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EXPERIMENTS IN EDUCATION

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THE OFFICIAL JOURNAL OF THE S.I.T.U COUNCIL OF EDUCATIONAL RESEARCH
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Special International Issue on Mathematics Education

The year 2012 has been declared as the Year of Mathematics and December 22, the birth day of the mathematical genius Srinivasa Ramanujan as the national 'Mathematics Day'. The SITU Council of Educational Research decided to pay homage to the memory of Sri Ramanujan in this 125th year of his birth by choosing Mathematics Education as the theme for the annual special issue of its official organ, *Experiments in Education.*

Dr (Ms) A.Renuka, one of the Members of our Editorial Board, took the responsibility for corresponding with most of the contributors and coordinating the collection of articles for the issue. Of course, the other members of the Editorial Team of this special issue also contacted contributors, solicited and obtained articles.

The range of the contributors in terms of age, nationality and professional background is very wide indeed. Their interest in mathematics is the common binding thread.

Soundararajan Srinivasa, formerly a high profile official of the Government of India, has given an encyclopaedic view of mathematics education in quite a few countries from a critical, interdisciplinary, global and cross-generational perspective, with due space for unique Indian contributions.

Dennis L.Carpenter, Deputy Superintendent of Operations, Newton County School System Covington, Georgia, USA, has reflected on the looming global shortage of competent teachers for Science, Technology, Engineering and Mathematics with focus on the US, the problems and prospects of recruiting teachers from around the world and has given practical suggestions about a mechanism for teacher recruitment and indicated, in the light of his experience, what teachers who aspire to emigrate to the US have to check and how they can be acculturated to the US school ***milieu***.

B.Viswanathan, National Centre for Catalysis Research, Indian Institute of Technology, Madras, has highlighted at once the importance and neglect of mathematics in research in chemistry, the costs and consequences thereof, and most important, how mathematics can and must be integrated in the curriculum for the preparation of a research career in chemistry.

Berinderjeet Kaur**,** National Institute of Education, Nanyang Technological University, Singapore, has reported a challenging but promising model of a field project in improving classroom practices in the teaching of mathematics. The university faculty who were mentors for school teachers took responsibility for *in situ* handholding towards institutionalisation of a functional innovative practice.

Adhithya Rajasekaran and Sri Madhavi Rajasekaran, student contributors in this issue, have utilised their exposure to mathematics education in India and the US to make perceptive comparisons of both systems and highlight the strengths and weaknesses of the two systems.

Philip G.Jackson, an 'amateur', as he describes himself, in mathematical research, argues that stark and pristine simplicity as the guiding principle of thinking, can lead to stunning and profound discoveries. He makes a meaningful distinction between proof and understanding in the mastery of mathematics. Understanding is a pre-condition for further discoveries. He appeals for due attention from the academia for amateur but serious scholars in mathematics.

Peter Barbatis, Vrunda Prabhu, and James Watson of Bronx Community College, NY,USA, report an action research project on generating a creative learning environment (CLE) in the teaching of Basic Mathematics at Bronx Community College of the City University of New York: a CLE is arrived at after several cycles of teaching-research experiments. It aims to provide avenues for learning of mathematics with enjoyment and mastery demonstrated through performance. The CLE is a microscopic exercise anchored in three supports: cognition, affect, and self- regulated learning practices.

Egan J. Chernoff, University of Saskatchewan, Saskatoon SK, Canada, has sought to probe and highlight, 'the fallacy of composition', a common fallacy in probabilistic thinking of mathematics teachers. It is a basic, logical fallacy consisting of attributing to the whole a property perceived in a part. There seems to be a propensity in the human psyche to go skewed in its thinking thanks to an inherent and peremptory need for closure and certainty. We believe, as the author himself observes, the composition fallacy is a promising new lens to explore the interface between an inherent propensity of the human psyche and cold and rigorous probability statistics. Perhaps the next, deeper step would be to probe the differences in the personality correlates of the subsample that does ***not*** succumb to this fallacy and the one that does and then account for the difference psychopedagogically. If and when we can do so we will be able to fortify the psyche and align—albeit asymptotically— the teachers' and through them the students' thinking to Nature's sublime, majestic, austere, and macrocosmic rhythms of probability.

Dr.Renuka. Life University,Marietta, GA, USA, has presented a report on a project about an icon-mediated, vocalized physical response based visual–verbal, aural/oral multiple representation method used as a creative strategy for teaching quadratic inequalities to high school students. It is cost effective, engaging and has been found promising.

Sivakumar Alagumalai, School of Education, University of Adelaide, Adelaide, Australia, has made a bold and creative attempt to extract the psychopedagogical trajectory of the development of the genius of Ramanujan from the scant material available on the books that he is known to have avidly studied and the structure of their content. Siva has also dealt with the need for adopting a probabilistic model to assess learning and make diagnoses of learning: the author states Ramanujan adopted only such a model for gauging his own progress in understanding mathematics. These are promising inputs for reforming mathematics education.

Vishal Sood, International Centre for Distance Education and Open Learning (ICDEOL), Himachal Pradesh University,Shimla, India, has reported a comparative, experimental study of the effectiveness of two mastery learning strategies namely Bloom's Learning for Mastery (LFM) and Keller's Personalised System of Instruction (PSI) with reference to concept attainment in geometry. He found mastery learning strategies more effective than the conventional method; and, Bloom's LFM more effective than Keller's PSI.

We hope this special issue will provoke thinking on the various aspects of mathematics education on which contributions have been received and also set a new standard of scholarship and presentation for our regular issues.

I thank Dr.Renuka, Life University,Georgia, USA, the coordinator of this special issue, and its two guest editors, Dr.E.Ram Ganesh, Director in-Charge, College and Curriculum Development Council, and Professor, Department of Educational Technology, Bharathidasan University,Trichy, and Sri.R.Athmaraman, Former Secretary, Association of Mathematics Teachers of India and a teacher committed lifelong to mathematics education, for academic support. I also thank Mr. Adhithya Rajasekaran,USA, a student contributor in this issue, for technical support in the launching stage of the project.

Dr. D. Raja Ganesan
Editor

REACH OF THE MIND - A CITIZEN'S QUEST: REFLECTIONS ON MATHEMATICS AS A TOOL KIT

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Abstract

Education is the enactment of an endeavor. The endeavor is scientific, leavened by artistry and enhanced by reflection. It undergoes subtle transformations. Society stands to gain immeasurably and durably in all spheres by investing in such an education. A reflection on Mathematics, as a versatile tool-kit for enabling education with an array of varied competences, is a natural corollary. We begin with numbers and go on to algebraic symbolism, geometrical proofs, trigonometric measures, econometric models, statistical analyses, random selections and so on. And, get lost!

This paper is a citizen's provocative quest for retracing our steps from the quick sands of lapsed educational detours! There is no tabulation to this soliloquy crawling toward such a quest, thin on pedagogic ruminations and thick on the perplexities, tensions and aspirations of a seeker. Engaged with arithmetic as the spine of learning and venturing to cut across the concentric circles of learning, through the linking chords of reflections thereon, this paper reviews a classic treatise on arithmetic, ventures to explore the application of John Dewey's educational philosophy, E. Thorndike's Law of Recency, Lancelot Hogben's "adventure" and Donald A. Schön's Reflective Practice, to the art of reaching mathematics to all. Throughout the discussion a global perspective is borne in mind.

Keywords: *education, citizen's perspective of math education, arithmetic, reflective practice, mathematics*

1. Opening Salvo!

At the outset, I place before the readers, the following comment of Hoffman, found on the back cover of his famous book, *Archimedes' Revenge* (Hoffman, 1988):

"*...But Mathematics is most of all a world of people, of Srinivasa Ramanujan, the great Indian Mathematician, who emerged from a tiny Indian village to dominate the great world of Cambridge...*"

Second, I wish to impress upon the readers, the mantra of Jean Le Rond d"Alembert (French Mathematician, (1717–1783): **"***Allez en avant, et la foi vous viendra***"** ("Go ahead and the faith will come to you!").

Exercising the mind is a continuous and constant process, almost always in vain and meandering pursuit of myriad imaginations. Seldom do we reach and reap from the inner recesses of the mind. Knowledge is a robust pursuit, most often guided (and, stalled!) by conventional schooling. Education is obsessed with its own borders, which are barricades, in reality. This leaves us in a limbo, when on the verge of adulthood. Most adults have not only stopped learning, but have forgotten whatever was taught to them. While this stunting of acquired knowledge is manifest in many spheres, it is appallingly so in the field of mathematics; "*people take a perverse pride in mathematical ignorance*" (Paulos, 1988). The *bonsai* of the knowledge-tree is a direct consequence of aversion to endowing mathematics with an ecumenical approach. The resultant deficit in organized knowledge, informed orientation, and applied science is a human calamity.

We are swayed by muddled surveys, hit by misdirected government policies and we mishandle own personal finances. In short, we lead ourselves astray. Worse still, we subjugate ourselves to false prophets: all because, the mathematics nail had not been shod.

As a person who stays in touch with people from different walks of life globally and who spends time with children, I could see innumeracy dogging the footsteps of the human learning curve, from the cradle to the grave, in most environs, globally speaking. This corrosive turn of events borders on a tragedy. Many scholars have articulated this concern. For instance Williams (2012) said: "*mathematical ability helps provide the disciplined structure that helps people to think, speak and write more clearly*." We need that structure in all walks of life. To cite Paulos again, *"Probability, like logic, is not just for mathematicians anymore. It permeates our lives."* (Paulos, 1988: 134).

Herein, I cite vignettes from South Africa, India, USA, the UK, and New Zealand, as an *ouvre*.

1(a). South Africa

The South African vignette borders on the Gandhian and takes precedence. A silver lining in the darkest cloud, it is the best salutation unto Srinivasa Ramanujan. Saul (2001) reflects in his *A Distant Mirror: Mathematics Education in South Africa* over his encounter with Theo Mokgatlhe, an oppressed fifteen- year-old high school student from the *apartheid*-ridden Orange Free State. Saul's host, Webb runs a successful system of national competitions in mathematics. Webb's teams, scoring higher and higher over the years, represent the country in the International Mathematical Olympiad. Theo, a rising star, has been invited to the summer camp. A silver lining, indeed!

1(b). India

The transition from the school to the college was the most traumatic one for me. Having obtained 100 percent in arithmetic and 95 percent in algebra and geometry in the School-leaving Public Examinations, I took to mathematics with confidence; my hopes were ruthlessly shattered by constant belittling, pretensions, and aggressive aversion-inducing antics. I speak about this with bitterness even six decades later, as the scar still hurts. My children suffered the same fate in high-end schools and one can imagine the pathetic condition in village schools. As a parent, I was involved in parent–teacher committees in most schools, to which my children went. Except in one, all schools frowned upon me, for suggesting more student-friendly mathematics. This curse is a defiant, persistent, and noxious weed even after seventy years, as I find Anivar Kannan, a third year engineering student, bemoaning in 2012, *"Theta, differentiation, integrals, probability— getting used to these terms takes a long time. We understand the concepts, but it takes a long time to register symbols and formulae. In exams, we lose out on speed itself...."*

Prabhakar Rajan (name changed) ranked high in the State, but, in vain. In college, he could not even understand a question, let alone answer it. Citing these tales of woe, Vasudha Venugopal and Aravind Kumar (2012) highlighted the harm done by rote learning and cited Robert Bellarmine, former English Studies Officer, British Council, in support of the view that most bridge courses are sympathetic to learners but not empathetic to them. Unhappy with the existing structures like the University Grants Commission and the All India Council for Technical Education, the Government of India is on the verge of establishing a National Commission of Higher Education and Research (Bhupendar Singh, 2012). One fervently hopes that it would not be old wine in a new bottle.

1(c). The United Kingdom

While I was glad of my prowess in handling simple quadratic equations while teaming with a child in her home work, I was gratified to find her more adept. The transition from the school to the college is less traumatic in the UK. First, there is no tantalizing bridge of medium of instruction. Second, teachers in the UK do not lord it over. Third, the steps are well-graded, even though the standards at every level are tough. The fourth reason is more important. Recognizing the need for providing national policy advice "on matters of 5–19 mathematics education", the Department for Education, the Royal Society, the Joint Mathematical Council of the UK, the Wellcome Trust, Gatsby, and a range of other organizations across the STEM (Science, Technology, Engineering and Mathematics) landscape, lend support to the Advisory Committee on Mathematics Education (ACME) set up in January 2002. The ACME 2012 Conference will discuss timely mathematics education issues inasmuch as "*there is no shortage of mathematics education policies to discuss, from the National Curriculum review to the A-levels and GCSE*[1] *reforms*" (ACME, 2012). Most importantly, there is increasing evidence of reflective practice gaining ground in the teaching of mathematics in the UK. One representative sample can be found in the book: *Theory and Practice In Mathematics Teaching Development: Critical Inquiry as a Mode of Learning in Teaching* (Jaworski, 2006).

1(d). United States of America

There has always been a persistently gnawing worry in academic and pedagogic circles in the USA about the deep-rooted presence of "*the notorious innumeracy of Americans*" (Siegeman & Niemi, 2001) not only in the educational trickles and cascades, but also in the stagnant pools of adult life, as lived. I trace this worry in modern times to 1922 when Thorndike came out with his *The Psychology of Arithmetic* (Thorndike, 1922). The mathematicians, however, began to articulate their concerns from 1947, when Zeisel (1947) came out with his much-reprinted classic manual, "*Saying It with Figures.*" The quest had begun a decade earlier in 1937 across the Atlantic in the UK, when Sir Lancelot Hogben came out with his trail-blazing "*Mathematics for the Million.*" As Albert Einstein put it, "*it makes alive the contents of the elements of mathematics.*" Yes. It still does.

The term "*innumeracy*" conveys a person's inability to make sense of the numbers that run their lives. This term was coined by Douglas R. Hofstadter, the cognitive scientist, for *Scientific American* in the early 1980s. It has since become a household word, thanks to the popular 1988 book, *Innumeracy* by Paulos (1988).

Brimming with anger at a society totally dependent on and cavalierly indifferent to mathematics, he explores and explains many numeracy concepts with felicity, including the Ramanujanian number: 1729.[2] The innumeracy worry is multi-fold in 2012, as only 32 percent of US students ranked proficient in mathematics in the latest international tests administered by the Organization for Economic Co-operation and Development (OECD).

Academia and the lay media are replete with hundreds of analyses that confirm that innumeracy poses a major national problem in the USA as of 2012. Imagine 71 percent of those polled by American Association of Retired Persons (AARP) believing that they did not pay fees on their 401(k)'s plan (a type of retirement plans in the USA)! For 2011, the average total administrative and management fees on a 401(k) plan were 0.78 percent or approximately $250 per participant (cited in Williams, 2012). Zeisel (1947) would bemoan that

[1] GCSE is the acronym for General Certificate in Secondary Education.

[2] Hardy used to visit Ramanujan, as he lay dying in hospital at Putney…Hardy, always inept about introducing a conversation said, probably without a greeting, and certainly as his first remark: "I thought the number of my taxicab was 1729. It seemed to me rather a dull number. " To which Ramanujan replied: "No, Hardy! No, Hardy! It is a very interesting number. It is the smallest number expressible as the sum of two cubes in two different ways." That is the exchange, as Hardy recorded, "the prolific ingenuity of Ramanujan can be seen in the fact that $1729 = 12^3 + 1^3 = 10^3 + 9^3$ (Snow in his Foreword in Hardy, 1967).

even when the figures speak for themselves, the populace is deaf and dumb!

1(e). New Zealand

Hannah Bartholomew *et al.* (2005) explored the issues surrounding the delivery of professional development to mathematics teachers in schools in low socio-economic areas in different regions of New Zealand. Finding "*the teachers relatively under-prepared for mathematics teaching in these environments, and isolated in their practice*" the researchers engaged them in reflective practice sessions and report that the community of practice, thus, formed by the teachers has gradually developed successfully through the project.

2. Indian Ink

"*…The ingenious method of expressing every possible number using a set of ten symbols* (*each symbol having a place value and an absolute value*) *emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of Antiquity, Archimedes and Apollonius.*"—–Laplace (http://www-history.mcs.st-and.ac.uk/HistTopics/Indian_numerals.html).

Jawaharlal Nehru (1946) wrote, "*The astonishing progress that the Indians had made in mathematics is now well known and it is recognized that the foundations of modern arithmetic and algebra were laid long ago in India….*"

In honor thereof and as a tribute to Srinivasa Ramanujan on the eve of his 125th birth anniversary, a time-line followed by a brief survey of the different realities in which the ancient Indian mathematical minds sojourned is presented. Young (1997) in his painstakingly compiled *Notable Mathematicians* furnishes the following time-line:

*c*500 BC: The *Sulvasutras*: A Hindu religious geometry text on construction of altars by rope-stretching. (p. 530)

- *c*250 BC: Ashoka Pillars with Number Symbols. (p. 530)

- *c*476–550: Arya Bhatta, the Elder: Evolved the Indian numerical system, found the accurate value for ð (pi). Algebraist. (p.499)

- *c*598–*c*665: Brahma Gupta: Authored *Brahma-sphittha-siddanta*. Applied algebraic methods to astronomical problems. (p. 628)

- 850 AD, Mahavira: Authored *Ganitha-Sara-Sangraha*, Noted for his Hindu geometry. His is the only known Hindu work on ellipses. (p. 534)

- c 1150, Atschiya Bhaskara (1114–1185): Authored *Siddantha Si'ronmani* and many approximations for ð (pi).

O'Connor and Robertson (2000) presenting *An Overview of Indian Mathematics* ascribe the architectural geometry for the altars attributed to the *c*500 BC *Sulvasutras* to the antiquity of the Vedic mythological texts in *Shatapatha Brahmana* and the *Taittiriya Samhita. Citing* Childe (1954) in *New Light on the Most Ancient East* that "*India confronts Egypt and Babylonia by the 3rd millennium with a thoroughly individual and independent civilization of her own, technically the peer of the rest...*", they attribute the composition of Sulbasutras to a stretch of 600 years (800 to 200 BC) and to the four authors, Baudhayana, Manava, Apatstamba, and Katyayana. They draw our pointed attention to the Sanskrit grammarian Panini.

Panini's ideas, they aver, are so modern as to be of *immense use* for a mathematician or computer scientist working with formal language theory. Jaina mathematics, in the petri-dish of cosmology, astronomy, and astrology, which flowered around 150 BC, covers the amazing array of *"the theory of numbers, arithmetical operations, geometry, operations with fractions, simple equations, cubic equations, quadratic equations, and permutations and combinations"* and not to be outdone, developing *"a theory of the infinite containing different levels of infinity, a primitive understanding of indices, and some notion of logarithms to base 2."*

Noting the imaginative Indianized translation of an old Greek astrology text in the second century AD by Yavanesvara with admiration, they pass on to record for posterity Aryabhata's remarkable astronomy and trigonometry and his Kusumapura Research Center. *"Brahmagupta is probably the earliest astronomer to have employed the theory of quadratic equations and the method of successive approximations to solving problems in spherical astronomy"*(O'Connor & Robertson 2000). Varahamihira's further contributions to trigonometry and astronomy made impressive gains for Indian mathematics.

Bhaskara I, leading the Asmaka School, concentrated on providing a commentary on Aryabhata and the nineth century saw a refreshing revival in the persons of Govindasvami, Mahavira, Prthudakasvami, Sankara, and Sridhara and in the developments in sine tables, solving equations, algebraic notation, quadratics, indeterminate equations, and improvements to the number systems. Aryabhata II and Vijayanandi worked on sine tables and trigonometry to support their astronomical calculations. The more we explore, the more astounding are the findings. For instance, *"the development of number theory, the theory of indeterminates, infinite series expressions for sine, cosine and tangent, computational mathematics, etc."* (O'Connor & Robertson, 2000).

I have extensively cited from O'Connor & Robertson (2000) as we find a survey of Indian contribution to mathematics, in a nutshell therein. And, from this zenith, we have fallen in 2012 into the nadir of innumeracy evidenced in Anivar Kannan's cries in the wilderness.

3. Plato's Cave

Elliot in his *Hollow Man* portrayed that "*Between the idea/and the reality/between the motion/and the act/Falls the shadow*." In a Socratic dialogue, Plato employs the powerful metaphor of the shadows being taken for the real by the cave-dwellers. Having seen only shadows and heard only echoes all their lives, they could not gauge them to be mere reflections of reality. Plato cleverly insinuates, with Socrates as his mouthpiece, that the shadows become the reality in their minds and that they would increasingly depend on the shadows on the wall.

Risking the academia rising in outrage, I venture to apply Plato's analogy of the cave to the global innumeracy problem. As a matter of fact, democracy has taken a beating in the hands of pretenders because of the average person lacking in mathematical acumen. The entire databank, on which the man in the street places reliance, is addled. Muddled thinking, as a result, misguides almost all human endeavors. It is worth our while to explore bye lanes of long-forgotten tutelage systems.

One such winding bye lane is the Indian *Guru–Shishya Parampara* (mentorship) exemplified centuries later by John Dewey's *"Right Kind of Telling"* and Thorndike's Law of Recency. Sir Lancelot Hogben, in his adventurous book called *Mathematics for the Million* has given an astounding model. Schön's trail-blazing "Reflective Practice" (Schön, 1991) and its impact on many disciplines, have since offered exciting pathways for pursuit of excellence in mitigating the ill-effects of innumeracy. This essay is, but, a mere signpost of where to look for.

4. *Guru–Shishya Parampara*

Bhaskaracharya's Lilavathi describes that *"Lord Shiva holds ten different weapons, namely a trap, a goad, a snake, a drum, a potsherd, a club, a spear, a missile, an arrow and a bow in his hands. Find the number of different Shiva idols. Similarly, solve the problem for Vishnu; Vishnu has four objects: a mace, a disc, a lotus and a conch"* (Nagaraj, 2005).

One could visualize the eagerness of the child, when the concept of permutations and combinations is introduced through puzzles like this, as in Bhaskaracharya's *Lilavathi*. The mentor relates to the chosen disciple in an intense manner in that tradition. He does not teach. He imparts knowledge and acts as the role-model. This textbook of arithmetic, geometry, trigonometry, and rudiments of algebra, excellent in methods, and intriguingly neglectful of proofs, is a living embodiment of the *Guru–Shishya Parampara*. Bhaskaracharya (1114–1185AD) was deservedly given the title of *"Ganakacakrachudamani"* which means "A Crest Jewel among Mathematicians." He was also an excellent teacher as indicated by the teasing and pleasing verses through which he tested his disciple's abilities to solve mathematical problems. *"Bhaskara was excellent at arithmetic, including a good understanding of negative and zero numbers. He was also good at solving equations and had an understanding of mathematical systems…"* (Nagaraj, 2005).

Legends apart, the *Lilavati* text is encased in a formal and sequential structure and is explicitly related to the day to day commerce, unlike the textbooks decried by Thorndike. The ritualistic invocation is followed by definitions of measurement unit, then by measures of gold, units of length, measures of grain in volume, and lastly the measure of time. Next comes the positional notation of digits and their values, closely followed by the methods of the four basic arithmetical operations, enunciated in an algorithmic fashion.

5. John Dewey (1859–1952)

The dawn of the twentieth century was an era of renaissance, indeed, in the USA. One of the foremost thinkers of that era, John Dewey, laid the cornerstone for the development of educational thinking, for urging informal education and for engaging with and enlarging upon experience. Most importantly, his passion for democracy is the umbilical cord of his educational philosophy. This passion, I reckon, should guide us in the task of mitigating the innumeracy problem, as democratic values are at stake. In the words of John Dewey, *"Learning involves…at least three factors: knowledge, skill and character. Each of them must be studied. It requires judgment and art to select from the total circumstances of a case just what elements are the causal conditions of learning, which are influential, and which are secondary or irrelevant. It requires candor and sincerity to keep track of failures as well as successes and to estimate the relative degree of success obtained…."* (Dewey, 1974: 181 as *cited* in Schön (1987: 313). We find the echo of *Lilavati*'s formal and sequential structure in John Dewey's knowledge, skill, and character and his emphasis on estimating the relative degree of success.

6. *The Psychology of Arithmetic* (1922)

Thorndike (1874–1949), the eminent animal psychologist, who spent nearly his entire career at Teachers College, Columbia University, is also one of the founding fathers of educational psychology. It comes as no surprise that he chose to write *The Psychology of Arithmetic*, with his characteristic thoroughness. He was already renowned for his puzzle boxes that made the animals respond, in a process not unlike the teasing *Lilavathi* riddles. Endorsing Dewey's functionalism, he contributed the stimulus–response component to his educational theory, which held sway for more than five decades. It looks as though the mathematicians' paradise took little notice of this book, which saw four reprints in quick succession. Such are the narrow confines of academia!

Thorndike held that learning was automatic and incremental and also that all animals learned the same way. Most important, he held out a practical tip to the teacher that the most recent response was most likely to re-occur. His central theme was that "*learning is essentially the formation of connections or bonds between situations and responses; that the satisfyingness of the result is the chief force that forms them, and that habit rules in the realm of thought as truly and as fully as in the realm of action"* (Thorndike, 1923: vi). Analyzing the "higher processes" of abstraction, general notions, and reasoning with his inestimable acumen, he challenged the orthodox pedagogy of sensations getting compounded into percepts, which got duplicated into images, which somehow got amalgamated into abstractions and concepts —*à la* Plato's Cave! He held the mathematical textbooks of his time in disdain—*so strong are mere use and wont."* (Thorndike, 1923:143). His insights are invaluable to our diligent pursuit of mathematics as humanity's life-line.

7. Mathematics for the Million (1936)

Sir Lancelot Thomas Hogben (1895–1975) taught to himself and the rest of the world, during a scintillating scholastic career. A versatile British experimental zoologist, he developed *Xenopus laevis* as a model organism for biological research, which led to the famous Hogben Pregnancy Test. Highlighting the "interdependence of nature and nurture" he attacked the eugenics movement. Imprisoned as a conscientious objector in 1916, he was the medical statistician of the Second World War. His everlasting service to humanity as "a scientific humanist" (in his own words) was his voluminous and lucid tutorials on science, mathematics, and language: *Mathematics for the Million* (1936), *Science for the Citizen* (1938), and *The Loom of the Language: An Approach to the Mastery of Many Languages* (Bodmer, 1944).

Hogben's papers are electronically archived. A gleaning arising therefrom viz., "…*Science educators should not just teach scientific facts, but present science as a practice and make students reflect on the nature of science, as this gives them a better appreciation of the ways in which intelligent design falls short of actual science"* (Ingo, 2012). This takes us to Schön.

8. Donald A. Schön (1930–1997)

Schön was an architect–philosopher. His seminars with Chris Argyris had won global acclaim for investigating the theme, "*learning as the detection and correction of error*" (Argyris & Schön, 1996).

Schön set the tone for Reflective Practice by integrating nuggets of wisdom from the past (Schön, 1983):

- Comte on the political and moral terrains of technology (p. 32)

- Rousseau's and Tolstoy's influence on Dewey's conviction that teaching required "not a method but an art" (p.65);

- Bacon on "commanding Nature to obey her" (p.163);

- Ivan Illich on the mystique of technical expertise acting as an instrument of social control of the have-nots (p. 288);

- Karl Mannheim's sociology of knowledge (p. 312);

• Gilbert Ryle on how procedure distinguishes "sensible" from "silly" operations in the intellectual and practical spheres (p. 51);

• Whitehead's distinction between a profession and an avocation (p.22).

Argyris and Schön, as a sequel, set up two models that describe single-loop learning that inhibits and double-loop learning that liberates.

Model I leads to deeply entrenched defensive routines (Argyris, 1990; 1993), impairing our potential for growth and learning and this applies to the individual, groups and organizations. Model II aims at sharing and participation and at guarding against, if I may term it, "received wisdom." Herein is the clue, I submit, to the reordering of the teaching of mathematics. Among the numerous reflective practice frameworks and models that have since emerged, the model of Boud, Koegh, and Walker for turning experience into learning (1985) and Kolb's experiential learning theory (1984) could help us in mitigating innumeracy problems.

9. Drama in Real Life

The very first move favoring mathematics is that of substituting joy for terror, a task admirably performed by Shakuntala Devi. She worked out the twenty-third root of a given 201-digit number in 50 seconds. A Univac 1108 computer took 10 more seconds and 13,000 instructions to confirm that she was right. Intent on "arousing the active state mind" (Shakuntala Devi, 1977: 157), she delights us by her 'secret steps" by infusing life into the 10 digits and by rendering calculations, painless. Puzzles had always been adding joy to mathematics. One sure way of ensuring retention of mathematics is to make it come alive, as in *Leelavathi*.

