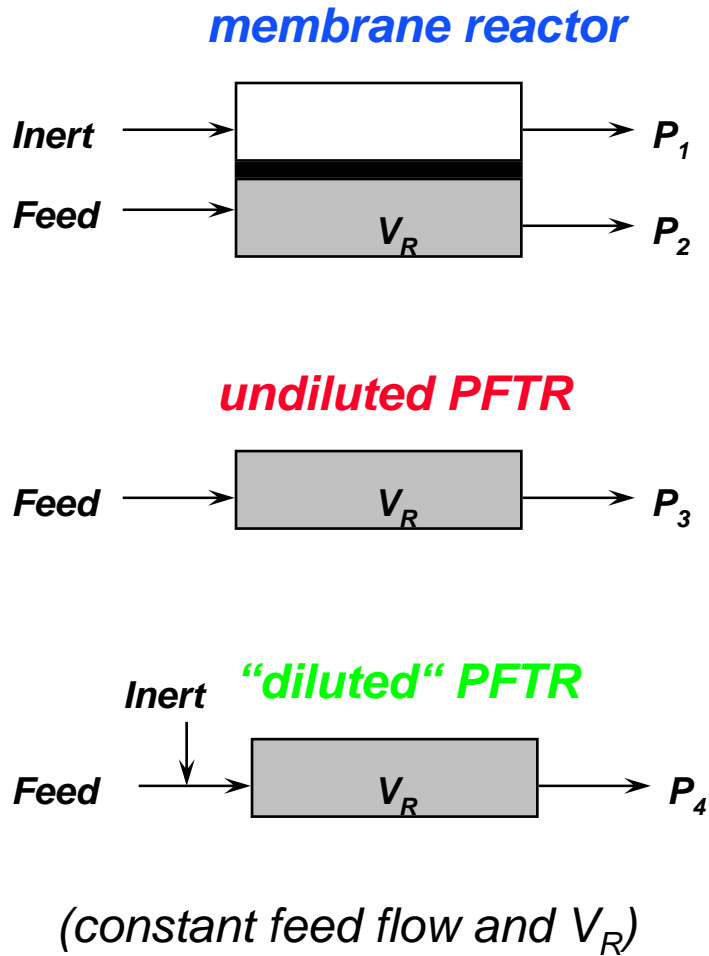
pressure conditions	$p_f = p_s$	= 1 bar
temperature	T	= 473 K
mole fraction (feed side)	$x_{f,cyc}^0$	= 3.73 ... 5.85 Vol-%
mole fraction (sweep side)	$x_{s,cyc}^0$	= 0.0 Vol-%
flow rate (feed side)	V_f^0	= 25 ml/min
sweep ratio	γ	= $V_s^0 / (x_{f,cyc}^0 V_f^0)$



(Tuchlenski, Ph.D., Magdeburg)

Evaluation of an „Extractor“ membrane reactor

(equilibrium limited reaction $A \rightleftharpoons B + C$)

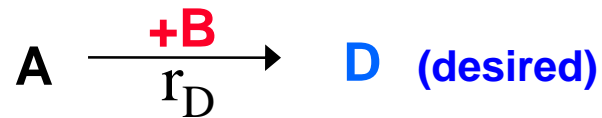


sweep gas must be included in reactor analysis

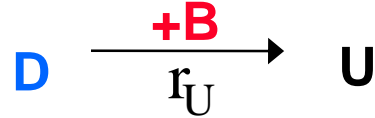
(Reo, Bernstein, Lund, *AIChE J.*, 1997, 495 and *Chem. Eng. Sci.*, 1997, 3075)

Example 2: Membrane as a “Distributor”

(impact of reaction orders on selectivity)



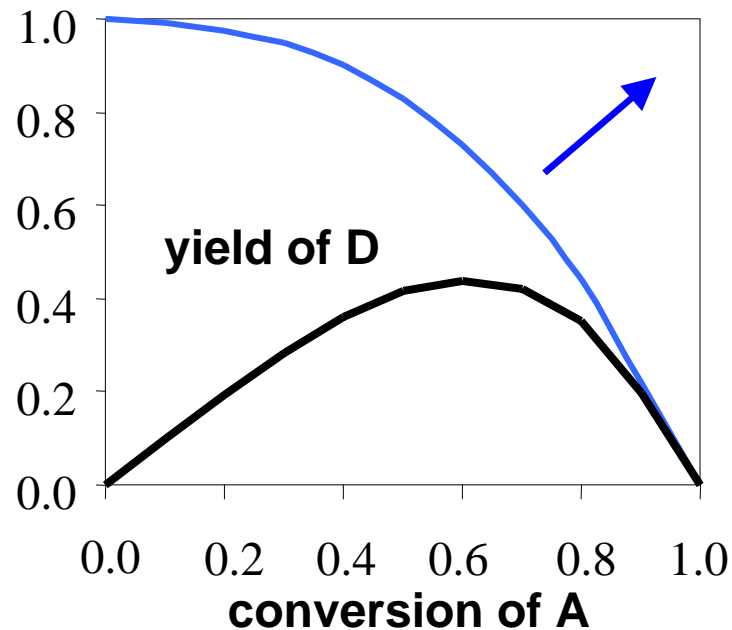
$$r_D = k_D p_A^\alpha \cdot p_B^{\beta_1}$$



$$r_U = k_U p_D^\delta \cdot p_B^{\beta_2}$$

$$S_D = \frac{r_D - r_U}{r_D} = 1 - \frac{k_U}{k_D} \cdot \frac{p_D^\delta}{p_A^\alpha} \cdot p_B^{\beta_2 - \beta_1}$$