Closure

At this point, I am reminded of Margaret Mead, who convinced us with: "*Never doubt that a small group of thoughtful, committed citizens can change the world.*"

Reflections on mathematics, as the versatile tool-kit for enabling education with an array of retained competences, have come to us from a philosopher, a psychologist, a zoologist, and an architect and it is a citizen's quest. New Zealand is not an isolated case of "*the teachers (being) relatively under-prepared for mathematics teaching in these environments and isolated in their practice.*" (Hannah Bartholomew *et al.*, 2005). The problem is ubiquitous. I risk the ire of the academic community by highlighting innumeracy as a global problem in my pursuit of a noble cause. The elegance of the set of 10 symbols, each having a place value and an absolute value, is like a thousand-petalled blossom. Laplace did not praise the Indian ingenuity in vain. *Leelavathi* is our secret treasure, to be globally shared. Arithmetic is but the edifice of mathematics. No science can employ John Dewey's desiderata (knowledge, skill, and character) with more telling effect, than mathematics. Nor can Thorndike's formation of connections or bonds between situations and responses be more refreshingly demonstrated by any other discipline. As Argyris and Schön had demonstrated elsewhere, defensive routines are our barricades; and, participation is our gateway to liberation. Margaret Mead, the pioneer anthropologist, had rightly heralded the co-option of thoughtful, committed citizens. That finds me here.

The overarching role of ancient Indian mathematics has also impressed upon us the enlightenment by cultivated scholarship. While John Dewey made us think, Thorndike made short work of set patterns. Hogben's instructions for the readers of his *Mathematics for the Million* are worth paraphrasing here. Rejecting the demonstration of logical sequence of the steps, he gunned for the historical sequence. Citing the Scottish mathematician

Chrysthal, he arranged his tutorial such that it could be read backward and forward. "Go ahead and the faith will come to you." is the veritable *mantra*.

Schön is a refreshingly welcome addition. His reflective *practicum* has yielded rich dividends in various disciplines and had helped in upgrading the teaching of mathematics in the UK and elsewhere.

In sum, a duty is cast on teachers, mathematicians, and academics from other disciplines to work together, particularly in close companionship with the citizen, to wean us all from "*mere use and wont*" and to go ahead with the "adventure" inherent in *Mathematics for the Million*.[3]

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[3] Online courses and tutorials are often countered these days. However, these courses deserve a critical appraisal on how they provide mathematics for the millions.

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The Association of Mathematics Teachers of India

(Registered body founded on 27th June 1965 and registered as SI43/1965 under the Societies Registration Act XXI of 1860 on 29 October 1965.)

The AMTI is an academically oriented non-profit premier organization of professionals and students interested in mathematics and mathematics education at all levels. Great mathematicians like Dr. A. Narasingha Rao, Prof P.L. Bhatnagar, Dr. J.N. Kapur, Prof K.R. Parthasarathy, Prof R.C. Gupta and (presently) Dr. Rajendra Bhatia have served as presidents of the AMTI.
Some of the main objectives are:

- To assist practising teachers of mathematics at all levels in improving their expertise and professional skills to make mathematics interesting and enjoyable.
- To spot out and foster mathematical talent in the young.
- To disseminate new trends in Mathematics Education.
- To offer consultancy services to schools.

PUBLISHES:

- The official Journal of the Association “The Mathematics Teacher (India)”- quarterly, published in English and is sent to members free twice a year now.
- The Journal for students - JUNIOR MATHEMATICIAN - is published tri-annually in English before the commencement of vacation(s) and is supplied only to the subscribers through the respective schools, wherever possible.
- Several books on Questions and Solutions for Olympiad type tests, Innovative teaching methods, Recreational mathematics etc. , have been published and are available for purchase.

CONDUCTS:

- National Mathematics Talent Contests. (NMTC)
- Exposure Programmes for talented students.
- Orientation Courses, Seminars and Workshops for teachers. Topics include, among others, Algebra, Geometry, Combinatorics and Number Theory; deliberations are on concepts as well as teaching methodology. (Many of these are residential).
- National Conferences *in different parts of the country* to meet and deliberate on important issues of Mathematics Education. The recent 47th Annual conference was held in Andhra Pradesh. **Delegates get a good opportunity to present papers, exhibit mathematical models, participate in group discussions, interact with mathematicians from different states of the country.**
- Sponsored programmes: The recent ones were Workshops for teachers held at Chennai, Bhubaneshwar and Guwahati, sponsored by the Department of Science and Technology.

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MEETING STEM TEACHER SHORTAGE INTERNATIONALLY

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Abstract

International professionals are the order of the day in a borderless global village. In US schools, the Common Core Standards Initiative and the Science, Technology, Engineering, Mathematics Initiative are being strategically implemented to prepare students for a global work force. At this propitious time, international teachers are emerging as a vital component of US school systems. In many cases, they have a higher level of content area expertise and dedication to work; thus they are providing to US students a snapshot of the global picture and various cultures. In this paper, some perspectives of employing international teachers to teach our children in US schools are discussed and possible suggestions are given for the prospective teaching professionals to help prepare them to serve in US schools.

Key words: *international teachers, human resources, stem teacher shortage, recruitment, teacher quality.*

1. Introduction

For the past three decades, the world has been going through the process of globalization, one that causes increasing economic, financial, social, cultural, political, market, and environmental interdependence among nations. Regardless of the definition of globalization, today's scenario reveals a mundane truth: the world is continuously changing. Education, as well, is inevitably affected by this process of change towards more interdependence.

One important off shoot of globalization in today's world is the emergence of evolved forms of interdependency (Elkus, 2010). One such evolved form is the global human resource (HR) needs. Robert *et al*. (2000: 94) delineates HR functions into meeting HR needs as follows:

1. Skill deployment: getting the right skills to where they are needed irrespective of the geographical location;

2. Information dissemination across all locations and talent identification and development on a global basis.

Given that the world has shrunk to a borderless global village, a time has come when countries look forward to preparing their children to the demands of the new world set up. For instance, in the United States, there has been much brainstorming about enhancing expectation levels in the kindergarten through 12th grade (K-12) schooling, on par with international standards. Common Core State Standard Initiative (CCSSI; www.corestandards.org) is an effort designed to improve educational outcomes for students by developing a set of common, voluntary, internationally benchmarked K-12 standards for college and career readiness in English language arts and mathematics. In fact, CCSSI is a first step toward leveling the playing field for the nation's students by increasing rigor in curriculum and instruction and setting equal expectations for all students. This will be an effort to get all kids in the United States to focus on learning the same material by grade level, by subject matter, in alignment with other successful education systems in the world.

Additionally, since 2000, there has been an increased understanding and support by many federal, state, and local policymakers about the importance of strengthening US STEM[1] concepts and skills in the pre-college (K-12) education curriculum.

Both CCSS and STEM initiatives are considered very vital because they are purposeful and are based on the characteristics of the evolving careers for our students who graduate from our schools and subsequently from colleges. "As the workforce becomes increasingly more global and technology-driven, it is essential that the United States align its K-12 core curriculum to the expectations of its 21st century workforce, ensuring that its future leaders remain competitive in the global economy" (ASME, 2010).

In driving the intended changes envisaged by both CCSS and STEM at the local level in individual schools, the most indispensable single factor is teacher effectiveness. We really need teachers who are not only good teachers but effective teachers who can enable students to meet the expectations set for them via both CCSS and STEM initiatives. In the twin perspectives of global workforce and global economy, it is a good option to employ global teachers to teach US students and this is a veritable case of dependence in US schools on other countries.

Professional dependence has a simple beginning. For instance, in a true business setting, whenever firms feel that it is less easy to "do it all and go it alone" (Ohmae, 1989), they go forward to cooperate with other firms. Through cooperation, firms tend to capitalize on and leverage their resource needs by capturing global opportunities (Adler, 1966; Robson & Dunk, 1999; Varadarajan & Rajaratnam, 1986). A search for prospective sources for US HR needs in teaching reveals that many countries have highly qualified and trained teachers to teach the subjects in the critical need areas such as special education, science, technology, engineering, and mathematics.

In this paper, the perspectives and current initiatives in employing international teachers in the United States will be discussed. Because math is an integral component to STEM and is closely linked to science, technology, and engineering, all facts and factors that are presented herein will apply to math, if not made explicit.

2. Are US Schools Prepared to Employ International Teachers?

The extent to which the HR availability of one country or culture could be translated into another country or culture is a subject of considerable importance (Adekola & Sergi, 2008). It has been of significant importance in American educational HR management (HRM) as well. Consequently, much thinking was devoted to harnessing the potential of teaching expertise of international teachers in American schools. Feasibility of employing international teachers in US school systems is fundamentally supported by some positive factors.

Foremost, the United States of America thrives on diversity. Herein you find a synthesis of the world's plentiful and varied races, religions, and cultures. Yes. America is a home to all, such that no one group can call itself more "American" than another. Particularly, the cultural regard in America is so exceptional that citizens can be just as proud of their original cultural heritage as they are to be an American.

Second, we have an established precedence in higher education of having employed a large number of teaching faculties from different countries. In fact, in some subjects, there are more international professors in US college classrooms than native professors.

Third, we have a shortage of teachers in critical needs areas such as special education, science and math.

[1] STEM Education is the acronym for Science Technology, Engineering, and Mathematics education.

A consideration of these three factors is evidently in favor of hiring international teachers in US schools. In addition, there is a more serious need. This is of the education of English Speakers of Other Languages in US schools. Recent estimates have shown that the number of Hispanic students in our schools is steadily increasing: as of now, they constitute 22 percent of all K-12 students (Shrestha and Heisler, 2011). However, they are not receiving the full benefits of education here because of language, cultural, and social barriers. The White House Initiative on Improving Educational Opportunities and Outcomes For Latino Students established in September of 1990 calls for focus on issues related to Hispanics and to address academic excellence and opportunities for the Hispanic community. In this context, it is worth mentioning that there is much literature concerning methods of improving the schooling success of "non-traditional" students in US schools. Such literature indicates that "traditional" educators lack the ability to deal with classroom cultural diversity effectively (Nelson, 1996).

Once this has been recognized, the next step is to find out various ways to address our ability to arrange for teachers who would teach effectively in the light of the cultural diversity of our students (Nelson, 1996; Wang and Oates, 1995). Some educators have devised innovative approaches to teaching diverse student populations. Ofori-Dankwa and Lane (2000), for instance, suggest employing what they call the "diversimilarity" approach. Diversimilarity involves exposing students to both similarities and differences of cultures. This suggestion brings out an important point: international teachers would prove to be real-time veritable ambassadors of diversimilarity in the process of educating the minority students in US schools.

3. Strengths of International Teachers

It is now understandable that some of the differences in the perceptions of international teachers, whether cultural and/or educational, are really strengths that could be tapped for the benefit of the diverse classrooms of US schools.

International teachers possess several strengths that characterize them as teachers with an enriched teaching style (Kumek, 2012):

1. Strong attraction for technology integration in teaching
2. Dedication to work
3. Mastery of different crafts
4. Depth of content knowledge
5. Expertise in problem-solving method of instruction
6. Their foreign background and diverse experience
7. Readiness to accept extra responsibilities
8. Multitasking nature
9. Desire to train students for Olympiads and Science Fair Competitions.

So it is not surprising that considerable thinking has also been devoted to explore how to create a pathway for the hiring of international teachers in American schools.

At the same time, it was of importance to see that there is substantial standardization in the identification of pre-determined needs and the corresponding HRM practices in total compliance with the legal requirements.

4. Outsourcing Yet Different

"Creating the next generation work environment—highly collaborative and capable of not just fostering, but also encouraging the instant, seamless movement of ideas and expertise— will present both intellectual and technical challenges for us as professionals" (MacDonald, 2003: 262).

While the intellectual accommodations are paramount, the technical challenges associated with hiring international teachers deserve a special mention. Traditionally, a small number of teachers have been regularly coming to the United States to teach in our schools on cultural exchange programs; however, their number as well as the duration of their stay was not sufficient to cater to our needs.

Therefore, it becomes necessary that teachers with credentials to work for US schools must also possess a permit to work as a teacher in the United States. Typically, what they require is a work visa, to work in the US schools as a teacher in the school system. A visa is a legal government document which shows that a person is authorized to enter the territory for which it was issued, subject to permission of an immigration official at the time of actual entry. The actual authorization is a stamp endorsed in the applicant's passport. A US work visa typically attaches various conditions of stay, such as the employer, territory covered by the visa, dates of validity, period of stay, etc.

However, the visa process is a time-consuming, laborious, and cost-involving process. In addition, arranging for a visa is a non-core activity for the local Board of Education and these entities do not like to assume this burden. While this is the overarching sentiment a few school systems prefer to directly sponsor visas for their incumbent international teachers (for example: some school districts in Colorado, Florida and Virginia).

Outsourcing is contracting with another company or person to perform a particular function. Almost every organization out sources in some way. Typically, the function being outsourced is considered non-core to the business. A publishing company, for example, might outsource its janitorial and landscaping operations to firms that specialize in those types of work since they are not related to the core activities of publishing business. The outside firms that are providing the outsourcing services are third-party providers, or as they are more commonly called, service providers. The hiring of international teachers is more or less like outsourcing. However, it is different from outsourcing in the sense that the services of the international teachers are core to our business—our business of educating our children. Therefore, this is outsourcing with a difference, given the direct correlation to the core business of schools.

Collaboration is a key factor in the process of seeking international professionals. A host of IT professionals are hired through collaborative arrangements via intermediary employers, who are the authorized work visa sponsors. So, in our effort to hire international teaching professionals into US public schools, three key questions emerged:

1. Are there international teachers equipped with necessary skills to teach children in US schools?
2. Are international teachers willing to move to US to teach our children?
3. Are there visa sponsors for placing international teachers in US schools?

While seeking answers to these questions, we found that we do have the extraordinary functionality to network with companies namely, the mediatory employers who provide highly qualified math, science and special education teachers to US schools.

However, it must be mentioned that besides fulfilling the certification requirements, international teachers have to go through the rigorous process of interviewing, and if selected, post-interview processes and necessary paper work involving legal documents through the employer or the respective visa sponsor.

5. Transition and Teething Problems

Despite their strengths, international teachers do face some challenges. Such challenges mostly emanate from their

1. Separation from their families and or extended families.
2. Language incompatibility (dialect and accent)
3. Cultural barriers
4. Social adjustments
5. Adjusting to the lack of respect for schools from some students
6. Inability to loom with the rest of the faculty and staff in the school
7. Rigorous teacher evaluation process
8. Job insecurity

Teachers from India and China are said to face a cultural shock as they land in US and further as they enter the US classrooms (Alder, 1997).

In between the two, the international teachers are usually disadvantaged by the hiring process they are subject to in the United States. During their interviews, they did not know many of the commonly accepted rules. International teachers might not seem confident due to their cultural humbleness. They sometime do not exhibit or understand common nonverbal cues. In these cases, some US administrators are not taking into account of the context of the culture of the international teachers and rate them as unacceptable, (Kumek, 2012).

Also, there are substantial differences in the educational systems of the United States, India, China, Turkey and Nigeria. The educational differences are said to constitute the core of the international teachers' challenges in the American schools. Such differences include:

(1) Differences in motivation of the students to education.

(2) Difference in the value education has in the society

(3) Differences in the nature of the special education programs and related services

(4) Demand on different types of instructional delivery

(5) Differences in grading procedure

(6) Differences in the academic level of students

(7) Differences in student–teacher relationships

(8) The provision for gifted education

(9) The provision for alternative education

The manner in which the international teachers get over these crises and successfully pass out of the initial high-stress periods varies from individual to individual. For instance, Indian teachers from southern states get over the accent problem more quickly than those from the northern states. Similar differences can be found in other areas of difficulty.

6. Teacher Preparation is the Key

Teaching in US schools is easier said than done. International teachers have very high content knowledge and have good teaching skills. More importantly, they have an extraordinary passion for teaching. However, this is not enough. In order to teach and be successful in teaching US students, additional skills are required. Given that teachers are emerging as global professionals, and are required to serve at a global level akin to the IT professionals, how they are prepared for the job is more important than their content knowledge and/or level of education. Teacher preparation programs should address these teacher needs. It is important to rethink the content of college-level teacher preparation courses so as to accommodate the necessary ingredients of student issues in international settings as well as the comparative analysis of educational structures across the globe.

Second, the visa sponsors of international teachers are required to take a proactive role in truly preparing the international teachers for teaching on a global scale. Visa sponsors are actually doing the job of global staffing. Therefore, they are required to professionally prepare the teachers for international assignments. This includes imparting a range of skills in: personal, interpersonal, professional, on-the-job, off-the-job, and legal matters. An effective training would be the one based on typical case studies and role play, with special focus on American non-verbal cues, colloquial conversations (including a good knowledge of versions of slang, at least to comprehend the context). Some prior exposure to US schools and US classrooms would also be a great asset to the international teachers. These prerequisites are essential but they can be fulfilled only by the visa sponsors.

Among the various approaches to international staffing, the integrative, polycentric, and regiocentric approaches are considered to provide the best practice options (Taylor, 1993; Hill, 2005). It is important for the visa sponsors of teaching professionals to follow the best practice approaches (Minchington, 2010) in training the teachers before they are placed in the US schools. These employers must carry out a SWOT[2] analysis for every individual teacher they sponsor and make a careful assessment of each teacher's individual strengths, weaknesses, opportunities and threats. It is paramount for them to polish and hone their teachers' individual skills for full deployment. Similarly, areas of weaknesses have to be clearly identified and addressed appropriately through remediation processes. They are also required to sponsor the teachers for various professional development opportunities. More importantly, the teachers need to be very effectively trained for meeting the challenges of the American classrooms.

Conclusion

Among the international teachers in the United States, Indians are in sizeable numbers; they teach math and science, or are special education teachers. There is an overall finding (based on their overall annual performance evaluations) that international teachers are doing great in their jobs. In consideration of the fact that our experience with international teachers is not even a decade old, it is hoped that we will have more data in the offing.

On the part of the schools, international teachers are provided with mentors and are encouraged to pursue various professional development courses. At the same time, it is also understood that they face some challenges at least in the initial years of teaching in US schools. However, we do not have systematic studies or data relative to the said challenges. Kumek (2012) has done pioneering preliminary studies on international teachers in US schools. However, more data is necessary for any action plan. It is important to recognize that many of the professional challenges faced by international teachers are purely related to language, cultural, and social barriers to acceptance both as colleagues and as teacher (Dinsbach *et al*., 2007). Therefore it is imperative that administrators play a proactive role in seeing that international teachers are accepted and respected by other teachers and staff. This can be best achieved by developing avenues for trust, cooperation, and support amongst and in between each others at school. One example of such an avenue is ensuring that teachers work collaboratively in highly functioning professional learning communities. Mutual trust, goodwill, and support will be fostered if American teachers and administrators create opportunities, events, or programs (both educational and general) that create ways of understanding the cultural and personal distinctions, and social habits of mind that international teachers possess. Just as we want inclusive classroom, inclusive staffrooms are a must. All of this calls for very innovative administrative strategies.

[2] SWOT analysis comprising the analysis of strengths, weaknesses, opportunities, and threats was invented at Stanford University by Albert Humphrey. It came as a result of a research project in the 1960s and 1970s that used data from Fortune 500 companies.

To enhance the level of acceptance from the students, elements of diverse/similarity of cultures should be encouraged in all subjects of curriculum in all possible manners. Periodical casual visits from administors into the classrooms of international teachers, and talking to students about their merits and the note worthy features of their social and academic experiences would go a long way in catalyzing the establishment of a bond between the students and the international teachers. This is a vital means of support that administrators must provide to international teachers.

It is worth mentioning here that there is a significant shortage of highly-qualified math and science teachers in the US public school system, the major provider of education to US children. The same is true of private schools, though they are lesser in number compared with the public schools. Over the last decade, the United States has begun making an earnest search for teachers over the English-speaking world. It is a great revelation that teachers of high caliber are available internationally and it is also equally pleasing to learn that they have a desire to teach in American schools to educate our children. When they land in the United States, they bring in with them the rich culture of their native country. This is yet another benefit of the search for international teachers, along with the opportunity to build lasting, diverse personal and professional relationships.

If you're a skilled teacher and if you want to teach in the US schools, you'll need to collaborate with those who are here already and get to know the nuances of teaching in the US classrooms. It is much better to arrange this before you put your application through the rigorous visa and related screening process, to save time, cost, and disappointment. Excellent international teachers with an interest in teaching in US schools should network with others to learn about the companies and local school districts that sponsor international teachers to teach in the United States.

To further your interest in teaching US students, it's a good idea to make a list of school systems who might be interested in your area of expertise, along with their requirements. It will also be paramount to conduct a self-evaluation of how well you meet these requirements. Finally it is important for a potential candidate to outline a personal professional growth plan as a means of addressing areas you identify during your self-evaluation process, as needing improvement.

All schools, whether in the United States or abroad, promote creativity and innovation when teachers have the frame of mind to devote time and resources to enable our students to experiment, create, and question, using technology or just tinkering on their own, or completing an original project. As teachers, we are aware that creative thinking grows not from guessing the right answer but from asking questions and exploring further. In no instructional setting is such an approach to teaching and learning more critical than today's STEM and special education classrooms across the United States; and international teachers provide yet another option in our ongoing quest to provide a high quality instruction in our nation's schools.

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About the Author

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A CHEMIST'S QUEST FOR MATHEMATICAL TONE

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Abstract

Many advances in the frontiers of research in chemistry have remained paralyzed for a long time for want of maths ingredients. Chemistry students lack essential math skills that are hampering innovations and breakthroughs in chemical research. In this paper, examples are provided to illustrate the influence of math deficits in chemistry mastery and the math phobia in chemistry students. A suggestion is provided for developing an Integrated Curriculum Model for designing appropriate math-laden chemistry content for chemistry courses to enable students to meet the challenges of the emerging world. At a time when India is celebrating the National Year of Mathematics, the math message must reach chemistry educators effectively. We need simple yet great personalities like Ramanujan to take charge of the situation.

Key Words: *math deficits in chemistry, connection between math and chemistry, integrated curriculum model, mathematical–chemistry or chemical–mathematics content knowledge*

1. Introduction

Students often choose chemistry as a safe refuge inside the "archipelago of science disciplines"[1], because they think mathematics in chemistry can be tolerated or managed. I am not surprised at this defensive path of students for there are reasons. Foremost, they are not confident to do physics as major because of its pure math orientation. They are not willing to go to biological sciences for the fear of lack of employability options. So they consider chemistry as a good option because they find that chemistry involves only some math and not a whole lot. However, all is not well with them as they get started with the chemistry course. Most of them really find math of chemistry hard, especially as they advance in the chemistry ladder to postgraduate programs and beyond.

Negative attitude of students toward mathematics is not unknown. I illustrate this statement with my own case. In my career as a chemistry educator at the Indian Institute of Technology Madras (IIT Madras) for the past four decades, I began with the attitude that every student who walks into IIT has a reasonable strength in math and is poised to take upon chemistry in its full ramifications from the biology-oriented natural chemistry through the neutral organic chemistry and caustic and inorganic chemistry to the math-rich frontiers in physical chemistry. For some years, at least, I was quite convinced of this expectation on my students, because all of them have graduated from top-ranking high schools, have gotten very high grades, and have all passed the IIT Joint Entrance Examination, and also have other credentials to boast of.

Later, however, I realized that I had a utopian idea of the entry level profile of my students. I was able to find for myself that during their struggle with studies by rote learning, the students have not gotten the live common thread in the sciences and between

[1] I am inspired to use this phrase from Sorgo (2010).

sciences and math (Baram-Tsabari & Yarden, 2009). In fact, I was myself a student of this category, when I graduated from school. I did make a personal transition from mathematics ignoramus to a user, if not admirer, only from my own trial and error pursuits, and my deliberate learning from the experiences of others. It is therefore a veracity that the transition in me did not happen as a moment of ecstatic enlightenment but progressively and slowly over years of work in the classroom and in the research lab both as a student and as a teacher.

Mathematics curricula mandate expectations for students to be able to simplify expressions, calculate and find solutions, solve limits, functions, derivatives, integrals, and infinite series, etc., to solve a variety of equations, define extremes and plot graphs, convert one trigonometric function into another, and many other skills just to pass the final examinations.

To pass the chemistry examination, students must know that hydrogen behaves both as a metal and non-metal, that sodium ions are very stable, understand why in the sea weeds, iodine is enriched, how in the Minamata disaster, cats, fish, mosses, algae, sea slug, and marine vascular plants were involved, what is laughing gas and when it can be anesthetic, and to be able to compare and contrast photosynthesis with photovoltaic power generation. However, the practical applications of chemistry involve not merely the theoretical knowledge but very many mathematical computations and evaluations as to how to design a reactor, how to optimize the reaction conditions, how to model the process, how to scale up the yield, etc.

The point I wish to stress is: standard curricula and syllabi make very limited connections between chemistry and mathematics. On the basis of the existing chemistry curricula, a very successful chemistry teacher only needs to know how to calculate percentages, construct graphs from the tables, and calculate half-life and reaction constants in reaction kinetics. On the other hand, an excellent mathematics teacher needs to know that radioactive disintegration is an exponential decay and how to calculate matrix and permutation groups in group theory, and derivatives in quantum mechanics. But this is not enough from the practical standpoint. *Teachers have to be extraordinarily proactive and possess the ingenuity to sense the math perception/ skill as an internal ingredient of chemistry and offer them both indistinguishably and simultaneously as a one-on-one recipe.*

However, teachers neither know nor are eager to know how really math must be incorporated in chemistry. They do not even make an attempt to learn to how to *seamlessly* infuse math components in chemistry concepts.

Teaching practices of this sort have remained in vogue for a long time. Eventually, what we see today is: very many advances in the frontiers of research in chemistry remain paralyzed for a long time for want of math fillip in appropriate measures.

In this paper, I endeavor to present practical examples of the intra-chemistry growth retardation owing to the lack/absence of math nutrients. Then, I propose to suggest a methodology for possible implementation in the educational process for connecting math with chemistry.

2. Math is the Language of Chemistry

Progress in science depends on the language one uses to formulate and expose the principles. It becomes obvious if one considers the equation for energy as $E = mc^2$; if the same concept were to be explained in words it would require multiple formulations and they may still not be complete. In this sense, mathematics becomes a form of language for the scientists though not for mathematicians. This realization has not been properly conveyed in our teaching methodologies and hence many students shy away from the study of mathematics which is an indispensable component of science education.

Mathematics can render concepts of science and technology easily understandable and comprehensible. In addition, mathematics makes it easy to define and state the basic concepts of science and engineering in a simple and concise manner. Without mathematics one would have to exhaust reams of paper to explain concepts like wave functions, spectroscopy, and other areas of science and technology. Consider if one were to explain the Stokes and Anti-stokes lines observed in *Raman Spectrum*: It is stated simply in mathematical form as í ± Äí,[2], if the same were to be explained non-mathematically that is, in words, any student will find it extremely difficult to comprehend..

3. The Math Struggle is Current and True

In this section, I present some examples from chemistry that illustrate the math-dependency of chemistry.

3(a). THE WAVE FUNCTION

There are two functions that dominate the quantum mechanical description of the behavior of electrons in any atom or molecule. They are conventionally termed as *Eigen Value* and *Eigen Function.* These are the two solutions that are obtainable from the so called Schrodinger *Wave Equation.* However, the solution for this equation is obtainable only when some trial wave function is plugged in and the equation is solved until self-consistency is reached. These days, this is done with some approximations and these are termed as *basis sets* which describe in some measure the contours for the behavior of the electron with respect to the nucleus under consideration. These can, at best, be imaginative contours and must be tracked in terms of mathematical functions— usually some kind of exponential functions. Chemists tend to believe that the wave functions or the contours drawn out of these solutions are in the real space in which the electron is found. This is considered to be so within the various approximations made in obtaining the solution. However, the solution can be just as good as the mathematical exponential functions used as trial wave functions but they cannot be the exact representation of the behavior of the electron in the system. However most often this is not realized and one tends to think that he/she has traced the real behavior of the electrons in the system and to interpret the subsequent behavior of the molecule accordingly. Most often this may become true in predictive exercises but not in reality.

3(b). GRAVITATION OF THESES IN CHEMISTRY TOWARDS NON-MATHEMATICAL TOPICS

We have another quickest and direct example for math lacunae in chemistry education. It is the qualitative levels of the PhD theses that are generated in Chemistry. Most of the countries have a good crop of PhDs in Organic and Environmental Chemistry; India is one such example. PhDs in other equally important branches of chemistry, namely theoretical, physical and even materials science are scarce especially in India because the PhD scholars are unable to cope up with the math demands and hence their contribution in these branches are wanting.

3(c) FUEL CELL STAGNATION

Math deficit in chemistry directly reflects in the innovations that arise and can arise in various branches of chemistry. One such example is the case of fuel cells. Though fuel cells are known for over 150 years now, successful deployment of this technology has not reached the desired levels simply because scientists are still not able to treat the electrode/electrolyte interface in quantitative terms taking into account simultaneously the effect of electric

[2] This means that with respect to the resonance line, one observes Stokes and Anti-stokes lines on both sides.

field, field-induced adsorption and also the solvation that will necessarily take place in the electrolyte. In fact the various models (Srinivasan, 2006) of this interface have only recognized qualitatively the presence of solvation and could not treat the same in quantitative terms. If this understanding and treatment have been rendered possible, energy conversion processes would have been in place and the world would have been spared the looming energy crisis.

3(d). THE CALCULATION–GRAPHING CONFUSION

The students of science have to carry out a number of experiments in order to comprehend the basic principles of science. In this process, they make a number of observations and collect data about a number of variables. These collected data have to be systematically analyzed to arrive at the appropriate deductions of the scientific principles. For example, in a number of instances current versus potential observations (or potential as a function of the amount of the titrating substance added) can give rise to S shaped variations and these observations have to be carefully analyzed since the initial variations are due to the changes in the capacitance of the system of the original redox system taken for examination while the end portions of the titration curve denotes the redox system generated by the substance added for titration. The absolute magnitude of the values of the potential at these two end positions have to be precisely known so that the student can understand what a potentiometric redox titration curve implies.

Other serious limitations that have been found in modern day students of science is their inability to recognize errors in their measurements and also to appropriately take this into account while deducing the principles involved. There are various numerical analysis methods for analyzing data and also to find out the relative weightage one has to give for each observation. In this requirement, students are not in a position to adopt the learned methodologies to the practical, empirical data on hand. This situation stems solely from the fact that the teaching methodology of chemistry and chemistry practicals does not bring out and highlight practical problems and they confine themselves only to the list of given topics.

The students of science courses most often lack the right perception to treat their observational data in the graphical format. This situation has been more agonizing with the advent of software usage for plotting graphs. For example, it is an unwritten rule that in the conversion time plot the origin is a sure point and this is not reflected in the graphs drawn and the data even indicate some intercept while the data should pass through the origin

Another alarming attitude among the chemistry students of today is reporting the collected data to most insignificant figures. This has greater repercussions on data analysis since one fails even to realize that the data reported can never be made to that significant figures.

3(e). THE UNIT PROBLEM

The use of appropriate units with the data is a necessary evil in science. For example in the Infra Red Spectrum the X-axis has to be wave number (cm^{-1}), while in UV-Visible Spectrum it has to be wave length (cm) and in resonance spectrum especially in NMR it has to be ppm. Though all these are different units, it should be recognized that a spectrum means energy versus absorption/transmittance. This basic principle in representing a spectrum is not realized by most chemistry and science students and this ignorance has its own adverse effects in the understanding of spectra by students. This is a very basic point for understanding, if it is missed in the early stages of education, it will carry itself forward throughout one’s educational career: one will not have the frame of mind conducive for advanced level learning of chemistry or science.

3(f) WATER PHOTOLYSIS STANDS STILL

The capacity to innovate (which is the cornerstone of any science subject including chemistry) stems from a sound background in mathematics. Most students are not well equipped with this math skills indispensable for innovation.

This handicap can be illustrated: The feasibility of photo-electrochemical decomposition (PED) of water [PED] was demonstrated in the 1970s (Fujishima & Honda, 1972) but till now no technologically viable process has been devised for its extensive application. This is another perfect example wherein governing parameters are known but one does not know how to mathematically track them down. The methodology adopted is to follow the conventional route 'from-the-known to the-unknown, the key point here is: the same substance, namely, titanium dioxide is used as a common material with possible tinkering in terms of doping, surface functionality, and coupling! Precise control on other possible parameters have been identified or implemented but for PED, the math has not been accomplished yet!

4. Math Deficiency is Pervasive

It is a truism that lack of math skills has resulted in dire consequences in chemistry. Even though organic chemistry can be practiced without mathematics, the principles of physical chemistry have to be completely known for applications involving organic chemistry. In organic chemistry, one of the essential requirements is the capacity to purify the products synthesized. The basic concept in this is to extract the necessary component in a suitable solvent and this can be easily calculated using Nernst Distribution Law which will facilitate one to optimally use the solvents and also in a calculated number of steps. In practice, this type of calculated manipulation is not done and the extraction is carried arbitrarily, mostly ending up in waste of solvents.

5. Establishing Connectivity is the Key

The non connectivity with the application of mathematics to scientific problems arises due to absence of dialogue and exchange of requirements between the teachers imparting scientific knowledge and the teachers dealing with mathematics. It is an unwritten law that mathematicians live in their own world enjoying abstract mathematical concepts and applications while the scientists dabble in practical, real life issues. Instead of hypothetical solutions, if the students can be led to practical solutions it may have some effect on their understanding of scientific concepts. One of the typical examples is the solution of differential equations

The general solutions can be expressed in the form of a mathematical equation but this solution is of no use for the scientists unless he/she can apply it to the issue on hand**.** This implies that the solutions must be rendered amenable to fit empirical phenomena. Unfortunately, students are not made aware of this need for adaptation of neat and abstract mathematical equations to the rough contours of empirical phenomena. So much so, they derive and present the solution for a problem in the domain of empirical science in the neat, abstract mathematical form, with the result the solution is lacking in goodness of fit. This situation is usually encountered by students of science while dealing with the effect of field and field effects with respect to interfaces. A typical case in point is the Debye– Huckel theory of electrolytes and the derivation of the conductivity relationship incorporating both asymmetry (or relaxation) and electrophoretic effects. Lack of appreciation of quantitatively accounting for these two effects on the moving ions in the electrolyte makes the understanding of the conductance of electrolytic solutions vague

There is dearth of scientists trained and prepared to understand both mathematics and chemistry equally. Even at the PhD level, researchers are not able to visualize chemistry problems mathematically or understand in terms of chemistry the quantification [mathematisation] process. This incompetence hampers the development and application of computer simulation methods in several branches of chemistry including chemical biology, portable power sources, biomimetic sensors, and neurogastrobiochemistry.

Currently, there are no texts for navigating the extensive and intricate field of mathematical and computational modeling through practical challenges in chemistry. We witness a lack of motivation in educators towards becoming pioneers, authors, and proponents of novel pedagogical methods to infuse mathematics during the students' learning of other subjects.

Mathematical vocabulary has to find place in the teaching of all other subjects. For example, when science teachers ask students to plot a graph or use data-loggers on a "plug and play" basis, (on online facility or in a local facility with preloaded data files), the only thing a student has to do is to sink a sensor into something and plot a graph to be inserted into a report that is being prepared, then there is no need for additional mathematics. However, a teacher can introduce mathematical domains of exponential functions, discrete values, and binary numbers by appropriately selecting the analog-digital conversion mode for plotting the data. Thus, in choosing computers to enhance understanding (Šorgo and Kocijanèiè, 2006; Šorgo_*et al.*, 2008), some knowledge about mathematics is indispensable.[3]

Chemistry being a central science, developing the connection between chemistry and mathematics is one of the most important ways to shift the paradigms of both math and science disciplines. However, in what has been done so far, mathematic content has been liberally tagged on to chemistry without the development of suitable pedagogical models for teachers to teach correctly the math ingredient of chemistry and possible chemistry ingredient of math.

This act of connecting these two disciplines should start as early as possible in the educational process, in order to produce prepared minds that will be able to trace the hybridization continually up to and beyond the graduate and postgraduate levels of study.

How can we achieve this? Because teachers are a crucial factor in introducing innovations in education, the first step toward such a goal should be the education of prospective and practicing elementary and secondary school teachers.

6. STEM Needs Support

Education system today is heavily skewed toward sciences, engineering, and technology and the practice of these disciplines depends solely on a sound foundation in mathematics. Successful professional career for all depends on how well one is equipped in all these disciplines.

Simultaneous learning of science, technology, engineering, and mathematics (STEM) has its own advantages. One has to realize that pursuing this combination of subjects and further acquiring competence in them can have beneficial effects for successful professional careers. However, it seldom seemed to be possible mostly because the fundamental core subject of mathematics has not been either taught or learned to a degree that is required to master the other three branches required for professional career. The serious flaw of this arrangement is: concepts on learning and teaching

[3] In the 8-bit analog digital converters, graphs appear in stepped lines, as a result of 8-bit analog-digital conversion. Nowadays, when most converters are 16 bits, this pattern is recognized mainly when students try to zoom in to the curve. By utilizing both the modes, teachers can easily explain such a pattern to the students with conversion of our experimental plots to lower 2- and 3-bit conversion entering the mathematical domains of exponential functions and binary numbers (Šorgo and Kocijanèiè, 2004).

mathematics have to be taken up separately, abstractly, and without the correct link to other subjects. This is the bitterest reality of education in a country like India.

Though many nations have realized the importance of the combination of four subjects in education, they have also been experiencing difficulties in imparting them simultaneously and effectively in their curriculum. The main reason for this mess up is that the curriculum in mathematics, though it is claimed as appropriate and adequate for the learning of the subject, has not enabled the learners to reach the desired, critical threshold. Math is not merely an austere abstract academic discipline but is very much an applied subject too; this means that as an applied subject its mastery must be demonstrated in other domains like science, engineering and technology.

There is generic connectivity among the four disciplines of STEM. It has already been pointed out that one of the generic connectivity is that mathematics is the language for other three disciplines namely, Science, Engineering, and Technology.

It was necessary, therefore, for us examine how lack of mathematics knowledge has adversely affected the practice of chemistry.[4] In fact, most of the chemistry students have already developed an aversion for mathematics and hence find it difficult to comprehend some vital areas of chemistry like quantum mechanics, structure and bonding, electrochemistry, and branches of physical chemistry.

Today, mathematics-averse students need to see those kinds of relations and applications with the same enthusiasm. Computers is not the only way, there are myriad ways of enhancing the math enthusiasm for students. For instance, total physical response (Renuka, in press), traditional Indian games, martial arts movements, puzzles and riddles as in Lilavati[5] or Meno.[6]

7. From Frustration to Action

The popular statement that there's an arithmetic crisis in the schools of our country is true. Numbers of students come from the elementary to high schools unable to do long and short division, problems involving fractions and decimals, and are deficient in skills of basic mathematical operations such as dividing a number by zero or raising a number to the power of zero.

We have now seen the causes of why chemists most often become frustrated doing math and develop an aversion to mathematics. In spite of this situation, it has been realized that the developments in the 20th century in chemistry mostly depended on mathematics and for the present century it can be both biology and mathematics which will dominate the progress in chemistry.

Most of the manufacturing plants require not only technological skill but also scientific bent of mind to be able to turn the manufacturing process to a profitable and successful one. It is therefore obvious that all levels of education should give considerable attention and concern to the knowledge transfer in this branch of science. I am reminded of Bransford *et al.* (1999) who identified four key characteristics of learning as applied to transfer. They are:

1. The necessity of initial learning;
2. The importance of abstract and contextual knowledge;

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[4] This is expected to be true of all sciences.

[5] *Lilavati* is an ancient book on arithmetic written in the twelfth century by Bhaskara in which techniques for the solution of problems are simple and easy to use (Nagaraj, 2005).

[6] Meno's Paradox, as presented in Plato's *Meno*, is an extremely interesting one as it calls into question the very ability of humans to gain knowledge at all. According to Meno's paradox, humans can never learn anything that they don't already know (Ferejohn, 1988).

3. The conception of learning as an active and dynamic process; and
4. The notion that all learning is transfer.

Lack of emphasis on transfer in the learning of mathematics has affected the learning and practice of chemistry, the branch of science which links all the branches of science.

Not to blame chemistry for its weakness, it must be realized that deficiency in our educational system for some time now especially ever since the computational capability has been increasing to unprecedented levels in the past two or more decades. Though software development has shown the capability of certain sections of the society, the adaptation of this software has given rise to some industrial growth in the software sector but not to skill development or knowledge increment in the core industry sector namely, chemical and allied industries. The possibility of doing the mathematical manipulations without knowing the actual procedure adopted has increased productivity within the current levels of technology but innovations have remained difficult thanks to lack of understanding at the conceptual level.

8. Integrated Curriculum Model

In my experience as a chemist and educator, I witnessed the awesome positive development in chemical kinetics with mathematics input. Yes. The chemical reaction kinetics has undergone a revolutionary change after the introduction of combinatorial chemistry. This situation not only enabled rigorous experimental manipulation of the data but also provided a means to a matrix type of representation of the parameters of the reaction and simultaneously optimizing the parameters in one go. However, this is only an isolated case. Not much has been registered in other areas of chemistry.

When I attempted to make the first connections between chemistry and mathematics, the enterprise was foredoomed to limited success or even to failure. Soon I realized that isolated efforts do not receive prominence, neither produce results. Connecting chemistry and mathematics requires many prepared minds to make our students to be successful.

If students take chemistry because they envision a science career but "don't like math," they are pre-determined to ignore mathematics. We all must take this condition very seriously. However, adding one or two math courses taught by math experts to the university chemistry curriculum won't work for them, because these students will not discover the math–chemistry connections by themselves. Even if the teachers were to show them the connections, students would probably stay within the safe field of chemistry while discussing chemistry issues and pre-selectively set aside the math-laden chemistry portions without even an attempt at them. As I mentioned already, connecting chemistry with math should therefore start as early as in the elementary school and continue throughout all pre-university or pre-college education.

This does not mean that chemistry teachers should teach mathematics or mathematic teachers should teach chemistry. Both subjects should be taught individually by concerned subject experts, but some overlapping zone must be found on the appropriate content level and pedagogy of teaching. Mathematical–Chemistry or Chemical–Mathematics content knowledge for all teachers should be drawn up. It is time to look at some reported models, for instance in biology: the model of pedagogical content knowledge (Shulman, 1986; 1987) and upgraded to technological pedagogical content knowledge by Mishra and Koehler (2006) and used in cell biology by Usak (2009).

Chemistry is a skill-oriented subject. Most of the students who major in chemistry are sure to head toward becoming industrial practitioners, small or big, and they are poised to becoming scientists in research labs. So, this subject is a talent area and is a fertile zone for innovation and creativity. I vouch here that many solutions for the present problems and those of the future of the world are expected only from innovations in chemistry. Inevitably, such a breakthrough involves a lot of transfer of learning and math is essentially needed if chemists need to bring out those expected innovations.

We must remember that chemistry is a daily-life subject. From the tooth paste to butter, through medicine, chemistry is involved. Chemistry is synonymous with life. Therefore, chemistry is the center of the industrial economy of every nation in the world.. Chemists gear the world and if they should continue to gear the world forward (which is getting more complex), chemists need to possess extraordinary math skills. It is truly an unprecedented time that we need more math for chemists in order for them to become successful entrepreneurs, technocrats, researchers, and teachers.

Given these salient features of chemistry, what we need for chemistry is not an advanced chemistry curriculum but an appropriate chemistry curriculum for talent development; such a chemistry curriculum must seamlessly incorporate essential mathematical ingredients. Herein, I propose a model of pedagogical mathematical chemical content knowledge as a feasible starting point for connecting chemistry (as well as other sciences) and mathematics in schools and universities: an Integrated Curriculum Model (ICM).[7]

The strong reason for my suggesting the ICM approach rests with the extensive body of current research on learning. In the context of transfer of learning, studies have documented that higher order thinking skills need to be embedded in subject matter (Perkins & Saloman, 1989); teaching should switch from teaching facts and rules to producing long-term learning (Marzano, 1992). Our understanding of creativity also tells us that students need to possess strong subject matter knowledge as a prerequisite (Amabile, 1983) for innovative ingenuity. In all, chemistry curriculum must be developed as a special curriculum and the principles of gifted education should be inculcated in chemistry pedagogy.

A second reason for my recommending ICM is it can lead to the development of a focus in the field of chemistry from the individual learner to the process of collective talent development for all learners. I am very positive about this proposition because of the promise associated with the ICM model (Table 1).

TABLE 1
FEATURES of ICM

Overarching Concepts	Advanced Content	Process–Product
Change	In-depth	Elements of Reasoning
Systems	Advanced Reading	Research
Patterns	Primary Sources	Problem-based Learning
Cause and Effect	Advanced Skills	Inquiry Skills

Source: Van Tassel-Baska (2003).

Since chemical industries are societal enterprises, a collective and uniform mental disposition is really what we need for running the chemical industries, which constitute the backbone of the economy and pathway to the prosperity of nations. We also need a collective mental frame in

[7] The Integrated Curriculum Model (ICM) first proposed in 1986 by Van Tassel-Baska is further explicated in subsequent publications (Van Tassel-Baska, 1992; 1993); it is comprised of three interrelated curriculum dimensions, responsive to very different aspects of the gifted learner.

our citizens for meeting the challenges of the burgeoning environmental issues (for which chemistry is responsible and the solutions for which also chemistry is responsible).

If it is true that classrooms are for making the right citizens of the world, I vouch that what we need to do is to strengthen the math of chemistry curriculum and chemistry pedagogy. We need to achieve the wedding of curriculum principles of math and chemistry as important for all chemistry students from both traditional and nontraditional domains, through interdisciplinary, concept-based curriculum and higher order thinking. When it is well developed, ICM (Van Tassel-Baska, 1995) is sure to provide a cogent exemplar for chemistry–math curriculum design.

Conclusion

For Aristotle, "all science (*dianoia*) is practical, poetical or theoretical" (Aristotle, Undated). His ideology on sciences is a universal truth and it still holds good. As a conclusion, I wish to present before the readers that we are living in an unprecedented time and are moving forward to embrace a future bristling with problems to solve. Chemistry as the central science is an indispensable subject and how well it is learned will have a lasting impact on how well our future generations will live for centuries to come. It is therefore our responsibility to ensure that:

- Learning and teaching evolve synthetically and directly reflecting day to day experiences.
- Education should not be centered on only the principles and applications *per se*; instead, the learners should adopt them to design newer paths of applications to obtain newer practices.
- Every concept enunciated in teaching has and must have a greater connotation; therefore, teaching should enable learned concepts to be enlarged by the learner.

In this paper, I have made a suggestion for employing ICM as a productive tool for curriculum development in chemistry. As such, there are seldom attempts to develop integrated curriculum models for chemistry. This field is a virgin area for future research and development. I foresee a bright future for educational research in this area. Chemistry ICM development calls for a close alignment of chemistry and mathematics educators along with education experts. A meaningfully hybridized subject matter with its higher order manipulation of skills and ideas is a need of the times. At a time when India is celebrating the National Year of Mathematics, the math message must reach chemistry educators effectively. We need simple yet great personalities like Ramanujan to take charge of the situation.

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'ENHANCING THE PEDAGOGY OF MATHEMATICS TEACHERS' PROJECT: A PARTNERSHIP BETWEEN UNIVERSITY SCHOLARS AND CLASSROOM TEACHERS

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Abstract

Enhancing the pedagogy of mathematics teachers (EPMT) project, a partnership between university scholars and classroom teachers, exemplifies a critical innovation in the professional development of teachers in many parts of the world. This development reflects a gradual shift in the centre of gravity away from the university-based, "supply-side", "off-line" forms of knowledge production conducted by university scholars for teachers towards an emergent school-based, demand-side, on-line, *in situ* forms of knowledge production conducted by teachers for teachers. This paper elaborates the EPMT project and examines the attributes of the partnership desired by the teacher participants of the project.

Key Words: *Professional development, mathematics teachers, partnership, university scholars, Singapore*

1 Introduction

1.1 Professional Development of Mathematics Teachers in Singapore

In 1997, the Ministry of Education (MOE) in Singapore launched the 'Thinking Schools, Learning Nation Vision' (Goh, 1997). This vision places emphasis on the need for teachers to be lifelong learners so that schools keep abreast of advances in knowledge and learning both at the national and international fronts. In support of the vision, as of 1998 all teachers in Singapore are entitled to 100 hours of training and core-upgrading courses each year. Professional Development (PD) of teachers is funded by the MOE. To support teachers in mapping their learning trajectories, in 2005 the MOE implemented an Enhanced Performance Management System (EPMS) (MOE, undated). The EPMS is an appraisal system that contains rubrics pertaining to fields of excellence in the education system be it teaching, leadership or senior specialist. These rubrics delineate very clearly the competencies deemed necessary at each level and hence teachers are entrusted with responsibility of their own PD. Inevitably, following pre-service education teachers embark on PD activities that fit their needs and many of these activities take the form of in-service courses, one-off type of training workshops, seminars, etc.

The most common type of PD in Singapore for mathematics teachers has been in-service courses of the type that focuses primarily on expanding teachers' repertoire of classroom activities or introduction to new initiatives of the MOE with regards to curriculum implementation. These PD activities may be said to belong to what Matos, Powell and Sztajn (2009) describe as a "training model of professional development" (p 167). They are conducted by specialist officers from the mathematics curriculum planning and development division of the MOE or academics from the National Institute of Education, the sole institute for teacher education in Singapore. These courses are conducted for about 3 hours per day spanning four to ten consecutive days or days spread over some weeks. After the completion of such a course there is seldom any follow up with the teachers about the use of the knowledge acquired and the impact that knowledge may have had on student achievement.

Research has shown that such courses are ineffective as teachers are likely to reject knowledge and skill requirements when i) the requirements are imposed or encountered in the context of multiple, contradictory, and overwhelming innovations; ii) they are excluded from the development of the courses; iii) PD is packaged in off-site courses or one-off workshops that are alien to the purposes and contexts of their work; or iv) they undergo them as individuals and are afraid of being criticized by colleagues or of being seen as elevating themselves on pedestals above them (Hargreaves, 1995). Smylie (1989), in his survey of teachers' ratings of opportunities to learn in the US found that district-sponsored in-service workshops were at the bottom of the heap, ranked last out of 14 possibilities in terms of what teachers considered most valuable. Although such workshops are often accompanied by evaluations, seeking feedback on the duration, satisfaction, etc., efforts to measure what teachers learned have not been part of typical evaluation fare. In the same survey, Smylie found that teachers ranked direct classroom experience as their most important site for learning. Furthermore, for some teachers PD may not be an autonomous activity, i.e. chosen by a teacher in search of better ways of knowing and teaching mathematics (Castle & Aichele, 1994).

In Singapore, the school mathematics curriculum is reviewed every six to ten years. Textbooks, which are key to the implementation of the curriculum, are also revised periodically but they appear to manifest the content more than the processes. In 2006, the school mathematics curriculum in Singapore was revised and the scope of processes in the curriculum expanded to include reasoning and communication (MOE, 2006a; 2006b). A study conducted in the US by the American Institute for Research (AIR) found that textbooks in Singapore were focused on practice exercises that emphasize procedural knowledge but lacked emphasis on reasoning and communication which facilitate higher order thinking skills (Ginsburg, Leinward, Anstrom & Pollock, 2005). This study provided a much needed outsider's perspective on the quality of curriculum materials that many teachers relied on for their teaching. In addition, Kaur, Low and Seah (2006) in their work with competent mathematics teachers found that the teachers were generally bound in their choice of learning tasks (tasks used by the teacher during instruction to develop a concept or demonstrate a skill or process) available in the textbook used by the school and that these tasks were not suitable to engage students in reasoning (logical—deductive or inductive) and communication (explaining the process / thinking either during oral presentations or in writing). Furthermore, they found that teachers did not make explicit the need to understand but rather placed emphasis on procedural knowledge, i.e. to remember algorithms and use them correctly to pass tests and examinations.

The 2006 revision of the curriculum and the research findings of Ginsburg et al., (2005) and Kaur et al., (2006) led the author of this paper, Kaur, and her colleague, Yeap, to conceptualise a project that would firstly provide teachers with the know-how of tasks that are suitable for engaging students in reasoning and communication and teach for understanding and secondly support teachers in implementing their new knowledge in their classrooms. This project known as Enhancing the Pedagogy of Mathematics Teachers (EPMT) was carried out in 10 Singapore schools for two years. It was the first of the kind in Singapore schools and teacher participants of the project hailed it as a highly appropriate PD model for their learning. The next section describes the project in some detail.

1.2 The Goal of this Paper

The goal of this paper is to describe the EPMT project and examine attributes of the university scholars and classroom teachers' partnership. The research question explored specifically in this paper is

What were the attributes of the university scholars and classroom teachers' partnership that were desired by the teacher participants of the project?

2.0 The EPMT Project

Enhancing the Pedagogy of Mathematics Teachers (EPMT) project, a school based project of the Centre for Research and Pedagogy at the National Institute of Education of Singapore, is a hybrid model of PD that integrates the "training model of PD" (Matos et al., 2009) with sustained support for teachers to integrate knowledge gained from the PD into their classroom practice. It exemplifies a critical innovation in the professional development of teachers in many parts of the world. This development reflects a gradual shift in the centre of gravity away from the university-based, "supply-side", "off-line" forms of knowledge production conducted by university researchers for teachers towards an emergent school-based, demand-side, on-line, *in situ* forms of knowledge production by teachers with support from university scholars.

The aims of the EPMT project were three fold. The first was to provide teachers with training on how to craft suitable learning tasks that engage students in reasoning and communication and teach for understanding. The second was to facilitate teachers' work (practice and feedback) at the school level by assigning them activities to carry out together with their fellow teachers who were also in the project. The third was to enthuse and support teachers to contribute towards the development of fellow mathematics teachers in Singapore.

2.1 Review of Literature

The conceptual framework of the EPMT project draws on research findings, specifically the characteristics of effective PD programmes. High quality and effective PD programmes have been found to have a purpose as teachers are involved in shaping the foci of the programme that is related to their school work (Clarke, 1994; Hawley & Valli, 1999; Elmore, 2002). These PD programmes are part of coherent programmes of teacher learning and development that support their instructional activities at school, such as adoption of new standards (Stiff, 2002; Desimone, 2009) and focus on how to teach and what to teach – the substance and the subject matter- are key (Stiff, 2002; Desimone, 2009). Ball and Cohen (1999) have argued that "teachers' everyday work could become a source of constructive PD" (p 6) through the development of a curriculum for professional learning that is grounded in the tasks, questions, and problems of practice.

Such programmes include training, practice and feedback, and follow-up activities (Abdal-Haqq, 1995). Ball (1996) claimed that the "most effective professional development model is thought to involve follow-up activities, usually in the form of long-term support, coaching in teachers' classrooms, or on-going interactions with colleagues" (pp 501-502). Effective PD programmes are sustained (Clarke, 1994; Abdal-Haqq, 1995; Hawley & Vali, 1999; Elmore, 2002; Stiff, 2002; Borasi & Fonzi, 2002; Desimone, 2009) and embedded in teacher work (Clarke, 1994; Abdal-Haqq, 1995; Hawley & Vali, 1999; Carpenter et al., 1999; Elmore, 2002). Teachers learn best when observing, being observed, planning for classroom implementation, reviewing student work, and presenting, leading and writing (Stiff, 2002) and therefore opportunities for teachers to engage in active learning are certainly related to effectiveness of PD (Wilson & Berne, 1999; Desimone, 2009). In addition, collective participation by teachers from the same school, grade or department allows for powerful form of teacher learning through prolonged interaction and discourse (Wilson & Berne, 1999; Desimone, 2009; Stiff, 2002). PD programmes that foster collaboration have been found to be effective (Clarke, 1994; Abdal-Haqq, 1995; Hawley & Valli, 1999; Elmore, 2002; Borasi & Fonzi, 2002).

2.2 Design of the Project

The five significant features of the project were:

i). Content focus

The project had a content focus. It was specific to the pedagogy of mathematics.

ii). Coherence

The project attempted to address the needs of the teachers in the following ways:

a) The revised mathematics curriculum of 2007 (MOE, 2006a; 2006b) placed emphasis on reasoning and communication in mathematics lessons. As textbook questions were inadequate for the purpose, there was a need for teachers to learn how to craft mathematical tasks that facilitate reasoning and communication during mathematics lessons.

b) With the Teach Less, Learn More (TLLM) initiative of the MOE (MOE, 2005) placing emphasis on teaching for understanding, there was a need for teachers to review and learn more about lessons that facilitate "understanding".

iii). Duration

The project spanned 2 years and comprised three phases. Teachers attended training workshops for a semester, followed by a semester of school based work guided and monitored by the university scholars (PD providers), followed by another year (2 semesters) of self-directed school based work.

iv). Active learning

The project engaged teachers in active learning – hands on work. They crafted mathematical tasks and planned lessons, worked in pairs to execute their lessons, video tape their lessons, critique lessons, and revise their plans, thereby engaging in iterative cycles of planning and implementing.

v). Collective participation

In the project there was collective participation at two levels – school and project. At the school level, at least 4 teachers with pairs of teachers teaching the same grade year and mathematics programme, participated. These teachers worked together during the training workshops and also at school when implementing their learning in their classrooms. At the project level, teachers also worked together building their knowledge by participating in sessions during which they critiqued their peer's work, shared their experiences and difficulties encountered during the implementation of their newly gained knowledge.