Selectivity with respect to D

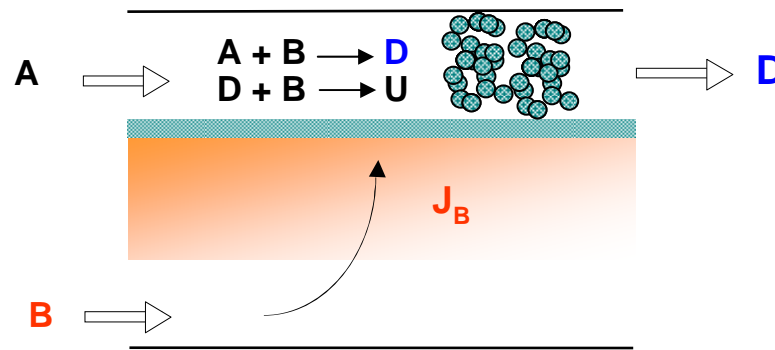


p_D small
(product removal)

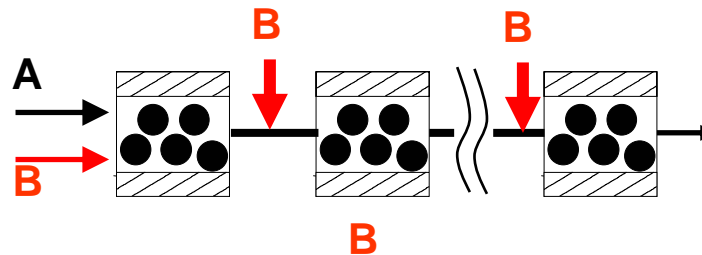
$\beta_2 > \beta_1$:
 p_B small (dilution)

dosing of B (“Distributor”)

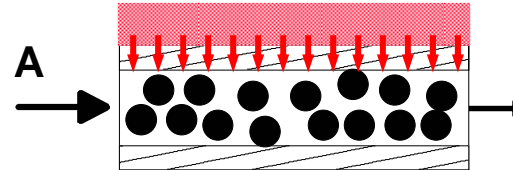
Controlled dosing of reactants („Distributor“)



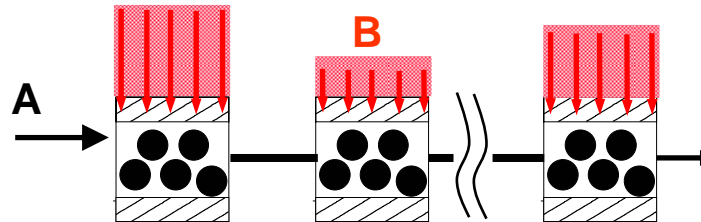
discrete
(series of FBR)



continuously
(single MR)