2.3 Implementation of the Project

The project comprised of three phases spread over two school years. A school year comprises of two semesters, each of 20 weeks duration. Details of the phases were as follows.

PHASE I

The duration of this phase was a semester of the school calendar year (i.e. from January till May of the first year of the project). In this phase, teachers attended training workshops conducted by the university scholars (the author and her colleague Yeap). The workshops were organised as two modules, the first centred around crafting of tasks that would engage students in reasoning and communication and the second centred around teaching for understanding. Each workshop began with the university scholar introducing the teachers to an idea. In the case of the first module, they were introduced to ideas of how typical textbook questions could be crafted into tasks that would engage students in reasoning and communication. In the first module they were introduced to 8 strategies. The second module focussed on the "why, what and how" of teaching for understanding. This module engaged teachers in planning lessons and crafting / selection of appropriate learning and practice tasks. Details of how the university scholars engaged teachers in creating knowledge during the first phase have been reported elsewhere (Kaur, 2011).

PHASE II

The second phase of the project took place during the second semester of the school year (i.e.

from July till December of the first year). In this phase teachers were encouraged to infuse in their lessons their learning from the training workshops of the first phase. Teachers were given specific assignments by the university scholars. They were assigned tasks to complete in their own time collaboratively with their fellow project participants in the school. The tasks are described in detail elsewhere (Kaur, 2011).

While teachers were working on their assignments, the university scholars facilitated fortnightly meeting sessions during which teachers shared their work with the others and invited critique. It was during these sessions that teachers' shared with the rest of the project participants their tasks, lessons (through video-records), students' work and students' voices. They invited both applause and critique. More details of the process are described elsewhere (Kaur, 2011). Towards the end of this phase teachers submitted their assignments. The assignments submitted by the teachers have led to the publication of the resource: *Pedagogy for Engaged Mathematics Learning* (Yeap & Kaur, 2010).

PHASE III

The duration of the third phase was two semesters of the school year (i.e. from January till May and July till December of the second year). During the third phase teachers were left to work with their project mates in their schools to advance the knowledge they had gained from the first two phases. The university scholars facilitated monthly meeting sessions during which project participants were engaged in a variety of activities:

a) They continued to share their "highs and lows" of lessons that engaged students in reasoning and communication and also lessons that "taught for understanding".

b) They worked on refining knowledge they created during Phases I and II and prepared exemplars of mathematical tasks, primary and secondary, for publication as print resources (Kaur & Yeap, 2009a; 2009b) for mathematics teachers in Singapore schools.

c) Teachers participating in national conferences, school based and cluster level forums prepared their presentations.

d) During the last meeting of the year, they completed the post-intervention survey for the project and some schools presented their "Way Forward" plans for the following year.

The university scholars also guided the teachers in making their presentations at meetings and conferences.

3.0 Methodology

3.1 Subjects

Table 1 shows the numbers of schools and teachers who participated in the project for two years.

TABLE I
NUMBER OF SCHOOLS AND TEACHERS IN THE PROJECT

Category	Primary	Secondary
Number of schools in the project	5	5
Number of teachers in the project for the 1st year	20	28
Number of teachers in the project for the entire duration	18	22

During the second year of the project some teachers were unable to continue participation due to maternity and child care leave, and change of schools.

3.2 Source of Data

The qualitative data presented in this paper comes from one data source, i.e. the post-intervention survey. The survey sought data about the usefulness of the content and implementation of the project. One of the prompts asked the teacher participants about the differences and similarities between the EPMT project and in-service courses they normally attended, and their preference for future PD programmes.

3.3 Data Analysis Methods

The qualitative data were analysed using techniques of qualitative analysis. For the survey data on the learning journeys of the teachers we adopted an inductive approach and carried out content analysis (Weber, 1990). The responses were first scanned through for common themes, following which codes were generated and the data coded. Inevitably "a progressive process of sorting and defining and defining and sorting" (Glesne, 1999, p 135) led to the establishment of the final list of codes for the themes.

4 Findings

A total of 33 participants of the project, 16 from primary schools and 17 from secondary schools completed the post-intervention survey questionnaire at the end of the second year of their participation in the project. The overall response rate was 82.5%. To answer the research question,

What were the attributes of the university scholars and classroom teachers' partnership that were desired by the teacher participants of the project?

the qualitative responses to the survey item:

Tell us how different or similar has it been participating in the project compared to attending a traditional in-service course?

were analysed. Table 2, shows examples of responses to the survey item, content analysis carried out and inferences made by the researchers.

TABLE 2
CONTENT ANALYSIS OF DATA

Teacher code	Response	Inferences drawn about attributes of university scholars and classroom teachers' partnership
P-1	In-service course is most of the time one-off session or may take a few sessions to complete. There rarely is time to evaluate what has been learnt and to feedback the success of the implementation strategies shared during the courses. EPMT project has shown a good example in trying to get teachers to implement their learning and evaluate it. This has been possible because the ***university professors worked with us hand in hand to help us create the tasks, implement them, evaluate them and refine them. They also helped us to showcase our work and put it into print form for other teachers in Singapore.***	University scholars and teachers worked hand-in-hand in school settings during which teachers created tasks, implemented them, evaluated them (self & peer), and revised them University scholars also provided a platform for teachers to showcase their work to colleagues in the fraternity.
P-7	It was a great experience to be part of this EPMT project. It gave me the opportunities to work in a team in an area close to our hearts. How to make our lessons more interesting and engaging for our pupils. To participate in a project like this is more useful	Teachers worked in groups with guidance from the university scholars.

P-7	though time consuming. It allows collaboration among teachers from the same discipline and interest. The sharing among schools and colleagues as well as the compilation of the work into a book is really great! In-service course does not allow so much of handholding and momentum to apply what was learnt! Thanks to Dr Kaur and Dr Yeap who motivated and guided us through the project.	University scholars motivated and guided teachers throughout the entire duration of the project.
S-12	Participating in the project has been more fruitful as the outcomes are more specific. We are able to work on assignments together as a group and then try out the tasks in our classes. Sometimes we are excited by ideas when attending a course but are unable to bring these ideas to practice due to time or manpower constraints in school. Another difference is that Prof. Kaur is able to guide us throughout the project whereas a trainer in a course may not be able to provide any other support after the workshop.	Teachers worked in groups on specific tasks with sustained support from the university scholars (experts)

It is apparent from the inferences drawn based on teachers' self-report data about how the EPMT project differed from traditional in-service courses they attended that some of the attributes of the university scholars and classroom teachers' partnership were as follows:

i) Although the university scholars were experts of knowledge they introduced to the teachers in Phase I of the project, the teachers co-constructed knowledge together with them hence forth. The teachers applauded the university scholars working with them on the ground, i.e. the classroom level.

ii) The teachers acknowledged the guidance they needed for implementing their newly acquired knowledge. The university scholars provided the guidance, *via* the design of the project, with much enthusiasm and willingness.

iii) The university scholars provided teachers with a platform to showcase their work to fellow teachers in the fraternity. A meta inference here could be that the university scholars empowered teachers to share with others knowledge they had co-constructed with university scholars.

5.0 Reflection and Conclusion

In reflecting on the design of the work with teachers by the university scholars several elements appear to be critical from the perspective of the teachers. The first is working with teachers in a concrete and explicit way, i.e. addressing their current pressing needs with regards to curriculum issues and working with them hand-in-hand as co-workers at the classroom level. This finding resonates with Ball and Cohen's (1999) argument that "teachers' everyday work could become a source of constructive PD" (p 6) through the development of a curriculum for professional learning that is grounded in the tasks, questions, and problems of practice.

The second is the structure of the project designed by the university scholars. It provided a means to support the teachers in ways they desired – regular meetings to discuss issues, use of video tapes to bring their lessons alive during discussions, etc. The teachers in the project valued the sustained support from the university scholars while implementing their learning in their classrooms.

The third and last one is acknowledgement by the university scholars that teachers should be empowered to contribute towards the development of fellow teachers by disseminating knowledge they created in meaningful ways. This is a significant shift from knowledge production by university scholars for teachers towards knowledge production by teachers in partnership with university scholars for teachers. The university scholars provided teachers with opportunities to codify, verify and disseminate their knowledge.

From the findings reported in this paper it is apparent that the five significant features of the project namely, content focus, coherence, active learning, duration and collective participation were critical in the success of the partnership. What has made the project a meaningful one for both the university scholars and the teachers is the role of both in the teacher learning. The university scholars have brought to the project expert knowledge which they disseminated through the two PD modules. The teachers have made sense of the knowledge acquired by working with peers and in their classrooms with the help of both the university scholars and their peers. Lastly, teachers in the project have seen their role beyond being mere participants in a project but as creators of knowledge for fellow teachers, capable of lighting fires in the classrooms of other teachers.

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TOWARDS A HIGH QUALITY MATHEMATICS CURRICULUM

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Abstract

The author, a regular contributor, lists a few recommendations towards a high quality mathematics curriculum. They are: content to diagnose and remediate pupil difficulties in ongoing lessons, appropriate developmental sequence, and instructional procedures, individual as well as group strategies of teaching, continuous evaluation and inservice teacher education as an integral part of this endeavor.

Introduction

There are many recommendations about how to develop an optimal, relevant mathematics curriculum. There are selected goals which are recommended by national and state study groups, The National Council Teachers of Mathematics (NCTM), Common Core State Standards (CCSS), and 'No Child Left Behind' (NCLB), among others. All agree that mathematics teachers must possess mastery of subject content and a range of instructional procedures if they are to feel more secure in teaching and learning situations. They must possess diagnostic and remediation skills too.

Students need to be able to utilize necessary content within a specific situation. Practical as well as theoretical knowledge is salient and vital. There are a variety of specific situations whereby pupils individually and collectively need assistance and this requires the mathematics teacher to be creative in finding ways of assisting learners to be successful in mathematical usage (Ediger and Rao, 2011).

Recommendations for a High Quality Mathematics Curriculum

Mathematics teachers need to follow selected criteria to develop self- efficacy in teaching and learning situations. Focusing on the following will help teachers become proficient in working with pupils.

1. *Collaborative endeavors*: within school and society, pupils need to harmonize ideas with those provided by others in a small group setting. Psychologist Len Vygotsky (1978), who is quoted widely in committee work, advocates strongly that learning is social and occurs within a group whereby subject matter is shared as in problem solving. Thus, ideas bounce off the minds of individuals and are modified while coming up with solutions in mathematical problem solving. Each pupil contributes in finding answers in a sharing session.

2. *Independent Work:* This might be preferable to those individuals who achieve best when working by the self in ongoing lessons. Pupils who possess initiative and motivation are enabled to progress in their own pace.. Perhaps, balance between cooperative and individual endeavor is the better approach since in society, persons work collectively as well as individually on projects, in leisure time experiences, as well as at the work place (Ediger, 2008).

3. *Groupig—Heterogeneous and Homogeneous:* Both modes of grouping should be emphasized in the mathematics classroom. There are times when the teacher needs to utilize the latter when teaching a specific process such as the commutative property of addition. At other times, a new lesson might stress mathematical subject matter whereby all in a classroom can benefit from the same/similar

content being presented. The approach used in grouping for instruction chosen must assist optimal learning, not grouping for the sake of grouping. Heterogeneous grouping might then be implemented.

4. *Continuous Evaluation:* Continuous evaluation of pupil progress must be in place. For example, a personal diary may be kept by the teacher, dating each entry pertaining to a pupil's difficulties/ errors in a mathematical lesson. It is a prerequisite for diagnosis and remediation,

5. In-service Education for Teachers: It must be a regular periodical exercise. Workshops, faculty meetings, grade level or academic area seminars, and reading professional literature on mathematics teaching, help the teacher to develop proficiency in the instructional arena. Additional areas of inservice growth include attending local, state, and national conventions on mathematics teaching. Enrolling in an advanced degree in mathematics education from an accredited college/university can be a means of acquiring increasing teaching efficacy. Within the school, teachers need to discuss ways of improving the mathematics curriculum. Learning from each other is vital such as meeting together to discuss results from standardized tests. Feedback should aid in improving learner achievement.

6. *Improving Parent-Teacher Conferences:* Quality standards must be used in these conferences in obtaining relevant and precise information from parents in cooperatively guiding pupil achievement. Good human relations whereby parents feel welcome and engaged during the conference is a prerequisite for the success of these conferences. A portfolio of the pupil's actual work products may provide an excellent basis for interactions in a conference

7. Community Schooling: In community schooling allstake-holders in a given region are involved in improving the school curriculum. Getting communities to accept and promote the community school concept takes time and incremental increases will come about as the school years progress. Each teacher, school administrator, as well as support faculty member, must be actively involved in being a promotor of public school education.

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STUDENT PERSPECTIVE OF MATHEMATICS EDUCATION: INDIA *versus* USA

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Abstract

In this paper we provide a comparison of Indian education system vis-à-vis the US education system in the aspects of teaching and learning arrangement and provisions for special services. We endeavor to cover the Content, Standards, Assessments, Gifted Education, and Special Education. We discuss both the strengths and weaknesses of these two systems and provide a basis for the readers to evaluate the merits of each system. Mathematics, though a compulsory subject of the educational programs in both the countries, is generally perceived as hard by students of both the countries. Despite extensive technology integration in teaching (in the US) and right student and parent attitude (in India) both countries have one common problem, namely teacher effectiveness.

Key Words: *US math education, Indian math education, comparison of US and Indian math education, teachers' math anxiety, math teacher effectiveness*

1. Introduction

Mathematics as a subject of study in today's schools is very different from the classroom experience parents and teachers must have had. The fundamental reasons being:

1. A vast revision in the content (syllabus).
2. Assessment modalities.
3. Indispensability of a continuous series of math courses for school graduation and for college graduation in career-oriented courses.
4. A revolutionary development both in theory and practice in the field of education.
5. Frequent policy changes in the State Departments of Education with respect to math education.
6. Inconsistent implementation of various approaches to math education.
7. Issues in remediation of struggling students.
8. Issues in special and gifted education.

Consequently, at the individual classroom level, we see inconsistency in teaching practices as well as teacher practices.

We both have had the opportunity to study math courses in India as well as the United States and herein we present from our experience a comparative analysis of the various aspects of math education in USA vis-à-vis India.

2. Content of Math Courses

No subject in school education might have gone through so much of content change as mathematics. By content, we mean both the actual themes (and the extent of study of the themes) and the structure of the content. In the US, in the recent past, unprecedented changes have been taking place in the way math courses have been constructed and offered to the students.

The revisions in math curricula took us from separate Algebra, Geometry, Trigonometry, Statistics, etc. to Math I, Math II, Math III, and Math IV, respectively. However, within two years of implementation, the integrated math curriculum is being substituted by the Common Core Math Courses: Coordinate Algebra, Analytical Geometry, Advanced Algebra, and Pre-Calculus.

The important point in these revisions is: math is no longer taught as separate courses, but as integrated courses that incorporate and delineate the connection between different key branches of mathematics, namely Algebra, Geometry, Trigonometry, Calculus (Pre-calculus), and Statistics.

Doubtless in the present arrangement there is so much of rigor built in the math courses. This approach certainly is more indicative of a strong math-oriented postsecondary experience by way of college-level courses and careers thereof. In fact, this is what was actually anticipated by the public at large, for the past many years in their objectives for college and career readiness for their children, especially in the scenario of globalized work force. "As the workforce becomes increasingly more global and technology driven, it is essential that the United States align its K 12 core curriculum to the expectations of its 21st century workforce, ensuring that its future leaders remain competitive in the global economy" (ASME, 2010).

At last, now we have gotten what we wanted — rigorous math curriculum at different grade levels of schooling. However, people doubt if this would stay for some time to see the impact. There are also other questions: "Is this content truly essential for all students?

Are we setting some students up for unwarranted failure? Will we water down these courses to give the appearance of success?" (Woods, 2010).

These questions are really thought provoking; as students we had always seen how our fellow students perceived the rigor of math content. Rigor is not necessarily perceived as worthwhile by all students. For some students, rigorous curriculum means they are adequately challenged and can remain motivated to do more and more of rigorous math; but for the majority of the students, rigorous curriculum is less pleasing. For some, it could prove repulsive too. Students with special needs (cognitive levels) would find the curriculum very hard.

In India, the revision in math curriculum is far more stabilized and occurs at intervals of four years. The key disparity is we have different Boards of Secondary Education, viz., State Government Board, the Matriculation Boards, and the Central Board. These boards follow a different syllabus each. Further disparity arises from the fact that schools under the Matriculation Boards have the autonomy of following any text book of their choice for a given course. The net result is there is no uniformity across states and within a state in the content of the math courses. Students who go to schools affiliated to the Central Board go to more advanced post-secondary institutions like the Indian Institute of Technology [IIT's], Birla Institute of Technology and Science [BITS], etc. Students who graduate from other Boards do not generally get into these institutions. There is a general feeling among students of both the State Boards and the Matriculation Boards that they lack the competence to get into IITs. This is an indication of how the difference in the rigor of the curriculum affects the mindset of the students about their destinations for college-level courses.

A further point about the content of math courses in Indian schools versus US schools is the abstractness of the problems taught and assessed on. By and large, Indian text books present abstract problems and teachers teach these abstract problems as such and students' attainment is also assessed based on tests that contain abstract type of problems.

On the other hand, in the US, math problems presented by text books, taught by teachers, and provided in assessments are applied problems —that is, they have day-to-day familiar contexts and provide concrete cognitive level compatibility to readers' mind. In our opinion, Indian students struggle most and even develop an aversion toward math because of the abstractness of the problems.

Differences in the content, lay out and presentation in math text books in India and the US deserve a special mention. The US textbook industry provides the teachers with a greater choice in quality textbooks than any nation in the world (Squire and Morgan, 1990). The text books are made in high-quality and durable art paper; the content is prepared in excellent English and big fonts are used. The content is so student-centered that the text is substantiated by figures/pictures in real-life situations. Important facts are presented in boxes. Illustrative examples are worked out in a format that enhances retention of knowledge. There are captivating boxes for Check Points for Review. Stand-alone Vocabulary exercises help master the key words and jargons. End-of Unit Assessments are student-friendly. Answers are provided for only odd-number problems. This arrangement promotes independent work as well as avenues for interaction with the teacher and fellow students. The text book has a note-taking guide, helping the students to write and practice the essentials and not the whole text book. Thus the text book serves as a dependable resource and not the only source. The most salient feature of US text books is the Teacher Edition. The teacher edition provides everything for the teacher. Syllabus, Standards, and Pacing Guide are presented. Problems are solved in the correct format. Solutions for all the problems are provided. Hands-on activities are delineated. Check point cues are provided. Quizzes and tests are offered. In addition, each teacher gets a box that contains transparencies of the warm-up activities, vocabulary worksheets, skill worksheets, lab worksheets, review worksheets, etc. Teacher edition also comes with a set of CDs, in which PPT presentations of each day's lesson are provided. The US teacher is thus so fully equipped in terms of the content and support materials to teach.

In the US, students do not have to buy the text books. Schools provide the text book. They also can check out the CD version of the book, in which lessons are presented in a simplified workbook format. In some states in India, the schools which are run by the State Boards of Education provide free books to all students. But private schools, which far exceed in number the State Board schools, require the parents to purchase text books. While this involves a financial burden for the parents, the need to carry all the books all the days to and from home is a real physical burden for the students.

Indian text books in general and math text books in particular are poorly written, designed, and printed. The poor-quality paper adds to the inconvenience. Whereas CBSE books have effected considerable changes in the layout and presentation of the content, the books adopted by the other boards are very ordinary. However, it must be mentioned that in India the Tamil Nadu State Board has made a revolutionary arrangement of providing all text books online for students, and during last year a colorful improvised version has been brought out.

Text books of State Boards in India come in two versions: in the vernacular and in English. The vernacular editions are for those students who study in the vernacular medium. These books present and discuss math and math terms in the vernacular. The vernacular or the regional languages generally do not have exact counterparts for mathematical terms or jargons. In order to make good this lacuna, math terms and jargons are constructed in the regional languages. This vernacularization of math with deliberately constructed new words makes technical terms harder than those in English. As such, students struggle with learning math through regional language (or mother tongue). This is another area of concern in math education in India.

3. Performance Standards and Assessments

A unique difference between the educational programs in the US and India is in the emphasis placed on standards or the specifications for students' academic attainment via series of formative assessments culminating in their performance in End-of-Course Tests (EOCT) or final examinations (at the middle school level, such tests are called the Criterion Referenced Common Tests, CRCTs) in the former. Every State Department of Education in the US publishes online the performance standards for every Unit of Instruction for every course. This is a wonderful arrangement because the Department of Education wants every stakeholder (the student, the parent, the teacher, the school, the community, etc.) to be aware of these expectations regarding students' mastery of the learned concepts. They also have mandated that these standards have to be displayed in the classroom as the Unit is delivered to the students. A step further, the elements of a standard are converted into Learning Goals and they too must be displayed in the classroom per day and even for a whole week. There is also a Central Question (also called Essential Question) or a set of Central Questions that will be displayed in the classroom; the Central Question reveals the big idea that runs as a common thread in the given unit of the content. This means that the student not only knows the expectations out of the learning in a given class for the given day but also of the whole week and further for the whole Unit!

Such a practice of explicitly displaying the performance specifications is not in vogue in India. But one thing that is missing in the US school systems and found in Indian school systems is the "Blue Print" of the State Tests, along with a model question paper. The Boards of Education publish the blue print of the State Tests in media and online in their respective websites. Every examinee is aware of what and how he or she will be tested on the State Test. Similarly, after every State Test, the question papers are released. This is a marvelous arrangement for the students to familiarize themselves with the types of questions that are asked in tests. It is also normal for students to discover the indispensable questions that are repeatedly asked and possible new questions in other areas.

Unfortunately, the US students are disadvantaged in this respect. Students are unaware of the questions that will be asked in the Benchmark Tests (the district-wide common tests) and the EOCTs that are the State Tests. There is also no practice of publishing the question papers after the test. The only exception is the free-response (essay type) questions in the Advanced Placement (AP) tests.

In the recent past, US teachers are challenged by the pressure of teaching the Standards to the students. As students, we are able to see the pressure on them in this respect. Honestly, we do not really see a value in the teaching of the Standard. If it is true that Standards are aligned to the EOCT, then what we like to be familiar with are the true pattern of the determinant tests namely, the benchmark tests and the EOCTs and not the Standards. It is a shock to us when we hear from our teachers in the US that they themselves do not know the blue print of the State Test. This is a sad part of US education system. When teachers are ignorant of what the final exam will be like, how are they expected to train the students for the final examinations? The same is the case with parents. Parents face this challenge more than any other stakeholder because they feel helpless about helping children prepare for tests, especially math tests. It is a further shock to learn that the EOCT and CRCT tests are prepared by private service providers and the teachers who actually teach the courses are not involved in the process. At the other end of the spectrum, in India, final examination test papers are constructed only by experienced teachers, who teach the subject.

We have been shocked by the fact that in the US education system what it claims is not what is really achieved. Let us illustrate this with one example: students with learning difficulties and other special needs have to take the same benchmark and State Tests as the other students. It is hard to accept this condition because the US schools boast of new tools and new teaching methods and have a greater awareness of learning differences and disabilities of students. But in the critical area of appropriately assessing students with special needs, nothing is in place. The existing practice for the critical assessments in fact intensifies math anxiety and the "math is not for me" attitude in students with special needs.

However, it must be placed on record that the US school system is superior to the Indian school systems in one major aspect of assessment namely, class work and class-based assessments. About 85% of the grades given to students are based only on the course work, namely what the student does in the class. The final examination/CRCT/EOCT amounts to only 15% of the grades. We consider this as the greatest advantage for the US students. This is an important lesson to be learned by the Indian school systems from the US school systems. In fact, Indian students do a lot more class work and class-based assignments than the US students; but they are not given due credit for these tasks. At this point, it must also be mentioned that US students face State Tests for math in grades 9 as well as 10, whereas Indian students face State Tests in grades 10 and 12. As of now Math 3 (grade 11) and Math 4 (grade 12) of US schools do not involve math State Tests but only school-based final examinations. From our experience, we consider this as a great stress relief for the US students.

Make-up tests and credit recovery arrangements constitute another support that theUS school systems provide to students but this is totally ignored by the Indian school systems. The US school system always provides a make-up day for those who happened to be absent on the scheduled day of the State Test or final examination. This option is provided to the students because students could be absent for the test on the scheduled day owing to so many exigencies. Therefore, they have mandated the make-up test. In fact, the make-up test dates are pre-determined and pre-scheduled by the school system and are widely publicized/announced. Another, more advantageous support for the US students is the quick arrangement for the next State Test. Suppose a student fails in a State Test, he or she has the chance to go for a remediation coaching and can take the State test within a short time, say within two months. This means that a student who fails in the State test in May will be able to appear for and get his/her result during the summer vacation and can join his/her classmates in the next grade level course in August!

4. Teacher Practices and Teaching Practices

It is amazing to see the host of technology applications in the US classroom and also for off the classroom practices both for the teacher and the student. This is called the technological pedagogical content knowledge (TPCK) for teachers. US schools believe that "TPCK is the basis of good teaching, which involves an understanding of the representation of concepts using technologies. Herein, pedagogical techniques incorporate technologies in constructive ways to teach the content (Mishra & Koehler, 2006: 1029). In the Indian schools, technology application in classroom instruction is almost absent. In the recent past, a countable number of schools are contemplating on technology use. However, power fluctuations, stringent power cuts, weak Internet connections, and cost factors are hampering such initiatives.

In this context, we must say that there can be no dearth of technology use in any US school; in fact, there is the problem of excess!

Teachers appear to be overwhelmed by the abundance of resources available online and the technological formats in which they are available. Sometimes, it also appears as though the teachers are using technology for technology sake and not for its appropriateness to a given concept. Sometimes, technology tools are so freely used just to engage the students without any instructional focus. We consider this as a not-appropriate use of technology.

A teachers' knowledge of the curriculum (i.e., what subject matter is to be taught and how it should be developed) influences student learning because the decisions teachers make about the given curriculum/ lessons can either enhance or hinder access to important mathematical topics. Although US teachers are given everything including the lessons, lack of mastery of content knowledge causes problems of confusion in technology use for many teachers, especially math teachers. We have also realized that when compared with their Indian counterparts, US teachers seem to have more anxiety while teaching math. We feel that: (a) inadequate content knowledge, (b) classroom settings and climate, and (c) students' and parents' attitude toward education in general and mathematics in particular are some reasons for US teachers' teaching anxiety. Their anxiety seems to be complicated by pressure from the administrators and annual performance evaluations. Added to this pressure are the high stress levels caused by legal requirements of fulfilling the requirements of the Individualized Education Plans (IEPs) of the special needs students and the severe punishments associated with the infarction of codes of ethics. We hear from high school math teachers that most students have been passed on to them from middle school without proper preparation and they lack the background knowledge to pursue the current course. US teachers attribute their highest level of stress to the lack of background knowledge in their students. It is said that teachers are asked to build the necessary background knowledge for the current course while they are teaching the current course. According to the teachers, building background knowledge is an intensive time-consuming process; neither the syllabus nor the pacing guide has provisions for it. In addition, when the students are far below the current level, efforts to build the background knowledge seldom yield results. In all, we feel that the stress on US math teachers is an important area for research.

At the other extreme of the spectrum, Indian teachers are almost scot-free and they are the most scot-free in government schools. In the private schools, depending on the management, there are some differences in the attitude of teachers toward teaching and toward their students. In the US, teacher evaluation and district funding are often linked to students' scores on the State tests. This is not the case in India. In almost all Indian schools, student attainment cannot be correlated to teacher/teaching effectiveness. This is so despite the fact that Indian teachers are academically more accomplished and qualified than their counterparts in US schools. It is very unfortunate that although from time immemorial India has contributed immensely to mathematics as a subject, many students struggle in math and fail in math because of lack of teaching focus with majority of Indian math teachers. They show a tremendous interest in covering the syllabus rather than discovering students' potentials and problem areas.

In India, there are very few pre-requisites for becoming a teacher. Individuals become teachers just because they have studied B.Ed courses. There is no license/certification which needs to be obtained to teach. In the US not only teachers need to obtain certification, it must also be renewed every 5 years through a rigorous process of professional development. Indian teachers often are not generally reprimanded for violations of the code of ethics as stringently as those in the US.

It is not an exaggeration if it is said that most schools in India thrive on academic attainment of students, which is the result of parental inputs and off-school academic programs and reinforcement. For US students, this is a great limitation because parents do not generally sit with students at home during studies/homework nor do they seriously monitor and support the academic progress of their children. Off-the school tutoring services are a rarity in the US; even if available, they are not generally affordable by the majority of the parents.

5. The "One Size Fits All" Dilemma

There are different levels of brightness among students. So they must be put to different levels of challenging tasks. Not all the students are bright in all subjects. Therefore, they must be put to differently challenging classes on the subjects they love or are good at. For example, a student can be really good in art. So she must be put in a challenging, advanced level art class rather than being forced into an advanced math class.

From our experience, we consider it important to separate the bright students from the normal class and provide them with tasks to hone their intelligence. Otherwise, in a regular classroom, they would feel frustrated and may even end up in giving up their interest (Colangelo and Davis, 1997).

The US education system has an established arrangement called the "Gifted Program" to cater to the needs of the bright and advanced learners. The Gifted Program provides extensive opportunities for bright students to exhibit their talents and maximize their attainments on par with their capabilities. In fact, in the US schools, we have different levels of academic concentrations: Regular, Advanced, Quest/Honors, and Advanced Placement. We are encouraged to take up any level provided we meet with the necessary basic entry-level criteria that provide evidence for our ability to withstand the rigor and academic pressure in an advanced level class. This is seldom possible in India even for the brightest student of the country because the Indian system adopts the "one size fits all" approach in education.

Typical Indian classrooms are a mix of bright, average, and struggling students. Theoretically, this is a good arrangement because bright students will act as motivators for other students to achieve higher. However, this is not really happening. Academically weak students become intimidated by the presence of the high achievers and even become jealous of them; instead of getting motivation from the brighter students, they withdraw from interactions and even turn into foes. So if you are bright, you just stayed bright and if you are failing, you will still be failing.

At the same time, the bright students are also not feeling good in Indian classrooms because their academic needs are not really fully catered to. Even though teachers can easily identify all the bright kids in the classroom, they do not provide any enrichment materials or higher order tasks that challenge the bright students and enhance their intelligence.

One more problem exists with respect to subjects that you can choose to learn in school, namely the Electives. Indian education system gives a solid focus on the sciences and mathematics or accountancy and commerce combinations. So, for a lot of students, the number of electives just boils down to one or two. Such a strict and narrow window denies the students the opportunity to explore additional — for instance: journalism, horticulture, parenting, consumer science, technology education/ engineering drawing, etc. US schools are remarkable in offering very many elective options for students.

In the US schools, not only are there elective options: a student can exceed the high school graduation requirements; that is, if, 20 courses are compulsory to graduate from high school (for the so called the Standard High School Diploma), a student can do any number of additional courses. A school district has Learning Academy, Career Academy, and other online study arrangements that encourage, facilitate, and support students to pursue additional courses, while they are pursuing the mainstream mandatory courses for graduation. These arrangements enable the students to properly and effectively invest their time in school; there are students who earn from 35 to 50 credits as they graduate from high school. Such students are in an advantageous position, as they apply to top-ranking colleges. This level of empowerment and support is nonexistent in the Indian educational system.

6. Exclusion in Inclusion

One major weakness of the US education system is the concept of "Inclusive Education" of the regular classrooms. Special needs students are taught in the same classroom as the students with normal/ regular needs. Inclusive classrooms are a wonderful concept but they do not become practically successful. In the first place, special needs students are not appropriately served in regular classrooms. This is because fully inclusive classrooms have students across the educational and developmental spectrum, ranging from typically developing students to students with significant disability. Such an exhaustive heterogeneity becomes a challenge for the teacher to find the balance to serve all the students. Although teachers are said to be trained in "Differentiated Instruction" they really struggle to differentiate the instruction and even if they successfully differentiate the instruction, it is not working because special needs students are not able to derive the benefit out of it. As we have seen it with the struggling students of the Indian classrooms, in the US schools, special needs students seem to be intimidated by the presence of regular education students. So instead of gaining in self-esteem in the presence of students with normal needs, students with special needs feel pushed down. We are told that many American parents do not like to put their children in special education service — they do not want the special education label hanging around their children. It is also important for us to point out honestly that the "Regular Ed Students" do not welcome the presence of the special needs students in the same classroom.

Indian education system on the other hand operates separate special schools for students with special needs. Special needs students are mainstreamed after they have been adequately trained in the exclusive schools to become fit to be accommodated in the regular classrooms.

In the context of special education, it remains a mystery for us to see that the US system, which has done so well for the education of the gifted has not been so successful with special education. The whole educational machinery is geared toward literally fulfilling the special education legal requirements rather than actually addressing the problems faced by special needs students. It is surprising why the special needs students are not treated special and given special attention. Why are they grouped with other students requiring normal course of instruction? How is it really possible for a teacher to be successful with a special needs student, while she has to attend to the needs of all students in the class?

As far as we have seen, co-teaching is also not really working for special needs students in inclusive classrooms. The original idea of integrating special education with regular education is to reduce various social stigmas and increase academic attainment of the special needs students.

It is also now an established fact that the root cause of most learning and reading difficulties are not handicaps such as deafness, blindness, and mental retardation. Rather, over 80% of students in special education services are there because of weak underlying cognitive skills. Therefore, identifying and retraining these cognitive skills is essential for overcoming learning struggles on a permanent basis. We are able to see that this is possible only in separate classrooms for special needs students. The instruction-based approaches in regular settings unintentionally overlook the underlying cognitive weaknesses in these students.

As we have already discussed, the more serious problem with special education is their inclusion in the State Tests, requiring them to pass the same State test as the "Regular Ed Students." This is very unfair because special needs students deserve special types of assessments, both formative and summative.

Conclusion

In this paper, we have made a comparison of the Indian educational system with the US set up in key areas in the light of our experience in both. The comparison is not complete though. Our findings show that the US educational system is superior in many respects. The benefits and drawbacks of such an arrangement are a subject of how well these are utilized and/or felt by the end users, namely the parents and the students. The niche that education has in Indian way of life keeps the educational machinery running. But this is not going to work in the long run. Students are not receiving due support from teachers in school settings. Primitive methods of teaching and learning continue to prevail. This does more harm than good. Technology integration, effective teacher preparation, and differentiated academic programs and instruction are necessary. On the US side, special education deserves a total rethinking. Teachers' lack of content knowledge and too many (frequently changing) expectations from the teacher deserve immediate attention and remediation.

Math is a universal subject and has no borders. Whether it is the US or India, we need new methods that are innovative enough to acknowledge students' ability to solve problems in different ways. Students must be facilitated to communicate about their mathematical thinking in ways that go beyond traditional homework and testing. They must be encouraged to understand and appreciate the connections between mathematics and other skills or subjects.

Whereas technology is abundant for use in the US classrooms, India has the abundance of very many innovative tools for math teaching: Vedic math, vocal math, play-way math, *kolam* math, and so on. Such methods are very traditional and have stood the test of time. They are well constructed for their robust use with students. But Indian educators have forgotten their own worth and are struggling with non-palatable models with self-inflicting, poorly written/printed math text books and printed notes to go with them. The net result is: students' poor performance in math at all levels.

For both good and ill, math is considered a hard subject in both India and the US. The US education system has devoted unprecedented attention on this single subject namely, math. In both India and the US we have a large number of students finishing with poor grades in the end-of-course tests.

We are aware that students' mathematical identities are shaped and developed by teachers (Cobb & Hodge, 2002). They influence the ways in which student's think of themselves in the classroom (Walshaw, 2004). We have witnessed in both the countries that effective teachers pay attention to the different needs of the students that result from different home environments, different languages, and different capabilities and perspectives. We have experienced in both the countries that if teachers have a positive attitude, it develops and raises students' comfort level, enlarges their knowledge base, and gives them greater confidence in their capacity to learn and make sense of mathematics. When students become confident in their own understandings, students will be more willing to consider new ideas presented by the teacher, to consider other students' ideas and assess the validity of other approaches, and to persevere in the face of mathematical challenge.

Success in teaching as a profession *versus* teaching as an occupation can be seen in the dedicated teachers both in India and the US. These teachers hold that "curricula are not collections of isolated pedagogical elements, but rather should function as coherent systems with a framework that lays out the underlying principles of learning and knowledge construction". Given this basic distinction, we feel that teaching of mathematics both in the US and India is confronted with the same singular problem namely, teacher effectiveness.

What we need in both counties is a new rethinking which involves a new environment for math education encompassing an assurance for effective teaching practices. What we rank as most important is the mental frame of teachers and parents. If they both realize the importance of math for their students/children, they would grab the best of the opportunities the present-day world order offers and they would support and stand by their children in accomplishing their educational goals. This would save the future generations of the world from being left in the dark vis-à-vis mathematics.

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AMATEUR'S QUEST FOR SIMPLICITY IN TEACHING MATHEMATICS

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Abstract

The author argues that applying teachable simplicity skills to students can lead to deeper understanding of the subject matter, and subsequently greater interest. Those students with a solid platform of these skills are poised to become teachers and lecturers themselves and thus be able to leverage that deeper understanding to reinforce this process. This paper demonstrates how simplicity skills led to discoveries in Number Theory and indicates that if simplicity skills are imbibed by students, they could make advancements in other fields such as Chemistry, Physics and Engineering. Finally, suggestions are made for unorthodox approaches to research and the role that universities can play to encourage amateur researchers and help them publish their research with the help of postgraduates and undergraduates at the academia. The suggestions presented in the paper emanate from the experience of his personal attempts to interface with the mathematics community.

Key Words: *prime numbers, prime number channels, simplicity, Goldbach's conjecture*

1. Introduction

Nothing excites a teacher more than seeing a student grasp a subject in its entirety while expressing his/her passion at the new found understanding. Mathematics is, after English, the second most important subject for most students. This prominence, I guess, is simply because it has provided an abundance of tools ready to be used in many other disciplines: sciences, business, day to day life, and so on. It is not an exaggeration that many branches of mathematics like algebra, geometry, and statistics are so commonplace.

Therefore if teaching methods are able to engage students at a deep level in mathematics and other subjects, those students will have their minds opened up to far greater possibilities and challenges than would otherwise be the case. The results are more likely to be beneficial to those subjects and to mankind in applied areas.

2. Proofs *versus* Understanding

Mathematics can be a difficult subject to grasp for students and the many abstract symbols and complex equations can become too high a hurdle. Although mathematics is seen as a science and is usually affiliated in the science faculty, there is an important and salient difference between mathematics and science, that can also act as a barrier to learning and understanding. That difference is that mathematicians search for logical proofs, while scientists search for plain understanding. This results in proofs that fail to provide a simple understanding of a problem and Andrew Wile's work on Fermat's Last Theorem ($Z^N = Y^N + X^N$ has no solutions where $Z, Y,$ and X are integers and N is an odd number > 2) is an example typical of this divide. This particular problem has simple demonstrable constraints (X^N has $Z–Y$ as a factor, and Y^N has $Z–X$ as a factor) that demonstrate that most examples cited by mathematicians as examples (and that almost fit there), are actually irrelevant. Andrew Wile's solution fails to acknowledge these constraints and so is far more expansive than it should be. The correct solution if it is ever found will show that the relationships between Z, Y and X are material and should be intelligible to anyone with some training in mathematics.

When a proof is complemented by an understanding of the same problem, not only is there a greater sense of accomplishment and satisfaction, but this understanding can also be more effectively used to provide understanding of related problems. Hilbert, the mathematician, once said that you can only consider you have completed a theory (or solved a problem) when you can explain it simply to the first man you meet on the street (Hilbert, Undated).

The following are examples of understandable proofs that demonstrate how they are much more effective than abstruse mathematical proofs.

Example 1: Why *Pi* is Irrational?

There are numerous mathematical proofs for why Pi is irrational but none is as simple and universal in understanding as the following.

Area is measured in squares. If you place the largest possible square inside a circle, you are left with 4 segments each bound by an arc and a straight line. Into each of these segments, the largest possible squares are placed, leaving additional segments bound by arcs on one side, and straight edges on the other sides. It is not a matter how many times smaller and smaller squares are placed into the remaining segments; there will always be smaller and smaller segments bound by an arc with straight edges on the other sides. Therefore the number that describes the area of a circle must be an irrational number when applied to the square of the radius. Once I demonstrated this to an English teacher who was visibly excited at being able to understand something that she had thought she would be unable to grasp.

The following diagram (Figure 1) shows the first step in placing the biggest possible square into a circle leaving four segments.

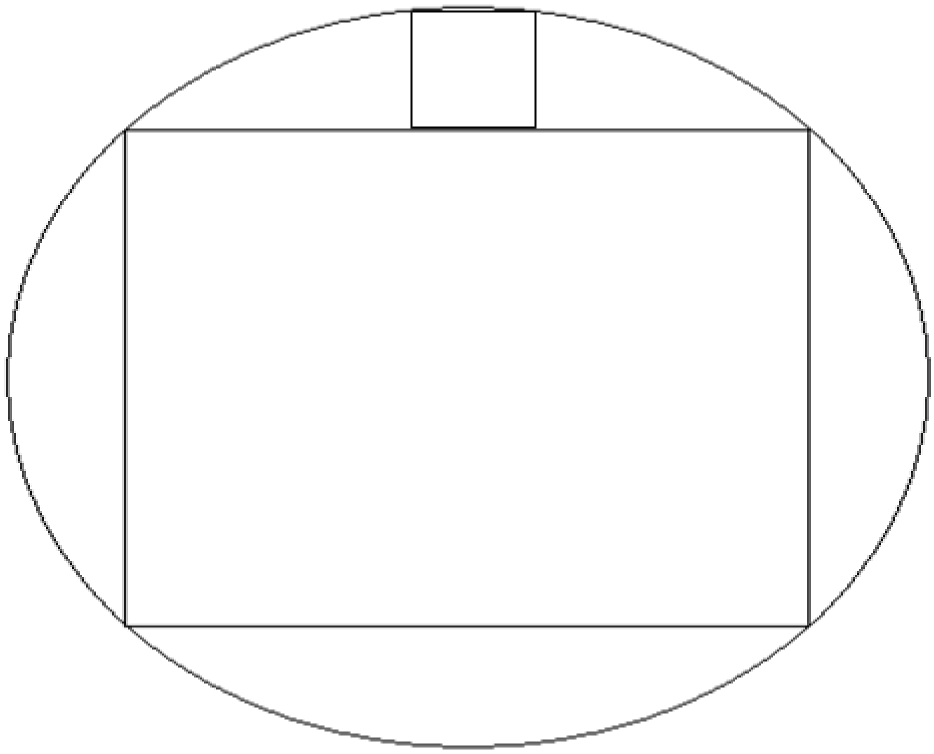

FIGURE 1
CIRCLE WITH THE LARGEST POSSIBLE SQUARE FITTED INSIDE.

Example 2: Why $X^2 + 1$ Cannot be Divisible by 3?

I had assumed this proof as universally known and it was only when I was challenged by an Australasian mathematician who incidentally said I was hopelessly lost in Number Theory, that I discovered it wasn't.

Either *X*^2%1 or X^2 must be divisible by 3 because *X*^2-1 is the same as (*X*-1) (*X*+1). Therefore, either *X*-1, *X* or *X*+1 must be divisible by 3 as they are all consecutive numbers and therefore *X*^2-1 or *X*^2 is divisible by 3, but *X*^2+1 cannot be. The traditional method for proving this is based on squaring (*X*-1), *X* and (*X*+1) which fails to provide three products that are consecutive.

Now just imagine how many mathematical problems have such universal validity. There are probably many published proofs that fail to provide a deep understanding in addition to Andrew Wile's work. Note that a deep understanding implies a simple description, not some feeling that there is some meaning in the work.

When we talk to students about which teachers they like most, we are able find that students like best those teachers who have a clear understanding of what they are teaching. Such teachers are able to make learning more engaging than those who are subject experts but lack the deeper understanding necessary to avoid having to say sometimes that, "that's just the way it is". Nothing in mathematics is really "just the way it is." If we want our students to excel in subject areas, teachers need to make students passionate about the subjects they learn, whether it is mathematics or something else. I consider generating creative tension in students is the most important quality of good teaching.

A mathematician makes his or her mark in mathematics by inventing a new mathematics which often equates to new complexity built upon abstraction. New mathematics is wonderful but this comes at a cost. The first cost is that understanding which lies in the realm of simple things, is effectively prevented, and the second cost is the obstacles to learning the new abstractive invention brings along with it, even though it may be an excellent piece of innovation. Very often, it is the obscure terminology that creates these barriers; if mathematics experts will be able to create in others' minds a deeper understanding of their mathematical innovation it would lead to successful dissemination of the new mathematical knowledge, down to the level of the layman. Such an effort calls for simpler descriptions of concepts (however intricate they may be) using fewer and handy terms.

Discoveries in Number Theory Made Using a Focus on Simplicity

The two examples I have cited above allow students to get a deep understanding of the math concepts. What I advocate here is the simplicity approach. By employing the simplicity approach, we can be very successful in revealing some astonishingly simple truths about prime numbers. I present below the following aspects of prime numbers I discovered using the simplicity approach.

Prime Number Channels—all multiples of 3 and 5 repeat their positions in each block of 30, leaving 8 Mod30 channels into which all higher prime numbers fall. This follows on from the observation (Jackson, Undated: 1) that after 2, the next most common difference between prime numbers is 30 and multiples of 30.

- All non-prime numbers occupying the prime number channels are paired when the equations creating them are threshed out and manipulated.
- The non-prime numbers are to all intents and purposes, evenly distributed between these 8 channels thereby implying that prime numbers are also evenly distributed with two channels with slightly less numbers.

Previously it was thought that prime number locations were entirely random, but this research now shows that their randomness occurs in eight distinct channels.

That prime numbers are simply not non-prime numbers which means that prime numbers acquire new simple attributes immediately even though some of those attributes seem obvious in retrospect (Jackson, Undated: 2).

When we employ the simplicity approach, we find that prime numbers appear to be a function of the differences between non-consecutive squares in that the density of prime numbers in the intervals between consecutive squares when divided by the square root of the lower square produces an asymptotic curve.

The simplicity approach takes us further to revealing that Prime Number Channels provide a deep explanation for the banding patterns of the scatter-plot of the number of prime number pairs plotted against even numbers in Goldbach's Conjecture (John, 2005).[1]

We are able to understand all of the above findings without a high level of training in mathematics. In fact this simplicity has also enabled us to unravel and expose to a much wider audience some of the very important but previously unknown constructs of Number Theory. In other words, people can understand complex things only when they are articulated by subject experts who have a deep understanding of the concept from simple mundane perspectives. Subject matter experts with sophisticated language and vocabulary do not and cannot produce effective learning in the learners. On the other hand, when subject experts become simplistic in their teaching methodology, then they need no complex or abstract vernacular to convey the concepts. Abstraction in my opinion is the consequence of lack of competence to see the concrete core inherent in concepts that are said to be difficult to grasp. Simply put, teachers must possess the basic simplistic understanding of the concepts they teach. Unless and until they acquire the simplistic understanding of the concept in hand, they will continue to use complex language and make learning harder.

2. Unorthodox Approaches to Research

In this section, I wish to stress upon the importance of unorthodox approaches to problems that are being taught whatever the field be. By orthodox I mean the conventional and unimpressive teacher-centered teaching methods. When traditional orthodox approaches to a particular problem or a set of problems have not provided any success or increased understanding, it may well be that in the teaching communication some hidden assumptions have been made and they are incorrect. The biggest obstacles in Prime Number research appears to be the prevalent assumption that prime numbers are directly-obtainable entities but with very little to show for the massive effort in terms of hours applied to them.

When I realized that prime numbers are not directly obtainable entities, I did not look upon prime numbers as indirect entities but I started to focus on understanding the non-prime numbers more and this eventually led to my discoveries.

When I first scribbled down a list of the prime numbers up to 100, I had a very strong sense that there was a pattern to do with 6 but was unable to deduce what it was. It was only when I did something that many mathematicians would not think of that I made my first breakthrough. Having been successful with this technique previously in an entirely different field, I wrote a small software routine to print all the even numbers up to 1000, and beside each one, the

1. *Goldbach's Conjecture* : all even numbers greater than two can be expressed as the sum of two prime numbers.

pairs of prime numbers adding together to give that sum. Over three sessions, I spent about five hours just looking at this list until I realized that there was indeed a very strong and impressive pattern. Not only was I reading across the lines, but also was making comparison between pairs of primes on one line and then another line. This pattern manifested as a very high occurrence of there being intervals of 30 between prime numbers. It took more than three years for me to go from this observation to the stalling discoveries I made. However, I wish to place on record that all the worthwhile discoveries became possible only because I began with a simplistic understanding of the problem and I continued to proceed with the simplistic understanding. In fact, I got into the prime number research when very big mathematicians of name and fame have been dominating in the field. What they could not see or express with their sophisticated approach became visible to me because of the simplicity approach.

I also wish to support the application of scientific method to mathematics (as well as other subjects). A look at the well-known scatter-plot of the numbers of pairs of primes against each even number up to one million (Figure 2; Jackson, Undated: 3), the distinctive bands must have made many a researcher wonder what was causing the patterns. Two years before I showed the reasons for the patterns, I had already thought that the bands were related to prime number channels. Conducting a simple classical experiment proved this very quickly and also revealed some of the reasons for the minor banding.

1. An Amateur's Experience Interfacing with Mathematicians

In my own experience, researchers and professors of mathematics can be distrusting of the merits of simplicity. Several times I have interfaced with various mathematics departments at three different universities in New Zealand and shown them

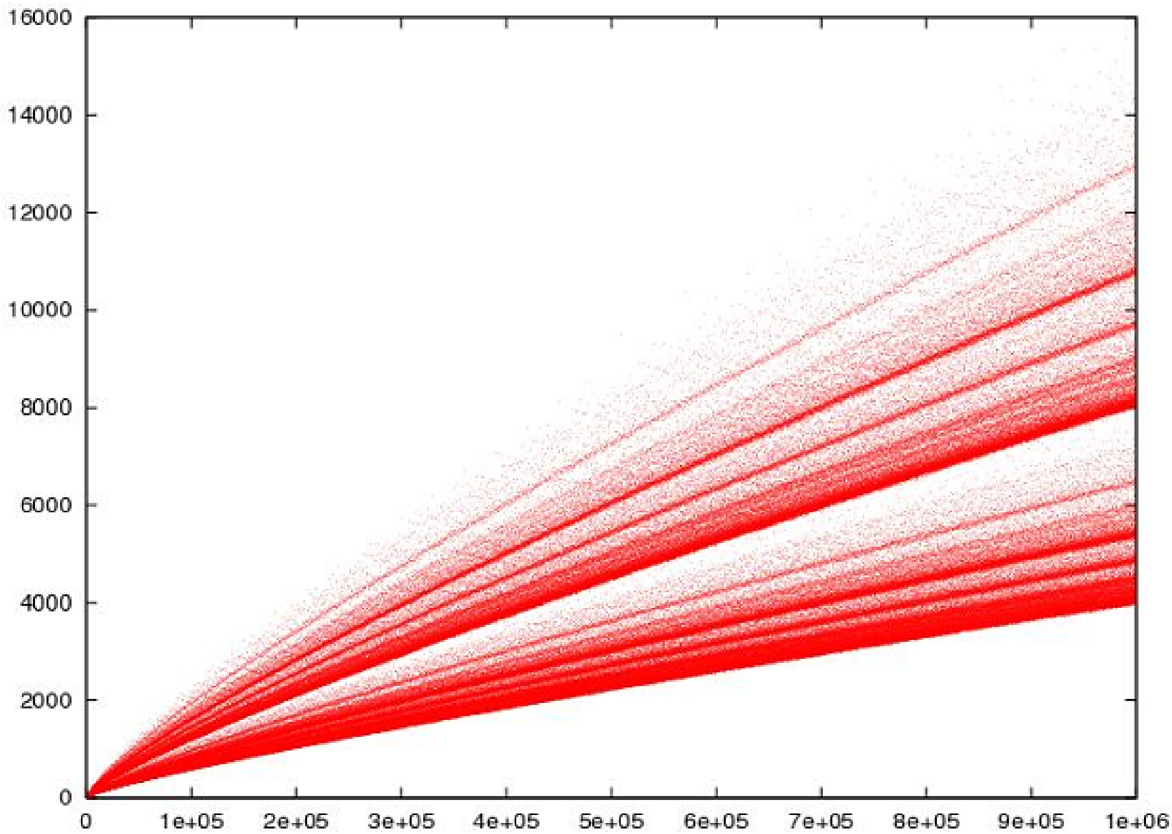

FIGURE 2

SCATTER PLOT OF EVEN INTEGERS AGAINST NUMBER OF PAIRS OF PRIMES (public domain graph).

the early stages of my prime number research as well as other novel research I was doing including the Four Color Map Theorem.[2] Excepting a retired senior lecturer at Auckland University, the response was only of mild interest with one case being negative. The negative case was with a professor who was regarded as the expert on prime numbers in New Zealand. His fellow researcher completely rejected the patterns of prime number channels I developed as not being significant in Goldbach's Conjecture; he also confused this with Dirichlet's work. This incident took place before I had discovered that prime numbers are evenly distributed throughout these channels. Never-the-less, these experiences were largely disappointing though fortunately I was little affected by this. Less robust individuals operating outside of the usual academic channels may have reacted less positively. My point here is that at least occasionally amateur researchers will make discoveries that have eluded professional researchers and therefore they should be encouraged. Such amateurs could be well inside our classrooms. Teachers need to be mindful of this and their simplistic teaching should be capable of motivating a student to explore the concept in simplistic yet novel styles without fear or intimidation.

The next time I visited the same professor, I showed him a new attribute of prime numbers. He spent nearly ten minutes trying to explain it to himself, on a whiteboard and then triumphantly turned around and said to me that it was obvious in a highly negative and scathing tone. Often a small observation is all it takes to develop new ideas to test and this was no different. This "obvious" attribute led me to plot a graph showing the prime number density between consecutive squares as described earlier on. Realizing that all non-prime numbers other than the squares of primes are the differences between non-consecutive squares, led me to believe that new prime numbers naturally occur between consecutive squares and the asymptotic nature of that graph (Figure 3) which follows gave evidence to very strongly support that claim. The graph represents the number of prime numbers in the intervals between successive squares against the square root of the lower square.

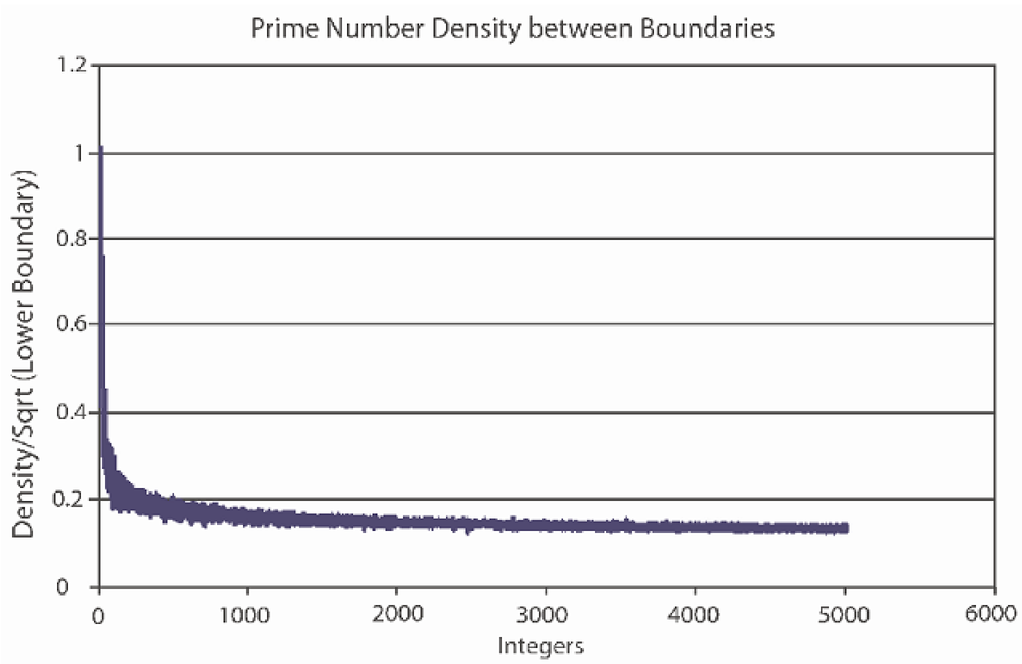

FIGURE 3. ASYMPTOTIC PRIME NUMBER DENSITY BETWEEN SUCCESSIVE SQUARE BOUNDARIES (own contribution)

[2] The four-color theorem -any map in a plane can be colored using four-colors in such a way that regions sharing a common boundary (other than a single point) do not share the same color.

6. Using Simplicity to Underpin Further Teaching

Chaos Theory has found many examples of seemingly complex behaviors being created by simple things (John, 2005) and therefore if one reasonably extrapolates the Chaos Theory out to cover most if not all problems, then if you want to understand a problem, you need to be in the domain of simple things.