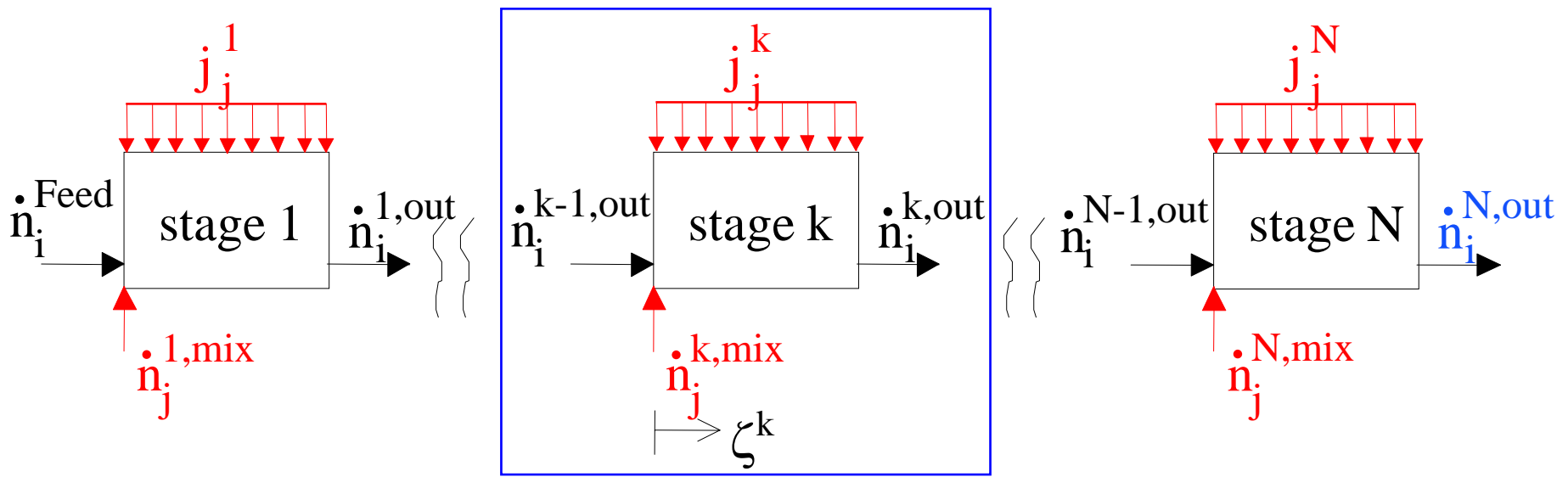


stagewise continuously
(series of MR)



➡ potential e. g. for hydrogenations, oxidations and oxidative dehydrogenations

Simplified reactor model: isothermal PFR stages, steady state

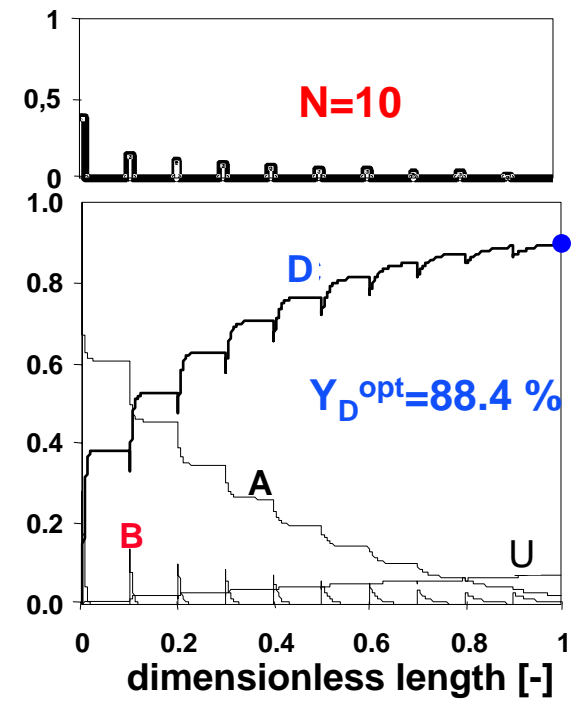
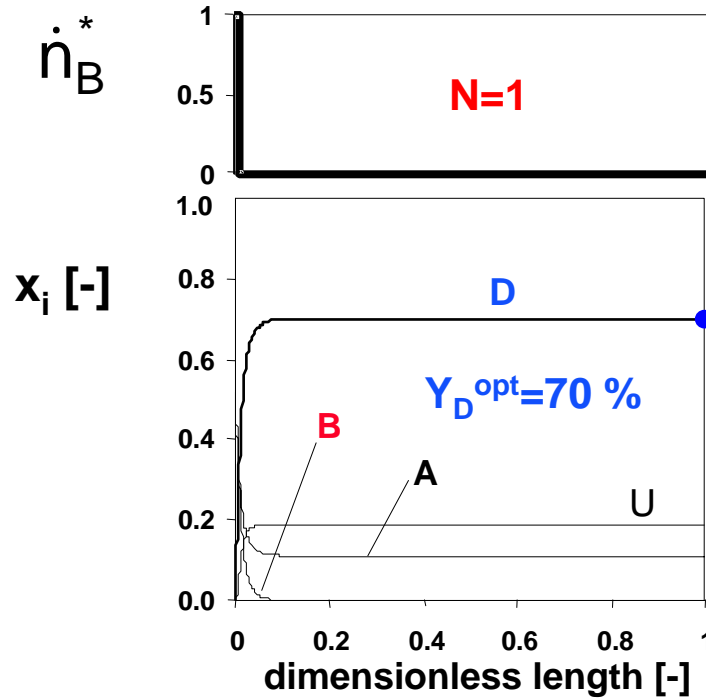


$$\frac{d\dot{n}_i^k}{d\xi^k} = P^k \cdot \underline{\dot{j}_i^k} + A^k \cdot \sum_{j=1}^M v_{ij} \cdot r_j \quad k=1,N$$