If teachers can demonstrate to students the importance of simplicity, then you equip them with a respect for the underpinnings of most if not all problems they will face. When I was a young man on a medical research project, I was most fortunate to discover a shorter method than normal for measuring an obscure fatty acid (phytanic acid) and during this endeavor, what was surprising to me was how simple it was to discover it. This was in late 1982 when the established method was to methylate an extract using heat and methanol so that subsequent gas chromatography could easily separate a complex mixture into its separate components in the elution profile. Looking at their initial results where they had tried two different column packing materials, I realized that a simple mixture of the two materials would do away with the need for spending an additional day for methlylation. Another research team in the local hospital head-hunted me and I wasn't able to complete this research and write a paper. That lesson taught me that simple things are overlooked by professionals and that despite my average qualifications, I was really able to contribute significantly because of the simplicity approach. So it is not what one has in terms of academic and or professional qualifications which is more important for doing worthwhile research. It is a truth that my tryst with phytanic acid was instrumental to my developing a successful approach to solving many problems and gave me the confidence to delve into areas like mathematics culminating in my contributions to Number Theory.

7. Suggestions for Encouraging Amateur Research

Many researchers operate within time and budgetary constraints and it is often the amateur, who has the time and he/she is the one who is able to follow lines of research that others will not see merit in following or simply not see. Indeed, there are many situations where untrained amateurs have made important contributions to Mathematics, Srinivasan Ramanujan perhaps being the most notable. My recommendation is that university departments must reach out to amateurs on a regular basis and get their undergraduates and postgraduates involved in this process so it opens them up to the role of amateurs as part of their own instruction. This could be in the form of open house days when amateurs are specifically invited to discuss their ideas with academics and receive constructive feedback.

The other way that undergraduates and graduates could assist is to help amateurs write their papers with them as a valuable learning mechanism in communication and to test and improve their ability to write up contributions from amateurs or in future from themselves in the style and language demanded by peer-reviewed journals. In my own personal case, I published my findings on the Internet, chiefly because I was unskilled in writing mathematical papers for peer-reviewed journals, secondly because of my experiences communicating with some individuals in the mathematics community, and thirdly because I wanted to expose to the widest possible audience my findings in a style that maximally enabled that.

At one stage I offered to share in some research with my old university, providing just a few details of what I had found but this offer was not accepted. This was another attempt to go mainstream which eventually contributed to my decision to self-publish in the end. My aim was not to get acclaim in the mathematics community, but instead to demonstrate what someone from outside the system, equipped with the right approach, could accomplish. In some ways I am most fortunate to be the one to reveal what I have found, when what I have done requires little training coupled with a belief that there are simple things to be discovered that previously were hidden. Any number of people with the same belief and stubbornness could have achieved what I did with the same tools I used, which makes me the first one, not the smartest one, to get this result. How many other amateurs working alone out there are on the cusp of discoveries?

Mathematics attracts some and repels others, yet we all use it in our daily lives. My hope is that mathematicians don't stop at solving problems but are courageous enough to venture forward to also deeply understand them and this is when they can build bridges with the wider community.

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About the Author

Philip G. Jackson runs a small software company in Auckland, New Zealand. He's been writing software for over 26 years with many challenges which has led to him to developing a unique approach to solving problems which he has applied to mathematics over the last seven years culminating in discoveries in Number Theory. His interests include history, politics, science, mathematics, writing, and chess.

A CREATIVE LEARNING ENVIRONMENT FOR MAKING SENSE OF NUMBER

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Abstract

This article reports on the theme, 'Creative Learning Environment' (CLE) in the teaching of Basic Mathematics classes at Bronx Community College of the City University of New York: a CLE is arrived at after several cycles of teaching-research experiments. The CLE is designed to fit existing learning needs of the classroom and its objective is to provide avenues for learning of mathematics with enjoyment and mastery demonstrated through performance. The CLE is anchored in three supports: cognition, affect, and self-regulated learning practices.

Key Words: *mathematics-teaching-research, integration of cognition, affect and self-regulated learning practices*

1. Context of the Teaching-Research Experiments

The work reported here is part of a series of Teaching-Research experiments. Teaching-Research TR-NYC model (Czarnocha & Prabhu, 2006), is the methodology of simultaneous investigation of teaching while investigating its impact on learning with the explicit goal of improvement of learning. Teaching-Research according to this model is cyclic, cycling every semester through four stages of:

(1) Diagnosing student learning difficulties
(2) Designing instructional intervention in agreement with craft knowledge and research literature base
(3) Implementing and assessing intervention
(4) Refining intervention and implementing again.

Stage 1	Diagnosing student learning difficulties.
Stage 2	Designing instructional intervention in agreement with craft knowledge and research literature base.
Stage 3	Implementing and assessing intervention.
Stage 4	Refining intervention and implementing again.

FIGURE 1
TEACHING-RESEARCH CYCLE

The cycle is repeated over the course of the semester over a particular diagnosed learning difficulty at least twice. This way, there is a definite possibility to address learners' difficulties in the same semester. Secondly, the teaching-research cycle in its passage across semester provides continuous learning for the teaching-research team, and this learning is made available and beneficial to those learners first, in whom the learning occurred. The research-based materials generated, in each iteration, are embedded in the instructional material made available to each new cohort of students in the ongoing process.

Interdisciplinary collaboration occurs based on classroom learning needs; hence an interdisciplinary approach as represented in the current report is standard in teaching-research experiments. Collaboration within the discipline of mathematics occurs in the context of the teaching-research team which is distributed over two colleges in the City University of New York (Hostos Community College and Bronx Community College), and this provides the mathematical development and the appropriate pedagogical interventions to suit the needs of the classroom.

In the work presented here, our objective is to address student disenfranchisement from mathematics, especially in classes termed as Developmental or Remedial. 'Remedial' means the content of the courses is Basic Mathematics (Arithmetic and Elementary Algebra) and students taking these courses have not mastered the concepts in question in prior education, as determined by the entrance placement tests of the college. Disenfranchisement from mathematics is explicitly expressed by students in questioning the relevance of the very mathematics taught. It might express itself in deeper issues, such as questioning the one-to-one correspondence between point and number on the "continuous" number line, traditionally accepted as a foundational axiom[1] in modern mathematics. Such issues pertain to the need for a cognitively sense-making learning environment. The second front along which disenfranchisement occurs is affective. It is based on repeated experiences of failure or non-success in learning of mathematics, leading to a mindset of not being able to succeed in the subject. The work through the semester has to change this mindset and thus instituting creative supports is an essential component of the learning environment. The third front along which learners need support is in the development of the skill set needed to succeed, i.e., the self-regulatory learning practices carried out independently by successful learners have to be a part of the daily classroom use and this use must become part of the learners' repertoire. The Creative Learning Environment, thus designed is aimed at students' making sense of number and in the process, enjoying learning of mathematics while simultaneously performing with mastery.

The Creative Learning Environment has evolved over a series of semesters and in each stage can be seen as an integration of attention to:

1. Well-scaffolded cognitive challenge
2. Just-in-Time elimination of affective inhibitors
3. Embedded self-regulatory learning practices.

The thesis on which the classroom teaching research and consequently this article is based, is that, student learning is predicated upon the creation of a useful learning environment as well as a satisfactory handshake to that learning from the learners themselves. Each of the three elements constituting the Creative Learning Environment, 'Making Sense of Number', is briefly sketched in the present sequel.

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[1] I find the essence of continuity in the converse" (Dedekind, 1963: 11) in the following principle: "If all points of the straight line fall into two classes such that every point of the first class lies to the left of every point of the second class, then there exists one and only one point which produces this division of all points into two classes, this severing of the straight line into two portions."

As already said I think I shall not err in assuming that every one will at once grant the truth of this statement; the majority of my readers will be very much disappointed in learning that by this commonplace remark the secret of continuity is to be revealed. To this I may add that I am very glad if every one finds the above principle so obvious and so in harmony with his own ideas of a line; or I am utterly unable to adduce any proof of its correctness, nor has anyone the power. The assumption of this property of the line is nothing else than an axiom by which we find continuity in the line.

'Making sense of number' is the practiced theme whether in classroom discourse or in the computations and homework assignments, i.e., students' attention is constantly brought back to whether the discussions, computations, etc., make sense. In actual demonstrations of computations, which follow after conceptual exposition, sense making is explicitly carried out. This sense making can be seen directly by imagining the two-column proof method[2] in elementary geometry classes. Each entry in the left column is a computation and the justification for this step is in the right column. The justification provided in making sense, could be by means of a rule or formula, say the rule of exponents, etc., when such a rule is available. By repeatedly making sense, students are being drawn away from the habit of randomly choosing some piece of a remembered formula.

2. Cognitive Challenge

The common difficulty that students in classes of Remedial Mathematics face is the absence of a coherent schema. The topics in the curriculum have been seen by learners, perhaps on several occasions before. At the time of being in the classes of the team, there is a remembering of pieces of formulas, i.e., the remembering too is scattered. Thus, instruction has to simultaneously assist in sense making of the remembered mathematics, while it also assists in organizing this and the new knowledge to be gathered during the course, in a form making learning possible and successful in the current semester, and providing clarity on the concepts and their interconnectedness to serve the learner in succeeding mathematics classes in general. This means that the concepts have to be presented in a manner that starts with fundamental and easily constructible notions, on which are built others which either follow by direct construction, and where construction is not possible, the underlying assumptions, terminology, and axiomatic structure are explicit for the learner to navigate through, and make sense.

Following the teaching-research cycle, the team diagnoses the learning difficulties from student interaction, class discussion, and student computations. This begins from the first day of classes. Standard student difficulties such as incorrect use of formulas are to be encountered and addressed; however, along with it are more pronounced and deeper questions raised by students which require a deeper look into the history of the development of mathematical concepts themselves and where possible, to locate any rifts in history of the development. By going to the essence of the difficulties through the means possible, one attempts to find the source and so the simplest way to address the difficulty generally.

The concept map of *Story of Number* (Figure 2) has evolved in this way of investigation and deliberation. Note that the one to one correspondence between point and number which casts doubts on the continuity of the line, in the minds of students is avoided and the route is freed as much of assumptions as is possible.

Note that in the concept map above, 0 and 1 form the foundational blocks. The unit, connecting 0 and 1 is the first construction. From these building blocks, the number line composed of real numbers is constructed. Such a construction bridges the rift, mentioned by Lebesgue (1966: 17),

[2] The two-column format is the method by which many students are introduced to formal proof-writing in mathematics. The student divides the page into two columns. In the left column goes a list of statements, each one a consequence of the one above it in the list. Adjacent to each such statement (in the right column) is the reason why this statement does indeed follow from the previous step. Eventually, some non-trivial result is obtained.

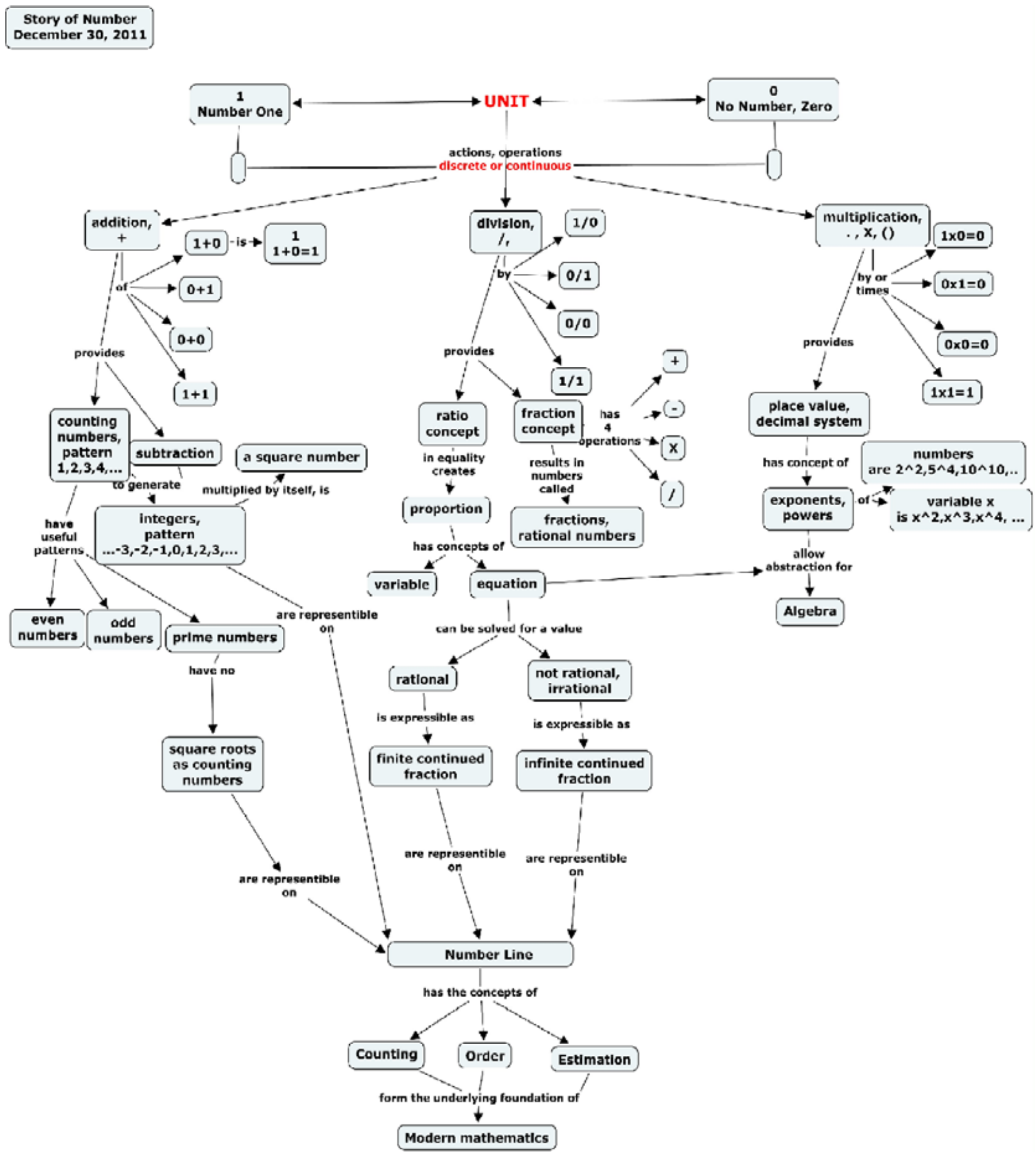

FIGURE 2
CONCEPT MAP OF THE STORY OF NUMBER

"… it may be more appropriate to ask why the decimal system is ordinarily used so little."

Primarily, the reason is that the Greeks, our models, did not use it. The decimal notation is not a heritage from the Greeks. As a result, everything dealing with that notation has been superimposed on Greek teaching and not incorporated into it. Quoting again Lebesgue (1966) "*Our teaching does not yet make full use of that historic event, which is perhaps the most important event in the history of science namely, the invention of the decimal system of numeration.*"

Note that in the construction of the concept map above, the usual difficulties pertaining to the irrational number by means of Dedekind cuts[3] have been avoided. This is done for two purposes. As

[3] In mathematics, a Dedekind cut, named after Richard Dedekind, is a partition of the rational numbers into two non-empty parts *A* and *B*, such that all elements of *A* are less than all elements of *B*, and *A* contains no greatest element.

Dedekind himself states in his *Essays on Number* (Dedekind, 1963), the assumption that he makes renders the continuous nature of the number line a mystery to students. Secondly, prime numbers, found so useful by S. Ramanujan, also prove their usefulness in bridging the gaps in student understanding of operations on fractions and in the construction of missing multiplication tables. After constructing a number pyramid of prime numbers with the help of multiplication tables, follow: prime factorization of composite numbers, finding the least common multiple to assist in finding common denominators for addition and subtraction of fractions. This provides ease of use with prime numbers. Hence, irrational numbers as the square roots of prime numbers are simpler constructions, without need for abstraction (such as those present in the construction of Dedekind cuts). The decimal system is continued to be used by means of continued fractions which offers a route to irrational numbers that can be understood by the beginning algebra student. Further, in this approach, the Archimedean axiom provides the natural connection with the decimal system.

In classroom teaching, depending on the specific nature of students' difficulties realized in each semester, discovery-based instructional sequences called *Story of Number*, are created; they are problem sets designed to provide a first principles grounded approach to problem solving, as much as possible. This way of designing problem sets is in accordance with the Moore Style Discovery Method (Mahavier, 1999) learned from W.S. Mahavier, student of R.L. Moore. The materials of *Story of Number* have the following features: they are designed in accordance with how development of concepts takes place, i.e., utilizing Bruner's recommendation that all of the concrete–iconic–symbolic stages (Bruner, 1966) be present in the instructional materials to allow learners to make the transitions back and forth as they make sense of the computations in question. Hence, for example, an instructional sequence for fractions would include these stages in the form of the fraction grids or pieces of the number line that allow learners to make parallel thinking connections with the drawings and their symbolic analogues. Further, with practice, by insisting on the drawings to accompany the symbolic calculations, students can find their own errors in thinking and correct them. The instructional sequences are made available to the students at the beginning of the semester as foundational learning and at the time of the midterm, the instructional sequence is modified to target the specific persisting learning difficulties.

3. Affect

The relationship between the cognitive and affective components of learning has been recognized (Falcão *et al.*, 2003; Gomez - Chacon, 2000). According to (Goldin, 2002: 60), when individuals are doing mathematics, the affective system is not merely auxiliary to cognition — it is central." Furinghetti and Morselli (2004) assert, "in the context of the discussion of mathematical proof, the cognitive pathway towards the final proof presents: stops, dead ends, impasses, and steps forward. The causes of these diversions reside only partially in the domain of cognition; they are also in the domain of the affect." Thus, along with the cognitive pathways, we have to consider [and impact] the affective aspect, which is described by DeBellis and Goldin (1997: 211) as "the sequence of (local) states and feelings, possibly quite complex, that interact with cognitive representations."

Affective inhibitors dominate in remedial classes where able learners see themselves as not "good at math." This attitude has to be reversed with learners' actively participating in the change. Interest in learning and change in attitude has to occur within the classroom, and has to be sustained by learners independently. Creativity provides the key for the required change by creating instances of cognitive dissonance (Festinger, 1957). The design of the

Creative Learning Environment is thus a mining of the mind, independently by learners, and with the support of the teaching-research team, to reach the capability set possible (Sen, 1992).

"Individuals come to "know" their own attitudes, emotions, and other internal states partially by inferring them from observations of their own overt behavior and/or the circumstances in which this behavior occurs" (Bem, 1967: 5). Prior experiences with learning of mathematics cloud student inclination to learn in the current mathematics classroom. In the learning environment with no negative classifications (Bloom, *et al*,.1975) despite repeated student hostility toward mathematics, over a short period of instructional intervention, cognitive dissonance (Festinger, 1957) is created with the supportive instructional team acting as the alternative new avenue to repeating the cycle of failure. Students' change in affect is visible. It is well known that the affective pathways and the cognitive pathways must act in unison, mutually benefiting the development of the other rather than inhibiting. In the mathematics classroom, daily living factors account for considerable absence of attention in the classroom. Mathematics thus sometimes has to take a competitive space with daily living. In the face of such difficulties, how can learners still find it possible to attend to the learning of mathematics?

Mathematics learning has to be enjoyable, and this enjoyment has to be experienced directly by the learner. In the classroom with two instructors team teaching the course (first and second authors), the required simultaneous attention to well scaffolded cognitive challenge and the mutually reinforcing attention to learner's absence of affective inhibitors' is made possible through their creation of multiple frames of reference (Koestler, 1964). Koestler posits that Humor, Discovery and Art are "three domains of creativity that shade into each other without sharp boundaries." Hence, while the central objective of mathematics is discovery, this process can be initiated through humor, and with a lighter mindset the learner can be led to discovery, and as art. This is what occurs in the team teaching approach where a counselor and a mathematics instructor pay attention to student learning. Through the team teaching approach learners remain engaged and hence eliminate momentary attachment to inhibitions and simultaneously through this involvement, create moments of understanding of the concepts under consideration. The team that is attentive to emerging moments of understanding of any individual student in the class makes these moments of understanding or discovery explicit. Learners, whether the person with the moment of understanding or others, learn to enjoy the moments of discovery and are encouraged to enter deeper into the discussion. The teaching research team is attentive to successfully navigate the learning or scaffolding the zone of proximal distance (ZPD) of the group. Consider for example the concept of fraction. Just the mention of the word is enough to have most of the class return to the state of "I'm no good at math, and I hate fractions." The affective inhibitors are strongly against learning. In such a context, in the team-taught environment, the counselor (first author) engages the students in using familiar representations such as cookies and then turning to pizzas, keeping the underlying problem, say 1/2 + 1/3 constant. The class, engaged in the shift of attention (cookies to pizza), keeps their attention on the problem and the mathematics instructor (second author) noting this increased attention span of the class, takes operations of fractions repeatedly by increasing the complexity from simple operations on fractions, to fractional exponents, solving equations involving fractional computations, and so forth. This progress in an Arithmetic class can allow the team to take students on an exploration and mastery of operations on fractions through the work on exponents, equations, and such complicated reasoning that generally appears in a second semester Algebra course.

Learning that our own craft knowledge of developing multiple frames of reference for learners to enter the thinking of the concept and sustain attention enough to create moments of understanding—is in accordance with the theoretical perspective on creativity (Koestler, 1964) and the development of bisociation,[4] the teaching-research experiments provided the following learning:

(1) A way to positively impact affect and cognition by the combined team teaching method;

(2) A way to connect with the theoretical supports developed by Koestler, such as the triptych as an explicit tool in the facilitation of bisociation.

Both are being introduced as standard features of the Creative Learning Environment, Making Sense of Number, facilitated by the Teaching-Research experiments being reported. Thus, while the discovery method of teaching had consistently been the objective in teaching-research experiments in remedial mathematics classes since 2006, there was need for a successful affective–cognitive intervention to understand how to create discovery as the means of sense making and learning by students.

In particular, to answer the question, what is creativity in the learning of mathematics and how can it be facilitated, we refer to Mahavier and Koestler, along with our craft knowledge as the route to its achievement in the classroom. With the use of the triptychs[5], bisociation is facilitated and the connections become explicitly clear to the learner as does the process of discovery and the independence of learning or ownership of learning to rediscover independently, is set in motion.

4. Self-Regulatory Learning Practices

Under the term Self-regulatory learning practices is included the mechanics of how to learn. What are the strategies employed by successful learners which have been left undeveloped for our students that we must incorporate into our instruction and materials in such a way as to make these habits second nature to their way of learning and hence, success. In particular, how can the semester serve as a means of creating the habit of learning how to learn?

The second and third authors have collaborated for numerous teaching research cycles on developing appropriate scaffolded exercises that can bring students' attention to the relevance of mathematics outside of the classroom setting. This has been through the embedding of Geographic Information System (GIS)-based mapping exercises into small-sized projects through which students can see the meaning of numbers that are derived from the census, see their representation on a map and compute simple percentages. Further, these exercises are so designed that students are able to communicate what they had found to their peers through an in-class written and oral presentation. The third author is a Librarian and his other role in the project has been to provide the infrastructural support for the project, i.e., where possible to find existing supports that lend themselves well to the pedagogical requirements of the teaching-research experiments. In this capacity, noting that the cognitive interventions being included in the learning environment did not always meet with warm reception by the students, his approach has been to find the scaffolding that could enable learners to enjoy the materials. The socio-cognitive theories (Vygotsky,

[4] Bisociation, a term coined by Koestler, is the creative leap of insight, sparking a new way of thinking.

[5] The triptych, for Koestler, (1964) connects the domains of Humor, Discovery and Art, and provides multiple avenues to enter thinking of the concept in question. In classes of mathematics, instructional triptychs deepen connections, providing for depth in learning.

1962; Bandura, 1993) with the goal of impacting the ZPD, led to consideration of self-efficacy and self-regulatory learning. As a result, the materials embed scaffolding, thus making them better utilized by learners.

In the team teaching by the first and second authors, the third author noted that it was cognitive apprenticeship that was being enacted, i.e, the team teaching was making thinking explicit and visible for learners and through the attention to students' responses, both cognitive as well as affective; students' in turn were making their own thinking visible and hence scaffoldable. The concept of a Cognitive Apprenticeship, described by Collins *et al.* (1991) states: "In ancient times, teaching and learning were accomplished through apprenticeship: We taught our children how to speak, grow crops, craft cabinets, or tailor clothes by showing them how and helping them do it. Apprenticeship is the vehicle for transmitting the knowledge required…" "We propose an alternative model of instruction that is accessible within the framework of the typical American classroom. It is a model of instruction that goes back to apprenticeship but incorporates elements of schooling. We call this model "cognitive apprenticeship." By participating in both positive affect and greater cognitive engagement, cognitive apprenticeship directly contributes toward the change from habit of dislike of mathematics to enjoyment and performance.

Mining minds, or creating the environment in which learners can mine their own mind to discover the underlying concepts and their connectedness to create the solution for the problem under consideration is actively facilitated through the cognitive apprenticeship of making the teacher-researcher's thinking and that of the students, visible, and hence approachable, by the learners independently.

5. Integrating Attention-to- Cognition, Affect, and Self-Regulatory Learning Practices

The 2001 edition of Bloom's Taxonomy has now the category 'Create' on the top instead of 'Evaluate'. To create is the as yet the apex of cognitive abilities which subsumes all the other lower order abilities. Coordination of the team's craft knowledge that resulted in positive change in the classroom (Fall 2010, Spring 2011), with Koestler's direct processes (triptych, multiple frames of reference, JiT attention to emotion/cognition/study habits) of facilitation of creativity through bisociation in the teaching-research experiments, has created a route to facilitate creativity and observe its development. The Just-in-Time instruction in the prepared learning environment provides repeated opportunity for moments of understanding to occur. Moments of understanding are many times publicly expressed in the classroom that becomes a safe space with the trust that develops rapidly in the team-taught open community approach. Moments of understanding allow for a pattern to form, and discovery becomes gradually an independent means of making sense. This causes students to change their attitudes of remembering how to add fractions, for example, and think through the given problem. The habit to original transformation referred to by Koestler, has become visible. Trust in a solo mathematics instructor in the classroom takes much longer to develop. The team-taught approach and the consequent CLE development provide a route to enhance group/class ZPD earlier in the semester, providing a greater duration of time for the enjoyment in own performance in mathematics to become a habit.

Conclusion

Facilitation of creative thinking is rewarding for the facilitators as well as for the participants, i.e., students in the classroom. Hence, the teaching-research environment provides learning with enjoyment for all the participants and the discovery approach toward learning propels new and interesting ways to integrate the educational enterprise to make mathematics interesting, challenging, and appealing to young and not young minds. Amabile (1983) states that domain knowledge, task motivation, and creativity-related skills determine creativity. The creative learning environment, 'Making Sense of Number, attempts to impact all three.

The standard exposition of basic mathematics as consisting of the four fundamental operations of addition, subtraction, multiplication, and division (and later square roots) is also being revisited to investigate the efficacy and clarity of thinking of division as the most basic operation. Division as the basic operation naturally creates the unit as the fundamental concept, comprising of 0 and 1 at each end. The number line is then seen as the replication of units, thus the continuity of the number line is not compromised or doubted as in the present case where students question the formulation of the one to one correspondence between numbers and points on the line. Further the number line seen this way is naturally continuous with the rational numbers interspersed densely.

Mathematics, whether described as "the queen of sciences", "the language of science", "technology undergirding technology", etc., in popular thinking, holds the role of developing the thinking technology in the mathematics teaching-research classroom. The thinking technology that all learners possess has to be aligned to make sense of the existing mathematical structures and learners have to be at ease with these mathematical structures so as to readily re-create them as required in their individual contexts, whether in the classroom or in general.

The Creative Learning Environment with its supports in three essential anchors attempts through iterative cyclic Teaching Research Experiments to learn from learners and create better conditions for their success.

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LOGICALLY FALLACIOUS RELATIVE LIKELIHOOD COMPARISONS: THE FALLACY OF COMPOSITION

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Abstract

The objective of this article is to contribute to research on prospective teachers' probabilistic knowledge. To meet this objective, prospective mathematics teachers were presented with a novel task, which asked them to identify which result from five flips of a fair coin was least likely. However, unlike previous research, the participants were presented with events, that is, sets of outcomes, as opposed to sequences, which have dominated previous literature on relative likelihood comparisons. Recognizing that previous changes to the task have resulted in new areas of research, a new lens—the composition fallacy—was utilized while accounting for participants' responses. Use of the new lens bolsters the contention that logical fallacies are a viable avenue for future investigations in comparisons of relative likelihood and research in probability.