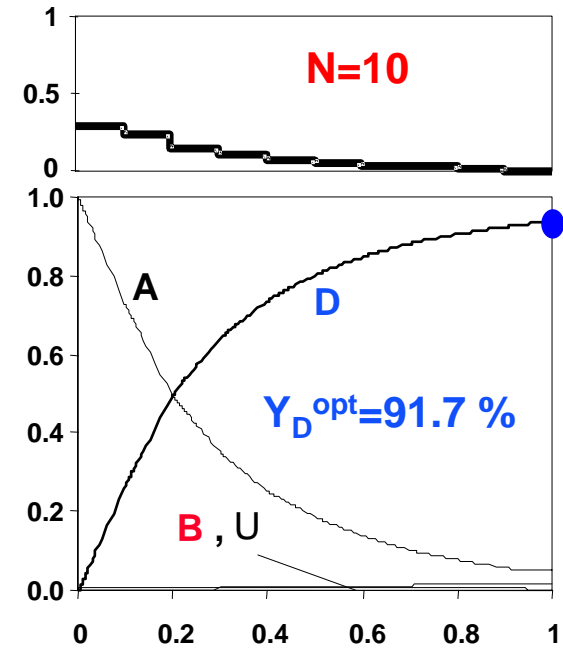
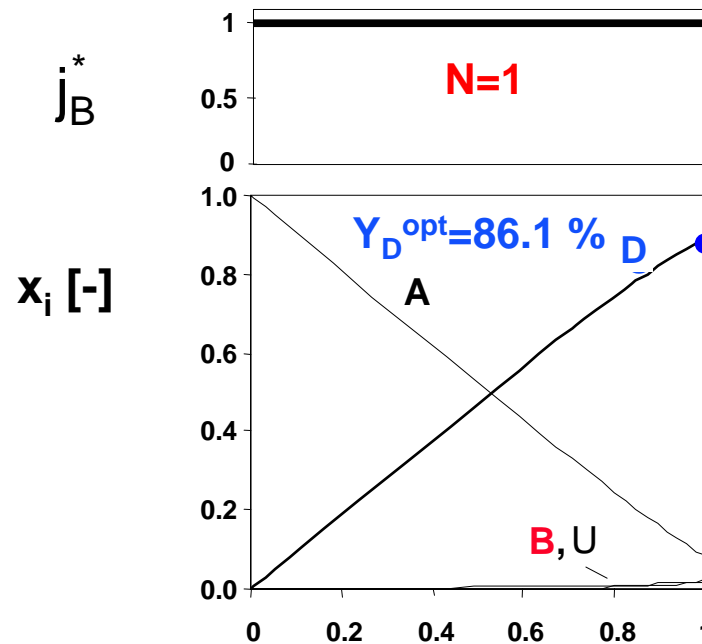
- Optimization:**
- analytical solutions for special cases
 - Pontryagin's maximum principle
 - numerical optimization (SQP)

Segmented reactors: Optimization with SQP ($\beta_2/\beta_1=2$)

FBR:



MR:



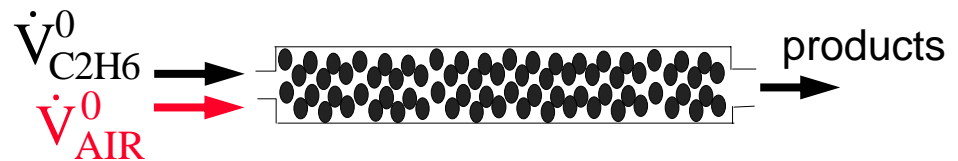
Oxidative dehydrogenation of ethane to ethylene

Catalyst: $\gamma\text{-Al}_2\text{O}_3/\text{VO}_x$ ($d_p=1.8$ mm, 1.4 % V)

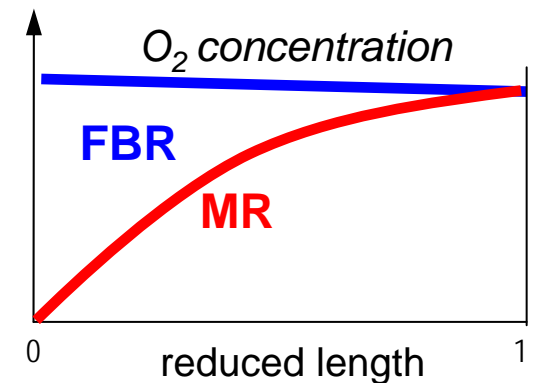
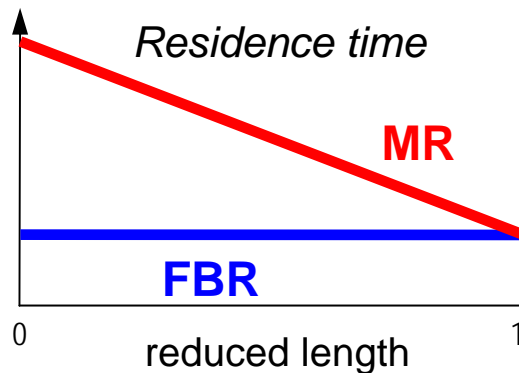
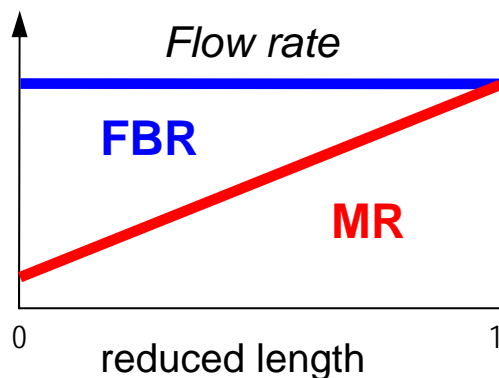
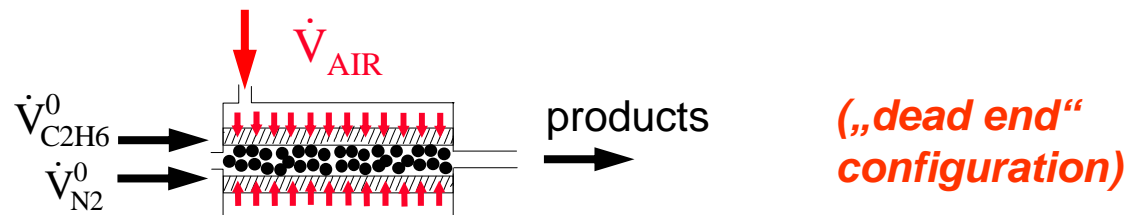
Membrane: Ceramic composite membrane (HITK, Hermsdorf)