Keywords: *fallacy of composition; logical fallacies; probability; prospective mathematics teachers; relative likelihood comparisons*

In a recent, comprehensive synthesis of research in probability, Jones, Langrall, and Mooney (2007) declared, "research on teachers' content knowledge in probability is sobering at best" (p. 934). They also noted, "research on teachers' [probabilistic knowledge] is limited" (p. 933). Recognizing the former point and given the dearth of research documented in the latter point and elsewhere (e.g., Stohl, 2005), the objective of this article, in general, is to contribute to research on teachers' probabilistic knowledge. More specifically, the objective of this article is to contribute to (1) an emerging thread of investigations into prospective teachers' probabilistic knowledge (e.g., Chernoff, 2008, 2009, 2011, 2012, in press *a, b*; Chernoff & Russell, 2011*a, b*, in press *a*, *b*; Chernoff & Zazkis, 2010, 2011) and (2) an established thread of investigations into comparisons of relative likelihood (e.g., Borovcnik & Bentz, 1991; Cox & Mouw, 1992; Hirsch & O'Donnell, 2001; Kahneman & Tversky, 1972; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; Rubel, 2006; Shaughnessy, 1977; Tversky & Kahneman, 1974; Watson, Collis, & Moritz, 1997).

To realize the general and specific objectives, a novel task for investigating comparisons of relative likelihood—the relative likelihood of events task—is rationalized and introduced. Further, this research will demonstrate that particular responses to the relative likelihood of events task can be accounted for with the *fallacy of composition* (i.e., because parts of a whole have a certain property, it is argued that the whole has that property). Accounting for responses with a logical fallacy also, potentially, paves the way for a new thread of investigations of research in probability—as responses in the past have traditionally been accounted for with normative, heuristic (e.g., Tversky & Kahneman, 1974), and informal (e.g., Konold, 1989) reasoning.

A Summary of Prior Research

Forty years ago, Kahneman and Tversky (1972) asked a group of individuals whether there would be more families with birth order sequence (using B for boys and G for girls) BGBBBB or GBGBBG. In a second, related question, the same individuals were asked whether there would be more families with birth order sequence BBBGGG or GBGBBG. Kahneman and Tversky argued that individuals who declared one sequence as less likely were reasoning according to the *representativeness heuristic*, where one "evaluates the probability of an uncertain event, or a sample, by the degree to which it is: (i) similar in essential properties to its parent population; and (ii) reflects the salient features of the process by which it is generated" (p. 431). Despite the subsequent permeation of the representativeness heuristic, there were concerns associated with the inferential nature of responses to the task.

Addressing the inferential concerns associated with the task, Shaughnessy (1977) introduced three developments: provision of an equally likely option, a request for reasoning and a reworking from a *least* likely version to a *most* likely version of the task. Despite these developments, the framework of the task remained, essentially, the same. For example, Shaughnessy had individuals compare the birth order sequence BGGBGB, first, to the sequence BBBBGB and, second, to the sequence BBBGGG. With the "provide a reason" development to the task, Shaughnessy was able to reinforce inferred results from Kahneman and Tversky's (1972) research and, further, establish new areas for investigation. For example, certain individuals determined, correctly, that the sequences BGGBGB and BBBGGG were equally likely, but according to their incorrect justifications, because both sequences had the same ratio of boys to girls (3:3).

Presenting an entirely different version of the relative likelihood task than had been seen in the past, Konold *et al.* (1993) provided individuals with four sequences and the equally likely option. For example, Konold *et al*. asked individuals "which of the following is the most likely result of five flips of a fair coin?" and provided them with the following options, "(a) HHHTT, (b) THHTH, (c) THTTT, (d) HTHTH, and (e) all four sequences are equally likely" (p. 395). Further, the researchers gave participants a most likely version of the task followed immediately with a least likely version. They found, for the most likely version, certain participants answered using *the outcome approach*—"a model of informal reasoning under conditions of uncertainty" (Konold, 1989: 59)—and for the least likely version subjects answered using the representativeness heuristic.

The framework of Konold *et al.*'s iteration of the relative likelihood task (Konold *et al.*, 1993) has, for the most part, been adopted by all subsequent research on comparisons of relative likelihood (e.g., Cox & Mouw, 1992; Chernoff, 2009; Hirsch & O'Donnell, 2001; Rubel, 2006). Alternatively stated, the relative likelihood task has not undergone any major alterations in nearly 20 years. To address the issue raised, the next section presents and rationalizes a major alteration to the relative likelihood task.

Research Instrument

As seen in Figure 1, the new task developed for relative likelihood comparison research—denoted the relative likelihood of events task—is heavily influenced by previous versions of the task. The new version of the task represents a unique blend of particular components found in the original task and the subsequent developments to the task.

> Which of the following is the least likely result of five flips of a fair coin?
>
> a) Three heads and two tails.
> b) Four heads and one tail.
> c) Both results are equally likely to occur.
>
> Justify your response...

FIGURE 1. THE RELATIVE LIKELIHOOD OF EVENTS TASK.

First, in a throwback to the original version of the relative likelihood task, the present task asks individuals to compare two events, as opposed to a larger number of events (or sequences). Second, two of the three task developments, introduced by Shaughnessy and used by all subsequent research (e.g., Cox & Mouw, 1992; Chernoff, 2009; Hirsch & O'Donnell, 2001; Konold *et al*., 1993; Rubel, 2006), that is, the equally likely option and the opportunity for response justification, are present in the current iteration. Third, the wording and framework of the task are similar to Konold *et al.*'s (1993) iteration of the relative likelihood task. Fourth, given that Rubel (2006) found "very few instances of such inconsistencies" (p. 55) between the least likely and most likely versions of the task and, further, given the lack of subsequent research confirming or denying Rubel's inconsistencies, the present iteration of the task asks individuals which event is least likely. In essence, the relative likelihood of events tasks is similar to all previous iterations of the relative likelihood task, except for one major difference: instead of presenting individuals with sequences of binomial outcomes, they are presented with events (i.e., sets of outcomes), which are subsets of the sample space.

Theoretical Framework

As demonstrated in the review of the literature, changes to the relative likelihood task have established new domains of research. For example, Konold *et al*.'s (1993) research, which asked participants to determine which of the sequences presented was *most* likely to occur, led to the now ubiquitous outcome approach. In a similar vein, given that a new iteration of the relative likelihood task is being introduced, a new theoretical framework is used for the analysis of results. Instead of using the "traditional" theoretical frameworks, a particular logical fallacy, the fallacy of composition, will be used to account for certain participants' responses to the relative likelihood of events task.

The theoretical framework for the current research will consist of one particular fallacy: the fallacy of composition. (See, for example, Chernoff & Russell (2011*a*, in press *a*, *b*) for research utilizing different logical fallacies and the fallacy of composition used to analyze traditional relative likelihood comparisons.) Put simply, *the fallacy of composition* occurs when an individual infers something to be true about the *whole* based upon truths associated with *parts* of the *whole*. For example: Bricks (i.e., the parts) are sturdy. Buildings (i.e., the whole) are made of bricks. Therefore, buildings are sturdy (which is not necessarily true). The impending analysis of results will demonstrate that certain participants inferred certain truths associated with individual coin flips to be true for events, that is, sets of outcomes. In other words, while, in the past, it has been demonstrated that participants' responses to comparisons of relative likelihood are a result of normative, heuristic or informal reasoning, the analysis of results will demonstrate that certain normatively incorrect responses (to the new task) stem from a flaw in reasoning.

Participants

Participants in the research were (n =) 63 prospective mathematics teachers enrolled in a methodology course designed for teaching middle-years (i.e., students aged 10 to 15 years) mathematics. More specifically, the 63 participants were comprised of two classes, containing 26 and 37 students, which were taught by the same instructor. Participants were presented with the relative likelihood of events task and were allowed to work on the task until completion. Of note, the participants had not answered any alternative (read: traditional) versions of the relative likelihood task prior. Further, the topic of probability had yet to be discussed in class at the time of the research.

Results and Analysis

Responses from the 63 participants fell into three categories. First, five individuals (or 8%) responded incorrectly that three heads and two tails is least likely to result from five flips of a fair coin. Second, 12 participants (19%) correctly responded that four heads and one tail is the least likely result. Third, the majority of participants, 46 (or 73%), responded incorrectly that both results were equally likely to occur.

Inconsistencies between responses and justifications were witnessed with both the normatively correct and incorrect responses to the task and helped further classify responses within each of the categories into subcategories. For example, of the 46 participants who responded that both results were equally like to occur, 20 of the 46 (or 43%) response justifications evidenced Lecoutre's (1992) equiprobability bias where the notion of equiprobability is misconstrued as anything can happen. Worthy of note, a consistency between justifications was evidenced for individuals who (1) declared four heads and one tail as least likely and (2) a sub-group of the 46 individuals who responded that both results are equally likely to occur—each of which are now commented on in turn.

Four heads and one tail

All 12 of the participants who declared, correctly, that the event four heads and one tail is least likely to occur after five flips of a fair coin were unable to provide appropriate justifications for their responses. Given consistencies associated with all 12 responses, three exemplary responses are presented for analysis.

Rupert: four heads and one tail are least likely to result because the coin is two-sided. Because the coin is two sided and has two different sides there is an equal chance that either side will result. The chance that the outcome will be tails is equal to the chance it will be heads.

Robert: Answer (b) is least likely to occur. It is unlikely that by flipping a coin five times your answer would result in four heads and one tail. Since the coin has a head side and a tails side there is a fifty percent chance you will get either heads or tails. It is just very unlikely that when flipping a coin it would result in four heads and one tail.

Amber: The least likely to occur is (b) because it would be more in favour of an equal end result.

As seen in the responses of Rupert and Robert, they pay particular attention to the "characteristics" of the fair coin. More specifically, they reference that the coin has two sides and that either side has equal chance of occurring or, as Robert states, "there is a fifty percent chance you will get either heads or tails." Further, the fairness of the coin, for Rupert and Robert, influences the ratio of heads to tails expected in the events they are presented. In other words, given that the coin is 50–50 or has a heads to tails ratio of 1:1, they expect the ratio of heads to tails in the event to be close to 1:1 (as exemplified in Amber's response). Given that the ratio of 4:1 isn't as close to the expected 1:1 as 3:2, they declare that the event with four heads and one tail is less likely than the event with three heads and two tails.

Presented within the fallacy of composition framework, Rupert and Robert's responses, declare that the ratio of heads to tails for fair coins is 1:1 (i.e., the brick). Further, they note that the event (i.e., the building) is comprised of five flips of a fair coin. Therefore, the event (i.e., the building) should also have a ratio of 1:1. For Rupert, Robert, and Amber, the expectation of a 1:1 ratio of heads to tails for five flips of a fair coin leads them to declare that the event with a head to tails ratio of 4:1 is least likely.

As seen in the analysis of the responses above, the fallacy of composition can account for *correct* responses to the relative likelihood of events task. In each instance, the equal likelihood of the fair coin, which is flipped five times is essentially transferred to the event. Worthy of note, the fallacy of composition was also present in certain responses from individuals who incorrectly declared that both results were equally likely to occur.

Equally likely

As mentioned, 46 of the 63 respondents declared incorrectly that both results are equally likely to occur and, while 20 of the 46 responses are accounted for with Lecoutre's (1992) notion of the equiprobability bias, 26 responses evidence the fallacy of composition. Given consistencies associated with all 26 responses, 4 exemplary responses are presented for analysis.

Randy: Because there is equal chance, one head and one tail.

Kelly: both results are equally likely to occur because you have flipped the coin five times, there is a chance each time that you can get either heads or tails, so there is an equal chance of coming out with the outcome of (a) or (b).

Randy and Kelly, in their responses, declare that both results are equally likely to occur because of the equal chance of one head and one tail for the flip of a fair coin. More specifically, Randy and Kelly's response each note that there is an equal chance of heads and tails for the fair coin (i.e., the brick). Further, the event (i.e., the building) is comprised of five flips of a fair coin. Therefore, the event should also have an equal chance of occurring or, in other words, both results (3 heads and 2 tails and 4 heads and one tail) are equally likely to occur. While (somewhat) implicitly presented in the responses of Randy and Kelly, Rudy's response is more explicit.

Rudy: Because each time you flip the coin there is a 50/50 chance of the coin being heads or tails. Therefore, each pattern created by flipping the coin answers (a) and (b) is equal in happening.

Rudy's response is presented, nearly verbatim, within the framework of the fallacy of composition. For example, (1) "each time you flip the coin there is a 50/50 chance," (2) "each pattern that is created by flipping the coin," (3) "Therefore each pattern...is equal in happening." Further, Rudy's response, as well as the response from Richard presented below, sheds light on how the participants take into consideration each individual toss of the coin (i.e., the brick) as part of the event (i.e., the building).

Richard: When flipping a coin there are only 2 outcomes: heads or tails. Therefore, there is a 50% chance of getting heads for one flip and 50% chance of getting tails for the same toss. It is just as likely to get 4 heads and one tail as it is to get 3 heads and 2 tails because looking at each individual toss the coin has equal chance of going heads or tails.

As presented in the four responses analysed above, the fallacy of composition can account for particular incorrect responses to the relative likelihood of events task. In each instance, the equal likelihood of the fair coin, which is flipped five times is, essentially, once again, transferred to the event.

Concluding Remarks

Demonstrated in the analysis of results, the fallacy of composition accounts for particular responses to the relative likelihood of events task. In particular, the fallacy is present in response justifications, which, *correctly*, declare that for five flips of a fair coin four heads and one tail is less likely than three heads and two tails. Correct responses with incorrect justifications note that the ratio of heads to tails for a fair coin is 1:1, that the event is the result of five flips of a fair coin, and, as such, the ratio of heads to tails in the event should be close to 1:1. (Of the options participants were presented in this task, the event three heads and two tails is closer, as a ratio, than the event four heads and one tail.) The fallacy is also present in the justifications for *incorrect* responses, which declare that both results are equally likely to occur. The justifications associated with (certain) incorrect responses, which declare that both results are equally likely to occur, note that a fair coin has equal likelihood of heads and tail, that the event is the result of five flips of that fair coin, and, as such, each of the two events presented are both equally likely to occur. In sum, the fallacy of composition is witnessed and can account for correct and incorrect responses.

Discussion

Research involving comparisons of relative likelihood has, historically, been focused on accounting for individuals' responses – both correct and incorrect. Particular developments associated with the task have allowed for parsing between the answer an individual provides and the justification for their answer. As a result, related research has developed a variety of theories, models and frameworks to account for incorrect, sometimes incomprehensible, responses. In more recent years, however, there has been a lack of developments to tasks that investigate comparisons of relative likelihood. In line with this point of view, the relative likelihood of events task has been introduced for use in future investigations. In addition, the analysis of the results has (re)opened a new area of investigation for future research on comparisons of relative likelihood: logical fallacies. More research involving more variations to the traditional relative likelihood comparison task (e.g., Chernoff, 2011) and different logical fallacies (e.g., Chernoff & Russell, in press *b*) will, eventually, ***determine to what extent logical fallacies are a part of teachers' knowledge of probability.***

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QUADRATIC QUEST: RAMANUJAN REMEMBERED- A MOVING SOLUTION TO THE INEQUALITY PROBLEM

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Abstract

Among functions in algebra, "quadratic inequalities" is posing the greatest challenge for students' learning, not only in USA but also in all countries around the world. Therefore the current methods of teaching quadratic inequalities must be drastically improved, newer strategies also have to be invented. Whereas multiple representation method was identified as a good strategy to teach functions, investigations in multiple representation methods have produced no success with students, although computers and specialized software have been liberally employed. The reason for this failure is not with the technology nor with multiple representation approach but with the lack of understanding of the true nature of the quadratic inequality problem. Although quadratic inequality is seemingly simple, there are about eight different inequalities in simple equation types and at least eight in the simple systems. The culminating point in quadratic inequality (as in linear) inequalities is: locating the solution on the graph and shading the region of solution, This is basically a visual-coloring exercise; but it is not simply choosing a color and smearing it in a region on the graph and declaring it as the region of solution. It involves a very thorough understanding of the complex process that precedes the coloring activity. Conflicts abound quadratic inequalities; These conflicts do require visual skills, but visual skills alone are not sufficient; students require spatial skills to move through the grid and arrive at the accurate evaluation of the graphic operation ensemble. This means that quadratic inequality does not really require any specialized technology but calls for the specialized human endeavor to examine and understand the nuances of the specific quadratic inequality, and tackle it with the appropriate spatial skills. In other words, we need to develop in our students the prepared mind and the necessary human visuo-spatial skills to first understand and appreciate the genesis of conceptual changes that are involved in the given quadratic inequality and then solve it. In the present investigation, an icon-mediated, vocalized physical response based visual–verbal, aural/oral multiple representation method is used as a creative strategy for teaching quadratic inequalities to high school students. The efficacy of this strategy monitored through a two-year study has been ascertained through standardized assessments, which provide evidence for student mastery in the concepts of quadratic inequalities.

The chief advantages of the present method are: (a) it eliminates the requirement for special software, (b) it is not a computer-based event, (c) it promotes cooperative learning, (d) it enables peer observation and peer teaching, (e) it incorporates visual-spatial learning, (f) it accords a pleasant feeling of accomplishment in the minds of the students, who are interested in some form of physical activity and oral voice in the class, (g) enhances aural/oral skills of students, (h) strengthens math vocabulary as well as processing and writing skills in math, and (i) it is inexpensive and can be offered in nay school, in any medium, in any country of the world. Ramanujan who started quadratic quest as early as in his childhood days and subsequently proposed (1916) the "Quadratic Ternary Form" is fondly remembered.

Key Words: *quadratic inequalities, creative math pedagogy, icon mediation, physical response, visual–verbal multiple representation, aural/oral multiple representation, inexpensive teaching method for math*

1. Introduction

"Quadratic Inequalities" is an important learning unit in high school mathematics curriculum of most countries. Being introduced at 9th or 10th grade, quadratic inequalities constitute the beginning of a learning continuum, which extends further both horizontally and vertically in various mathematical topics including algebra, trigonometry, linear programming, and calculus both at secondary and postsecondary levels of education (e.g. Hardy *et al.*, 1934/1997). However, data-driven evidence from student performance shows that students do not effectively learn the concept of quadratic inequalities (Allan, 2007; Champagne *et al.* 1993; Clement, 1982, 1985; Confrey, 1982; Ginsberg, 1977; Hoines & Fuglestad, 2004; Janivar, 1987; Rebecca, 2009; Zaslavsky, 1987, 1990; Zodhiates, 1988 and references therein). Chief of the problems daunting students (in quadratic functions in general and inequalities in particular) are the confusions in the algebraic procedures and the vague linkage between the algebraic form and the graphical representation, location of a test point, deciding the boundary, and deciding where to shade in the graph (Kaput, 1987; NCTM, 2000; Renuka, 2010*a*).

Mastery in quadratic inequalities is a pre-requisite skills for many other subsequent skills (mentioned above) and the deficit in it hampers students' academic attainment in a chain of math topics mentioned, affecting the overall attitude of students toward mathematics and the need to learn it. It is also necessary to point out here that in the last two decades school students' interest in mathematics has been declining very sharply (Dossey *et al.*, 1988). This trend is very marked in developed countries such as the USA (Anderw *et al*., 2009; Symonds *et al.*, 2011), where schools are said to be failing far too many children, especially those not of the majority culture (Hilliard, 1991). These realities urged the politicians to start the Math Reform movement and with active participation of organizations such as the National Council of Teachers of Mathematics (NCTM) and the Mathematics Achievement Partnership (MAP); these establishments after investigation made it clear that "current teaching methods must not only be drastically improved, we also must invent newer strategies that have never existed to meet our current needs" (NCTM, 1989).

Notable among the math reform efforts in quadratic inequalities is the 28th Conference of the International Group for the Psychology of Mathematics Education, which was convened in Italy in 2004. The published proceedings of this conference is a valuable document (Hoines & Fuglestad, 2004) that brought to focus the state of student difficulties in the quadratic inequalities. Pessia and Maya (2006) and Nikita Collins (2004) found that in quadratic inequalities, errors were common in both student and teacher work; this finding indicates that not only students, teachers also have problems with the cognitive demands of quadratic inequalities.

In the light of these issues, a systematic investigation on quadratic inequalities was carried out by the author over a period of 2 years at the Newton County School District, Georgia, which is a US Public School System. The outcome of the investigation is the development of a novel, dependable and effective method to teach quadratic inequalities: an icon-mediated, vocalized physical response based visual–verbal, aural/oral multiple representation.

In this paper, the issues with quadratic inequalities are first presented and then the creative teaching methodology is discussed; the efficiency of the method is demonstrated vis-à-vis the conventional method of teaching the quadratic inequalities.

2. The Issues with Quadratic Inequalities

From literature cited above as well as from direct observation and interviews with students, the following causes have been identified for the problems in quadratic inequalities. The top most difficulty is the convention of setting the function equal to zero. Because students are being taught to solve inequalities by setting the algebraic equation equal to zero, students force themselves into this habit whenever they see an inequality. However, there is no one-to-one correspondence between the graph and the set of solutions in *x*, for quadratic functions. The T-chart is totally unsuitable for solving quadratic inequalities because it does not help one to fix the solution on the graph. Also different graphs can lead to the same set of solutions and *vice versa*. One has to abandon information, namely the precise graph of the functions, to focus only on abscises of these points. Above all, students are not even sure that they learn and do the same mathematics when they solve inequalities algebraically or graphically.

The mathematical standard for academic attainment in the case of quadratic inequalities requires that students should be able to:

(i) Locate the solution for a graph by shading the region where the solution is;

(ii) Match an algebraic function with its graph from the many other graphical forms.

In general, the instruction for solving quadratic inequalities (Math 2 or the Algebra 2 course) is as follows:

1. Find the zeros by solving the equation you get when you replace the inequality symbol with an equals sign.

2. Find the intervals around the zeros using a number line and test a value from each interval in the number line.

3. The solution is the interval or intervals which make the inequality true.

In order to teach these skills, students are asked to take a test point, solve the equation for it and, if the obtained solution is true, the graph will be shaded inside; otherwise the graph will be shaded outside. If the inequality symbol has the "equal to" also (d" or e"), the parabola will be drawn in a solid line; on the other hand, if the inequality symbol is without the "equal to", it will be set in a dashed line.

This approach is fundamentally vague both for the students and the teachers. Student performance in assessments showed that students invariably gave incorrect answer for the questions on quadratic inequalities. When multiple-choice questions were given, those who selected the correct response did so by sheer luck of chance and not because they knew that was the correct answer. This means that students are conditioned to embrace failure in quadratic inequalities because of lack of understanding, caused by improper teaching. On the teachers' side, they say that they are following the methodology that is offered to them in the prescribed text books and the various professional trainings they received and the curricular outlines provided in the manuals issued by the Board of Education. However, they admitted that they are aware that with the existing methodology of teaching, students do not catch the "point" to answer the questions related to quadratic inequalities.

3. Quadratic Inequality is a Concept Change with Conflicts

If there are so many issues with quadratic inequalities, what is so especially hard with them?

Foremost, there exist some conflicts/dualities:

- The leading coefficient has a negative or positive sign

- The curve can go up or down
- The curves can have different vertex positions
- The curves can be broken or solid
- The inequality sign can be greater or smaller
- The inequality sign can have the equality symbol or not

Seemingly separate entities; each of these contradicting situations gets incorporated combined in the learning of quadratic inequalities, and they can occur in various combinations (there are about eight different inequalities in simple equation types and at least eight in the simple systems). It must also be noted that students have no prior knowledge on how to link these separate entities to arrive at these combinations. Added to this compounding effect is the influence that comes from the following three actions required to match the algebraic form and the graphical form of the quadratic function:

(a) Identification of a test point

(b) Finding whether it is a true or not

(c) Shading of the graph either inside or outside in accordance with step (b)

Thus the culminating point in graphical solution to quadratic inequality (as in linear) inequalities is: locating the solution on the graph and shading the region of solution, This is basically a visual-coloring exercise; but it is not simply choosing a color and smearing it in a region on the graph and declaring it as the region of solution. It involves a very thorough understanding of the complex process that precedes the coloring activity. Thus we understand that conflicts abound quadratic inequalities.

Conflicts yes, but these are natural phenomena in any educational situation especially when we have to extend prior-learned concepts in a new assembly or a new mode of representation. When a prior-knowledge is extended and manipulated, it is sure to lead to a conceptual change and such conceptual changes are associated with conflicts (Roschelle, 1995). Given that in any educational situation, there is likely to be some conflict owing to extensions to prior knowledge, Dewey indicates that educators must capitalize on such situations. In Dewey's words, "under the right conditions, a learner engaged with a problematic experience can effect a transformation of prior knowledge. This transformation restructures thought, perception, and action elements into a more integrated, coherent whole" (Dewey, 1938*a*,*b*) resulting in accommodation of the mathematical conception (Posner *et al.*, 1982). Extension of prior knowledge thus forces a theoretical shift to viewing learning as "conceptual change" (Strike & Posner, 1985; West & Pines, 1985) as opposed to the previous views of learning as a process of accumulating information or experience. In order to address issues with conceptual change, Scott *et al.* (1991) show two successful strategies, one based on explicitly working with conflicts, and the other based on building on correct prior knowledge. Herein, we must also remember that a conceptual change is continual and incremental and it is mediated by physical tools, and regulated by social discourse (Toulmin, 1972). So if students' prior knowledge needs to be corrected and the conflicts have to be tackled, we need to start with students' prior knowledge or use student ideas and/or familiar everyday concepts. At the same time, as Toulmin (1972) shows, students' every day concepts need to be coupled with some physical activity/tool and a social discourse must be initiated among the students and between the students and the teacher. It is worthwhile to recall the classroom technique of "Anchoring analogies" by Clement *et al.* (1989) and of gradual restructuring of student conception by Minstrel (1989) both of which use everyday concepts that students transact in the classroom.

What everyday concepts we need to start with?

To answer this question, teacher should ask, "What everyday concepts are being transacted in the classroom?" The everyday concepts are topic dependent. Quadratic Functions is the biggest of the math topics in Math 2 curriculum, spanning for 5 weeks. In this duration, students write, speak, and think certain 20 basic concepts everyday, which are listed in Table 1.

TABLE 1

EVERYDAY CONCEPTS IN QUADRATIC FUNCTIONS

"a" is positive	Function has greater than
"a" is negative	Function is greater than or equal to
a>0	Function is smaller than
a<0	Function is smaller than or equal to
Curve goes up	Parabola/curve
Curve goes down	Include/Exclude
Maximum	Inside of parabola/Outside of parabola
Minimum	Dotted line/solid line
Positive infinity	Overlapping
Negative infinity	System of equations

While beginning to start with everyday concepts, it must be recognized that conceptual change is a cyclic phenomenon (Figure 1) in which everyday concepts lead to new concepts through physical tools and social discourse, refinement of prior knowledge, and transformation of prior knowledge. A transformed prior knowledge becomes a new knowledge, which then becomes the everyday concept, and the cycle repeats. In the case of quadratic inequalities, from 20 individual prior learned everyday concepts, students have to develop 16 different new concepts (in fact, they are higher order composite skills). New concepts from the equation combinations will be as shown in Table 2.