Conditions: $T = 400 - 650$ °C, $p = 1$ bar, $\text{GHSV}_{\text{ov}} = 6000 - 38000$ h⁻¹
 $c_{\text{ethane}} = 0\text{-}1$ vol%, $c_{\text{O}_2} = 0\text{-}20.3$ vol%

**Fixed-bed reactor
(FBR)**

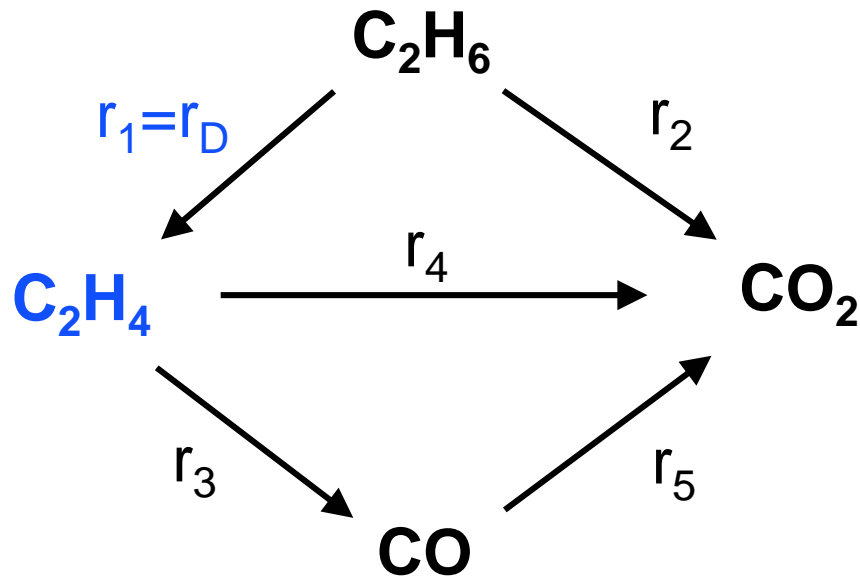


**Membrane reactor
(MR)**



Reaction scheme and kinetic parameters

Analysis of 70 FBR runs



	α [-]	β [-]
r_1	0.88	0.02
r_2	0.75	0.24
r_3	0.84	0.11
r_4	0.87	0.2
r_5	1.12	0.13

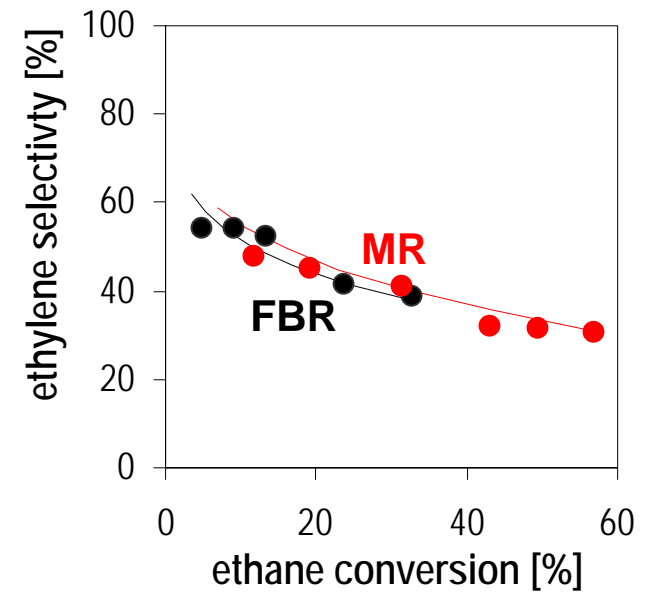
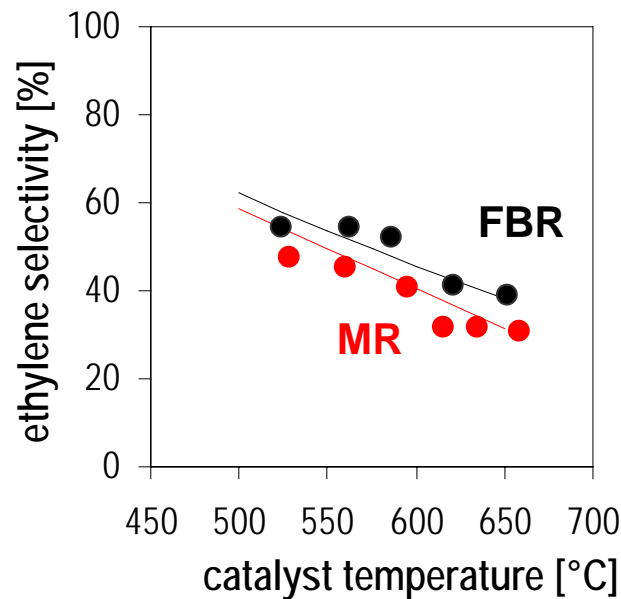
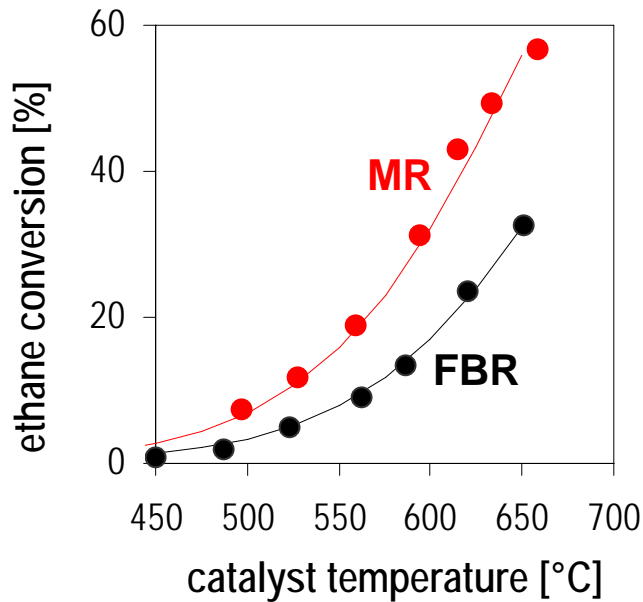
!

$$r_j = k_j(T) \cdot p_C^{\alpha_j} \cdot p_{O_2}^{\beta_j}$$

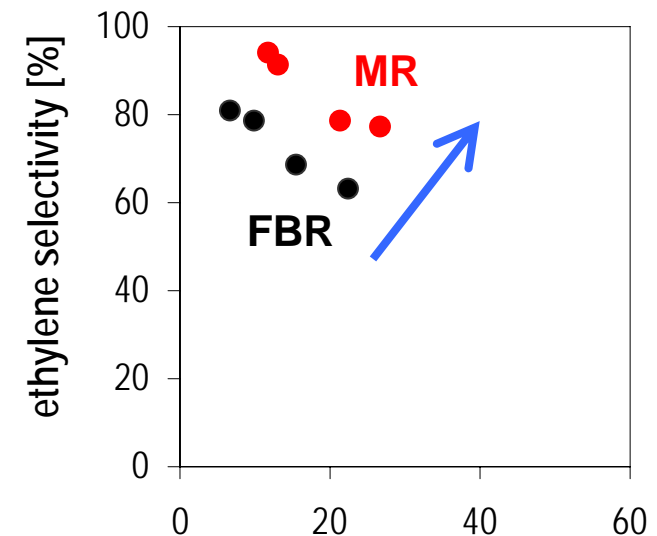
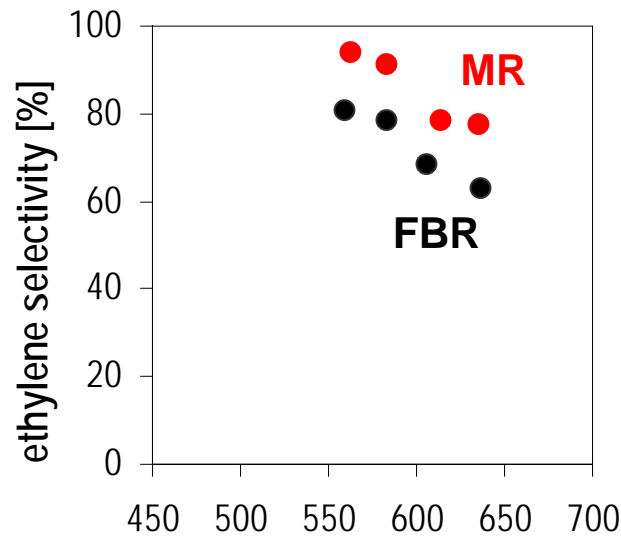
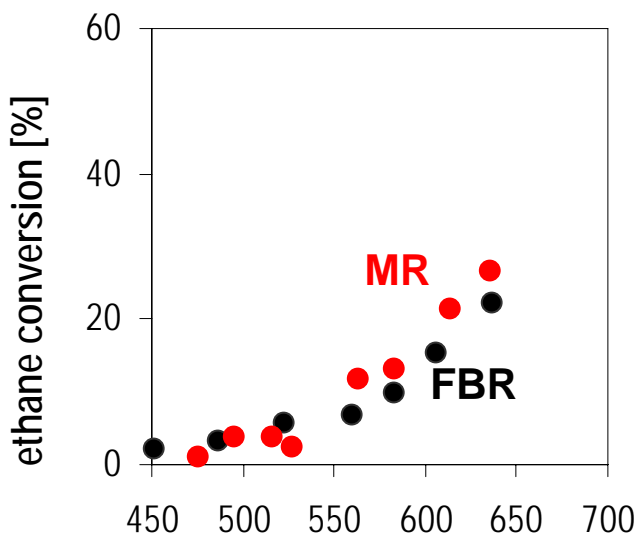
(Klose et al., Appl. Catal. A., 260, 2004, 101)

Comparison between FBR and MR

oxygen excess ($x_{O_2}^0 = 19-20 \text{ vol\%}$, $x_{C_2H_6}^0 = 0.7 \text{ vol\%}$, GHSV 38000 h^{-1}) $\rightarrow \tau$ relevant

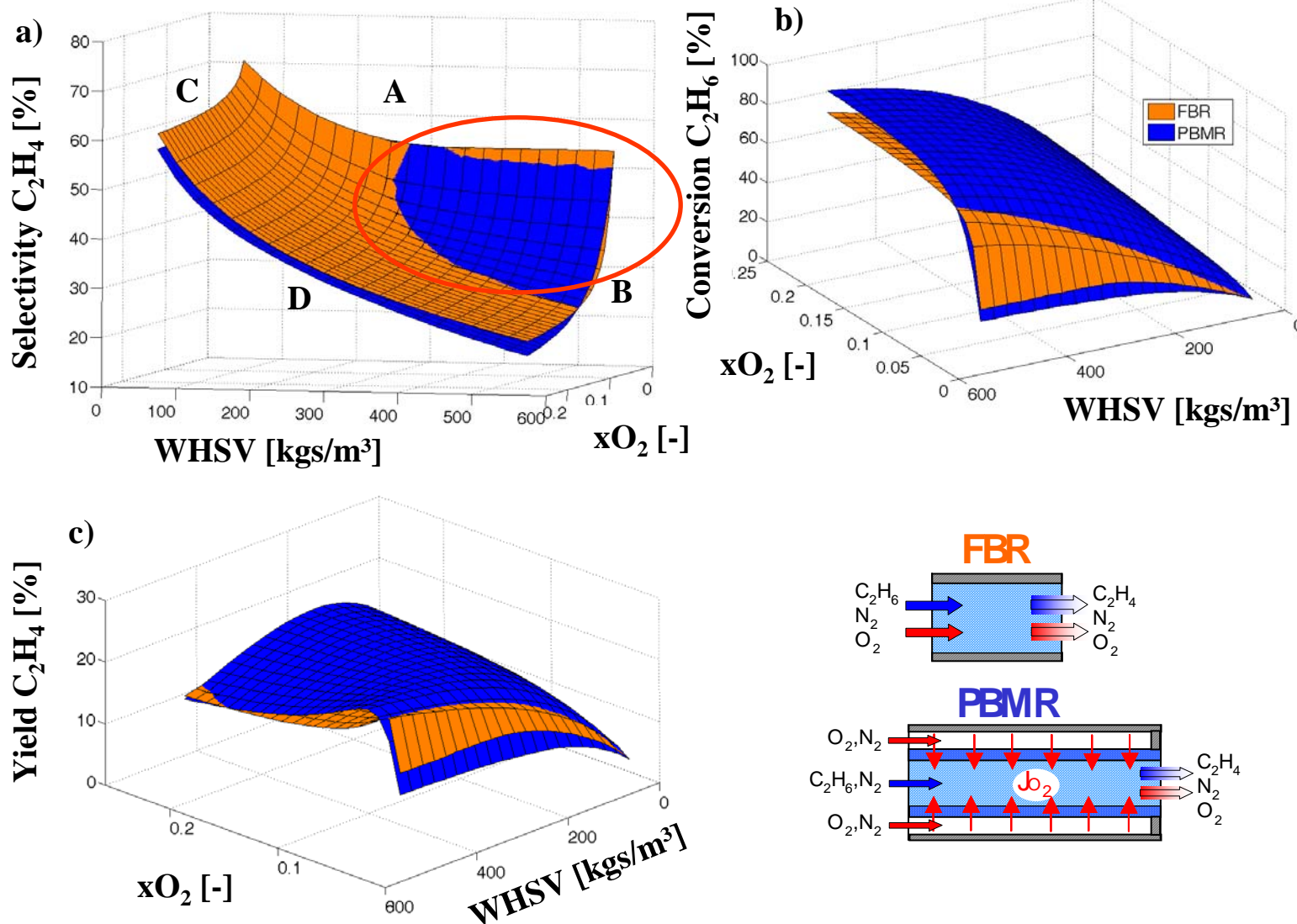


oxygen shortage ($x_{O_2}^0 = 0.4 \text{ vol\%}$, $x_{C_2H_6}^0 = 0.7 \text{ vol\%}$, GHSV 38000 h^{-1}) $\rightarrow x_i$ relevant

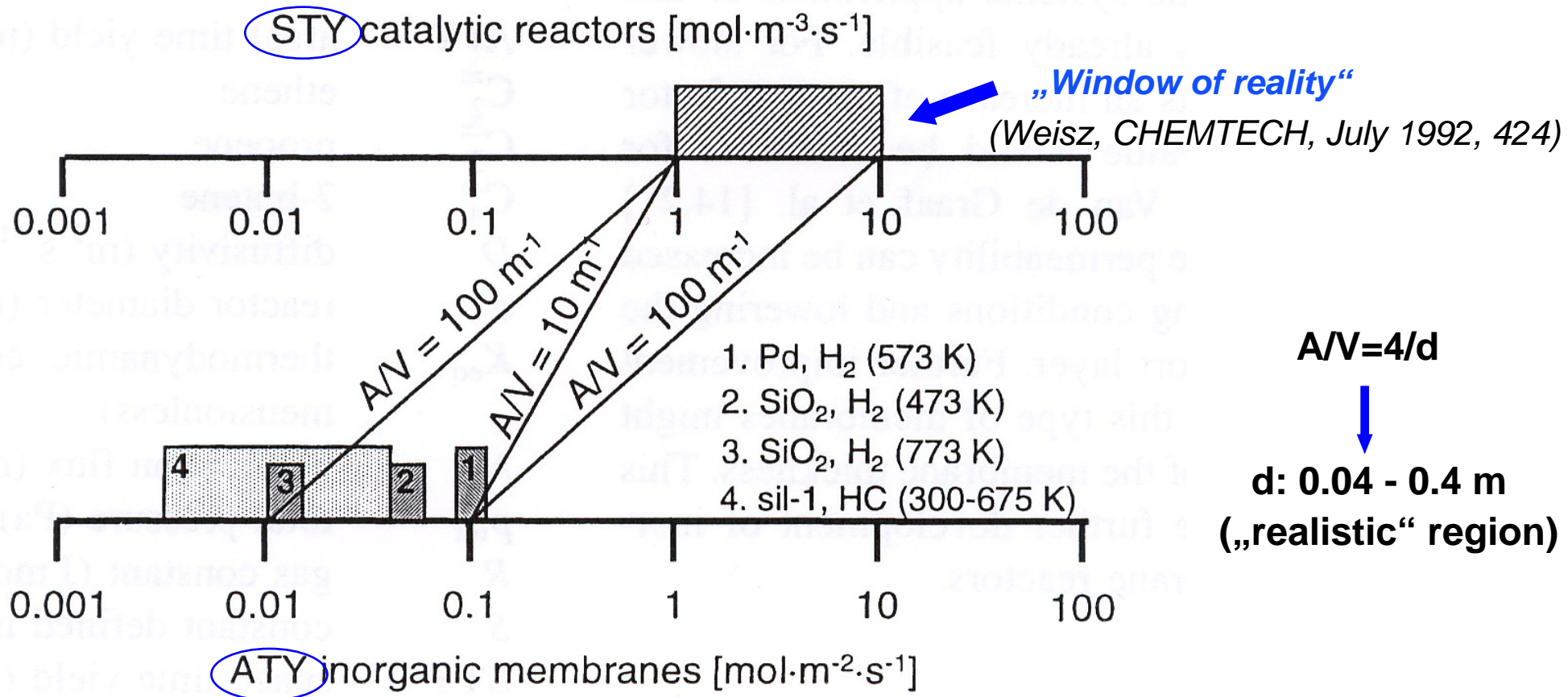


Comparison between FBR and MR – Simulation study

oxygen excess ($x_{C_2H_6}^{in} = 1.5\%$, $x_{O_2}^{in} = 0.5-21\%$, $WHSV = 50-550 \text{ kgs/m}^3$, $T = 600\text{ }^\circ\text{C}$, $F_{TS}/F_{SS} = 1/8$)



Compatibility between reaction and transport



(van de Graaf, Zwiap, Kapteijn, Moulijn, Appl. Catal. A, 178, 1999, 225)

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