TABLE 2

NEW CONCEPTS DEVELOPED IN QUADRATIC INEQUALITIES

Function has greater than symbol, solution is inside, dashed boundary	Function has greater than symbol, solution outside, solid boundary
Function has greater than or equal to symbol, solution inside, dashed boundary	Function has greater than or equal to symbol, solution outside, solid boundary
Function has smaller than symbol, solution inside, dashed boundary	Function has smaller than symbol, solution outside, solid boundary
Function is smaller than or equal to symbol, solution inside, dashed boundary	Function is smaller than or equal to symbol, solution outside, solid boundary

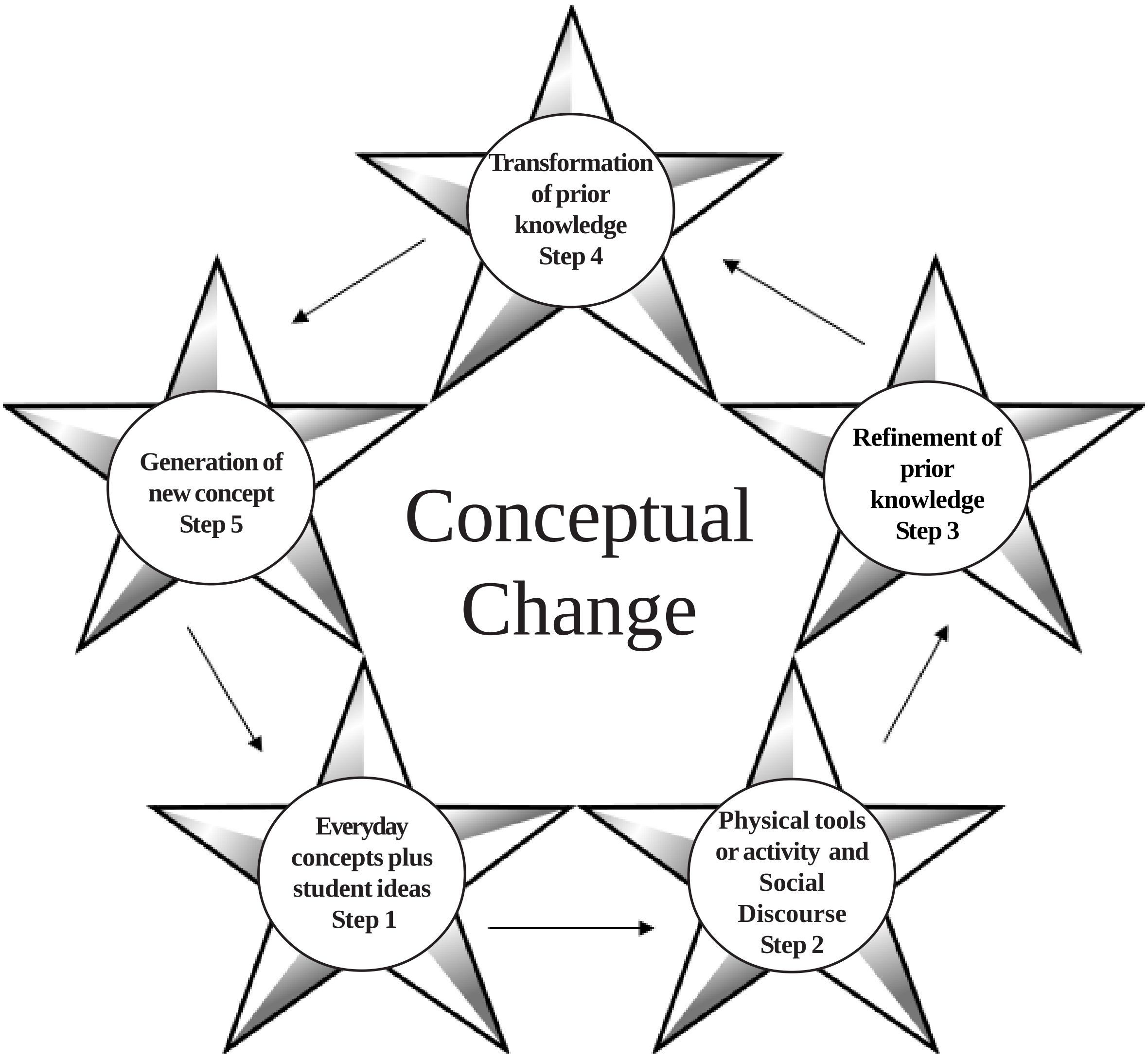


FIGURE 1. COMPONENTS OF CONCEPTUAL CHANGE.

4. The Need for Motion: Kinetics of Conceptual Change

Drawing upon Toulmin (1972), a key question arises: Why should be a conceptual change mediated by a physical tool? While attempting to answer this question, I was enthralled by two questions, "Why should mathematics and science teachers be interested in cognitive research findings?" (Mestre, 1987) and "How does dancing to music help promote creative expression?" (Stockman and Squires, 2006).

Dewey (1896) and Rousseau (in Dewey, 1916) showed that physical movement in the teaching/ learning process is a pathway to the cognitive processing of subject matter. Tapping, clapping, beats, gestures, etc, are encouraged in music learning. If teachers employ physical movement as a pathway to achieving students' musical expression, performance, and demonstration of overall knowledge of the subject of music, why not apply physical movements to math? After all and first of all, math and music are so related (Benson, 2006; Loy, 2006; Reid, 1995). Toulmin's echo of Dewey and Rousseau was sound.

In the present context of quadratic inequalities, there are conceptual switches involving extension of prior knowledge (i.e. everyday concepts listed in Table 1), understanding of and execution of such extensions, refinement of such extensions, and then the emergence of new concepts. Therefore, the incremental and continuous nature of the conceptual change was evident and the essentiality of physical movement was also evident. There was an additional reason for the physical movement. In the quadratic inequalities, graphing the function and shading the region of solution requires visual skills, but visual skills alone are not sufficient; students require spatial skills (Lean & Clement, 1981) to move across the grids to arrive at accurate evaluation of the graphic operation ensemble.

This uniqueness of the inequality situation explains why technology use and special software failed. This means that quadratic inequality does not really require any one to use technology but it demands the human endeavor (Goldenberg, 1988) spatially find the solution by visuo-kinesthetically examining and understanding the nuances of the specific quadratic inequality graph, and then tackle it with the appropriate spatial and physical skills.

In other words, like typical physicochemical phenomena, in quadratic inequalities, we are in a composite multi-step cognitive process in which the kinetics of movement in the individual steps of the sequence determines the rate of transformation of the prior-learned concepts (Step 1, Figure 1) to the new concept, namely the solution to the specific quadratic inequality (Step 4, Figure 1) .

5. Identification of Student Ideas

It is now clear that to establish and anchor conceptual changes, we need to begin with student ideas (prior learned or background knowledge) on their daily concepts of the topic. This is very important because student ideas could be genuine knowledge or misconceptions; also teachers may be aware of only some of these ideas. Authentic reasoning in mathematical contexts can be developed or occur if three conditions are present (Whitin and Whitin, 1999): (i) Real contexts; (ii) Mathematical reasoning motivated by the teacher; and (iii) Collaborative communities.

Teachers can motivate mathematical reasoning by encouraging a skeptical stance and by raising key questions about numerical information; for instance: (a) What information are we still not confident about? (b) What relationships do our data fail to reveal? (c) What questions did we fail to ask? (d) What information did we never monitor? While collaborative communities can be organized within the classroom by cooperative grouping strategies, teachers should be very careful in choosing real contexts. Most real phenomena are associated with a number of emotional, political, aesthetic, and psychological distractions that have to be pre-selected and eliminated because such issues of real phenomena are not linked to math standards and math academic attainment. They will also amount to waste of time in a math class. Anchoring math concepts and indispensable problem-solving heuristics are common across groups and have to be established well ahead of time before the actual intended learning experiences are provided. Therefore, when math teachers are involved in situations involving conceptual change contexts, such as the quadratic inequalities, it is worthwhile to check the strength of the background knowledge that the students have and begin discussing the background knowledge straightaway. Symbolic and verbal inequality notations, drawing pattern for the boundary, etc. are nothing but the real contexts directly associated with the conceptual change scenario. Teachers therefore, do not have to search for extraneous examples to build and or strengthen the background knowledge.

Therefore, in accordance with the above discussion and projection thereof, it is a good exercise to design an instruction for quadratic inequalities that would include students' daily concepts of the theme, physical tool, and discourse. To decide what physical tools can be chosen, the answer must be sought from the students. To provide the discourse component, the discourse has to be done with the students.

When a teacher decides to invent a new instructional method, it is essential to run a pilot study with a small sample group before presenting it to the whole group.[1] Such a pilot study was carried out in the present work with a set of students (mixed ability group) randomly selected for the purpose from the main group.[2] The pilot group was asked to suggest appropriate physical activities for each ordinary day to day concept they were encountering in the quadratic inequalities (Table 3).

TABLE 3

SIGNALS OFFERED BY THE STUDENTS FOR THE FEATURES OF THE GRAPH OF QUADRATIC FUNCTIONS

	Feature of the Graph	**Signal Suggested by the Students of the Pilot Group**
1	Dashed line	Single clap
2	Solid line	Double clap
3	Upward parabola	Standing
4	Downward parabola	Sitting
5	Matching	Facing forward
6	Not matching	Facing backward

Then using rulers and construction paper, the different graphs of the quadratic functions were prepared as iconic placards. Then the students were explained what they should look for when they relate an algebraic form of a quadratic inequality with its graphical form (namely the inequality symbol) through the physical activity. They were taught how to find whether the inequality symbol of the algebraic function matches or not with the overall trend shown by the graph. Once they learned it, the teacher showed each of the graphs along with the corresponding algebraic equation. At this point of time, the students were able to provide all the six signals in different combinations as determined by the curve and the inequality equation.

In the next step, the discourse with the students was on the representations for the solutions and shading of the graphs for quadratic inequalities. The details of this discourse are presented in Table 4.

[1] Through pilot studies, teachers may become motivated to make changes in their constructions, either to accommodate or to assimilate the experience (Millsaps, 2000). Ultimately, their instructional practices will have an impact on their students' learning.

[2] To avoid pre-contamination and cross-communication, the members were invited casually and without letting anyone know of the others who were coming for such a meeting.

BOX 1

DISCOURSE ON SOLUTIONS TO QUADRATIC INEQUALITIES AND SHADING OF GRAPHS

The culminating step in quadratic inequalities is identifying the region of solution on the graph and shading that region. In order to facilitate learning of these twin actions, students were taken through a discourse:To begin with, they were asked to explain what they understand by "matching." The spontaneous reply from the students was "agreement." Then they were asked what quality brings two people together in agreement. They were given various items such as beauty, stature, skin color, wealth, thoughts, feelings, love, and affection. They discussed and came out with the following answer:With agreement in beauty, stature, skin color, and wealth — two persons match well only externallyWith agreement in thoughts, feelings, love, and affection — two person match well internally. They were then asked which of the two matching would give a true agreement. They invariably selected the latter. If the two were in excellent agreement in their thoughts, feelings, love, and affection – how they would describe the bond between the two to be. The response for this was: "solid." The next question was: about how they would represent a bond between people that is not solid. The response was 'broken, fragile'. The next question was which two individuals would find solution to their problems? Students said people who have internal compatibility will find solutions internally and those who have external compatibility will find solutions outside. They were asked to visualize a quadratic function and its graphical representation as two human individuals and they were asked to shade the region of the graph where agreement takes place. They were suggested to treat the curve as the bond between the two. With these, the students perfected the graph with a solid line or a dashed line with shading inside or outside.

The next task for them was to represent a system of quadratic inequalities. For this task, they were enabled to recall the behavior of a system of linear equations and then project the behavior for two parabolas one being upward, one being downward. Eventually, they were successful in transforming the concept of point of intersection as applied to a set of linear equations to a set of parabolas. However, to arrive at the "region of overlap" there was a time lag, yet the discovery came abruptly. There was obviously a refining of the prior concept, reshaping of the prior concept, expansion of the prior concept, modification of the prior concept, and birth of a new concept (Figure 3).

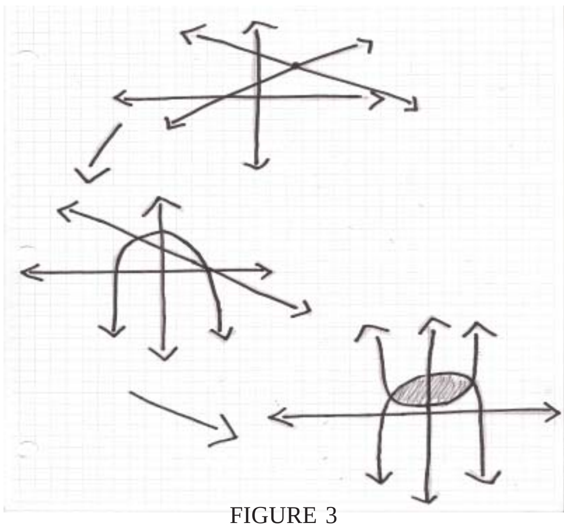


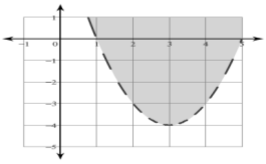
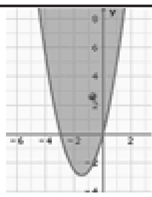
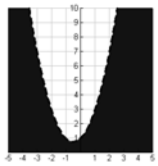
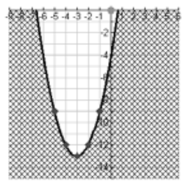
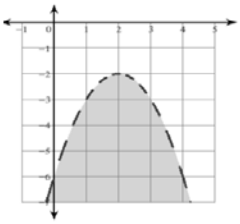
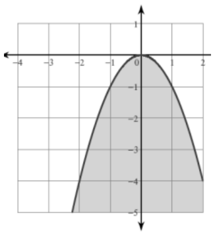
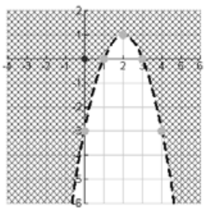
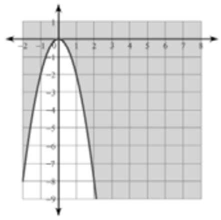
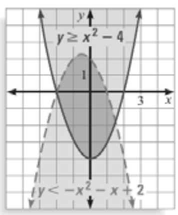
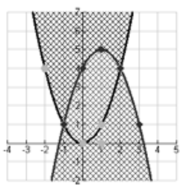
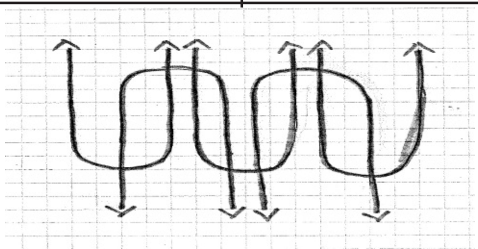
FIGURE 3

STUDENT WORK DEMONSTRATING THE BIRTH OF A NEW CONCEPT FROM PRIOR CONCEPT

Thus, the students were able to identify the region of overlap of the parabolas as the solution for the system of quadratic functions; subsequently, they also brought out the appropriate physical activities for the overlapping of parabolas: "give me five" if one parabola had a solid line and the other had a dashed line and a "handshake" for the overlap of two parabolas that had solid lines. One student went a step further querying, "Why should we restrict the overlap of only two parabolas? If several parabolas overlap, we can have a human chain as a good representation of such an overlap." The physical activity suggested by this student along with the icon is presented in Table 5.

BOX 2

STUDENT-CREATED PHYSICAL RESPONSE FOR THE ICONIC REPRESENTATION OF THE QUADRATIC INEQUALITY

Icon				
PR	Standing facing forward and clapping once	Standing facing forward and clapping twice	Standing facing backward and clapping twice	Standing facing backward and clapping twice
Icon				
PR	Sitting facing forward and clapping once	Sitting facing forward and clapping twice	Sitting facing backward and clapping once	Sitting facing backward and clapping twice
Icon				
PR		 		

Best results are obtained by repeating this exercise (Icon ”! PR) spirally both in the horizontal and vertical directions. Oral statements such as the following add further strength to the efficacy of the strategy

“up, greater than, match” “up, greater than, mismatch” “up, greater than or equal, match” “up, greater than or equal, mismatch” “down, greater than, match” “down, greater than, mismatch” “down, greater than or equal, match” “down, greater than or equal, mismatch”

Picture Courtesy: Google Images; Reproduced with Permission from the Source Owners.

6. Multiplicative Power of Iconic%Vocalized PR%Visual%Verbal Multiple Representation

Given the fact that student’s construction of knowledge is a result of psychological *and* physical activity on the external environment. (Bybee & Sund, 1982), the present scheme of learning experience was designed (as shown in Figure 5) by a judicial combination of (1) Iconic representation; (2) Physical response (PR) to iconic representation; (3) Visual representation of the icon for which PR was given; (4) Verbal representation of the visual representation that was made in the previous step as shown below.

Icon	Vocalized PR	Visually Representing the Icon	Aural/Oral basedVerbal Representation of the inequality.

The power of each of these four techniques is elaborated in Tables 6 to 9. The order in which these representations are arranged in the multiple representation scheme is noteworthy. At each step, the power of the technique enhances because of the power of the immediately following step. Mason (1987) suggests a spiral movement based on the distinction between iconic and internal representations of a concept. Therefore, techniques (3) and (4) have been repeated both forward (3 to 4) and backward (4 to 3). So also were techniques (1) and (2) repeated forth and back to strengthen the visual and aural/oral skills.

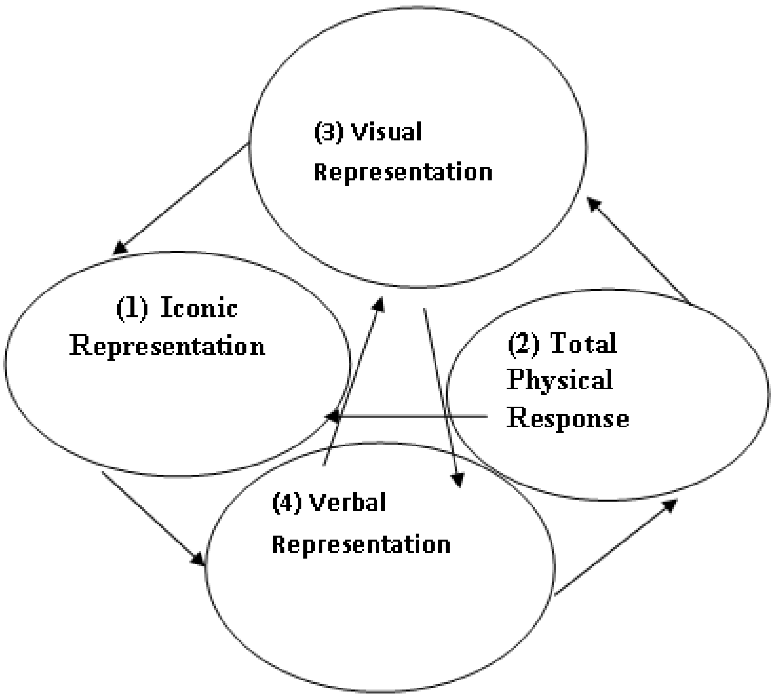


FIGURE 5
THE SUCCESSIVE SERIES OF TECHNIQUES USED IN THE PRESENT STUDY

BOX 3

THE POWER OF LEARNING THROUGH ICONIC REPRESENTATION

Students seem to choose a composite symbolic framework to process mathematical information rather than a simplistic symbol (Hambdan, Undated; Janivar, 1987; Kaput, 1987). An icon is not a simple symbol but it is a specially custom-designed unique composite visual that is accurately specific for a given concept. Icons are not pictorial scenes or maps that can be interpreted in different ways by different people or by the same person at different times/situations. An icon has one and only straightforward knowledge/concept associated with it. There are no ambiguities with icons. Icons take advantage of people's visual recognition by tapping into the innate hunting skills of recognizing the visual for its single meaning instantaneously. Text does this as well, but to a lesser extent than icons because text needs more time to process and isolate than an icon. The power of iconic representation in pedagogy was first recognized by Bruner (1966). Since then various applications of icons have emerged; however, these applications are abundant in non-academic settings as opposed to academic settings, typical of classroom teaching. According to Harvey (1991), students can acquire significantly deeper understandings of the function concept by working more directly with carefully designed visual representations of functions. Although he has not mentioned icons specifically for this purpose, there exists abundance of opportunities to employ icons in our classroom teaching for math functions. However, it must be remembered that icons are meant for initiating actions. This means that icon has to be coupled with an action, especially a physical response typical of the internal representation of the concept transmitted by the icon. Physical movement in the teaching/learning process is thus a pathway to the cognitive processing of subject matter.

BOX 4

THE POWER OF LEARNING THROUGH PHYSICAL RESPONSE

Physical response (PR) is not a new strategy but is well known from time immemorial in all cultures. Both ethnomathematics and place-based pedagogy are based on play-way mathematics typical of traditional games in localities, regions, and cultures of the world. PR is distinct from kinesthetic hand-on math in the sense that it is not learning by doing but it is learning by playing wholesome. As a more personable, joyful, self-satisfying psychological excitement, PR ranks superior to a hands-on activity. PR is aptitude-free, working well with a mixed ability class, and with students having various disabilities including English Language Learners. Class size need not be a problem, and it works effectively for children and adults. In modern times, PR has been employed in classrooms for teaching language/ vocabulary but has not been widely adopted in math.

BOX 5

THE POWER OF VISUAL REPRODUCTION OF LEARNED CONCEPTS

Students use visual thinking and reasoning to represent and operate on mathematical concepts that do not appear to have a spatial aspect (Lean and Clements, 1981). Students can acquire significantly deeper understandings of learned concepts by visually reproducing the particular learned concept by drawing and sketching (Harvey, 1991); this is because internal representation is intimately linked to short-term or working memory. Based on organization of visual computation into WHAT/WHERE modules of physical response Jeff (1995) shows that visual perception is closely linked to motor skills and working memory. Therefore, in this study, visual reproduction of the iconic graph is arranged

Contd ....

BOX 5 - Contd

immediately after PR to enhance student's working memory and to further retention of the knowledge about the quadratic inequality associated with the icon. Visual reproduction of learned concepts is prevalent in many subjects; in math for example, in early math courses this strategy is widely used in fraction concepts, geometry, and operations (Whitin and Whitin, 2011). Key–Information–Memory Cue (KIM) charts also serve the same function of visual representation.

BOX 6

THE POWER OF VERBAL REPRODUCTION OF VISUALLY ACQUIRED CONCEPTS

Aural/oral is the most basic level of skill learning sequence, the foundation upon which all higher level skills are built. Although popular in language settings, in math efficacy of aural/oral learning is exploited in multiplication table mastery. In the present case, since there is a unique verbatim for each quadratic inequality, verbal reproduction/description of visual drawing or sketch the student prepared becomes indispensable. Orally telling aloud and writing the correct label after the visual representation has been incorporated in the present scheme because it is an effective strategy that enhances inference learning and in turn further enhances retention of knowledge. As a further measure to strengthen verbal representation, questions in the assessments were composed of verbal statements about problems. Similarly the multiple answers listed for a given question are mostly verbally constructed.

As such what is prescribed through the present multiple representation is a semiotic activity, (a technique introduced by Charles Sanders Peirce (1839–1914)). Semiotic activity is defined as: "the activity of relating a sign and its meaning, including the use of signs, the activity of investigating the relationship between sign and meaning, as well as improving the existing relationship between sign (or sign system) and meaning (or meaning system)" (van Oers, undated). Young children involve often themselves quite often in semiotic activity based on the construction and use of self-made diagrams of real-life situations and objects (Van Oers, Undated). In the practice of semiotic activity for children, schematic multiple representations are often used as a starting point, to provide potentially meaningful objects for conversation and study.

7. Evaluation of the Efficacy of the Methodology

To evaluate the efficacy of this learning method for quadratic inequalities, two comparable groups of students of Math 2 with were selected (Renuka, 2010*a*). One group was taught by the routine instruction, while the other group was taught by the present method. The conventionally adopted routine method of instruction consisted of explaining to the students, (with the help of a power point presentation) how to solve quadratic inequalities by the location of a test point. This was followed by a worksheet on which the students were asked to match the algebraic equation with the graph. The four problems on one side of the worksheet were guided-practice; the four problems on the other side of the worksheet were given as independent work (Renuka, 2010*a*). During this instruction, it was transparent that the students of the Control group, taught by the conventional routine method were really struggling to match the algebraic equation with the graph. Some of the students did not even make an attempt. Even after the teacher re-taught the procedure, students still fumbled to match the graph with the function.

The latter (Experimental) group showed enthusiastic participation and they scored remarkably higher than the former group (Figure 5). As a next step, the present methodology was adopted school-wide in all Math 2 classes (totally four groups). Not only did all the four groups secure high scores in the class test, they all answered correctly the two questions on quadratic inequalities in the District-wide Benchmark test.

This method has been highly useful in teaching quadratic inequalities to not only for mainstream math students but also for students with learning disabilities and other special-needs students. Excellent results have been obtained with at-risk student in alternative schools.

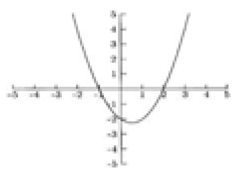
In the present time, we are talking much about the achievement gap (Williams, 2003); academic gap is generally given as the difference in the academic performance between students with disabilities and students without disabilities in State Tests. The gains in academic attainment realized with special needs students in the present investigation are very encouraging.

Many mathematics educators are advocating methods of assessment other than frequent tests, quizzes, and daily worksheets (Williams & Wynne, 2000). In fact, the NCTM's Curriculum and Evaluation Standards states, "The assessment of students' ability to communicate mathematics should provide evidence that they can express mathematical ideas by speaking, writing, demonstrating and depicting them visually" (NCTM, 1989). The post-pedagogy in-class exercise used in the study to evaluate the mastery of learning (Table 10.1 and 10.2) was designed incorporating all the requirements of NCTM evaluation standard.

Math Teachers have conventionally been spending a lot of time teaching quadratic functions mainly by algebraic form (Dickey, 1993) and of late additionally in graphical forms. However, it has been shown that students struggle to establish connection between the algebraic and graphical representations; also graphical display on graphing calculators poses additional challenges to students' understanding of graphical representations. Schwarz *et al.* (1990) and Schwarz and Dreyfus (1995) proposed computer-based instruction to address these issues; however, such methods do not have the built-in human element/ human endeavor to ponder the nuances of quadratic inequalities, and develop the human skills to understand and appreciate the genesis of conceptual changes and convolution of new concepts from the prior-learned mundane day to day concepts.

PR, the efficacy of which is demonstrated in the present study can be introduced in many different ways, ingenuity is the key. If the teacher has the open mind, if the schools are determined to instill the right math attitude in students, and if countries look for inexpensive and efficient pedagogical methods, then PR would prove to be an indispensable technique. PR is not a new teaching tool; PR is everywhere in all cultures of the world freely present as part and parcel of many traditional games and arts of the cultures. Tapping their potential for math teaching is in the hands of the educators.

TABLE 10.1
IN-CLASS EXERCISE ON QUADRATIC INEQUALITIES.

Sign of *a* and Direction	**Equation**	**Line pattern**	**Shading**	**Exercise (add dot when needed and the right-most box has the five characteristics one below the other**	
a is positive or a > 0 curve goes from minimum to positive infinity.	y >	Dotted	Inside parabola		1) a is: 2) Range: --- to -- 3) IEQ: 4) Line: 3) Shading:

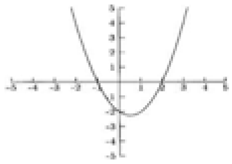
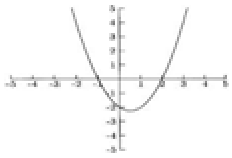
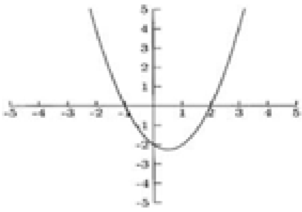
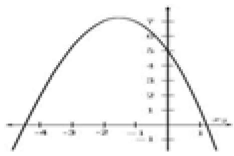
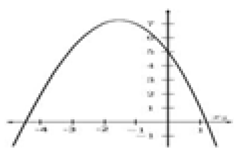
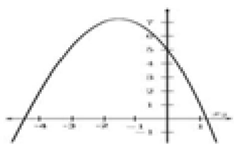
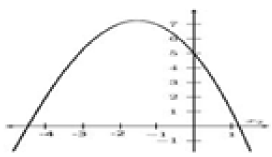
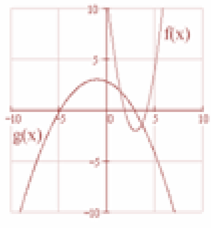
a is positive or a > 0 curve goes from minimum to positive infinity.	y≥	Solid	Inside parabola		1) a is: 2) Range: --- to -- 3) IEQ: 4) Line: 3) Shading:
a is positive or a > 0 curve goes from minimum to positive infinity.	y<	Dotted	Outside the parabola		1) a is: 2) Range: --- to -- 3) IEQ: 4) Line: 3) Shading:
a is positive or a > 0 curve goes from minimum to positive infinity.	y≤	Solid	Outside the parabola		1) a is: 2) Range: --- to -- 3) IEQ: 4) Line: 3) Shading:
a is negative or a < 0 curve goes from maximum to negative infinity.	y>–	Dotted	Outside the parabola		1) a is: 2) Range: --- to -- 3) IEQ: 4) Line: 3) Shading:
a is negative or a < 0 curve goes from maximum to negative infinity.	y≥–	Solid	Outside the parabola		1) a is: 2) Range: --- to -- 3) IEQ: 4) Line: 3) Shading:
a is negative or a < 0 curve goes from maximum to negative infinity.	y<–	Dotted	Inside the parabola		1) a is: 2) Range: --- to -- 3) IEQ: 4) Line: 3) Shading:
a is negative or a < 0 curve goes from maximum to negative infinity.	y≤–	Solid	Inside the parabola		1) a is: 2) --- to -- 3) IEQ: 4) Line: 3) Shading:

TABLE 10.2
IN-CLASS EXERCISE FOR A SYSTEM OF QUADRATIC INEQUALITIES

a is negative or a < 0 curve goes from maximum to negative infinity. a is positive or a > 0 curve goes from minimum to positive infinity.	y≤– and y≥–	Think of a system of linear equations and the corresponding graph. In the given graph for a system of quadratic functions, shade the region where you would find the solution.		Where will the solution be: If one of the curves is a dotted line? ------------------ If both the curves were dotted lines? ----------------------

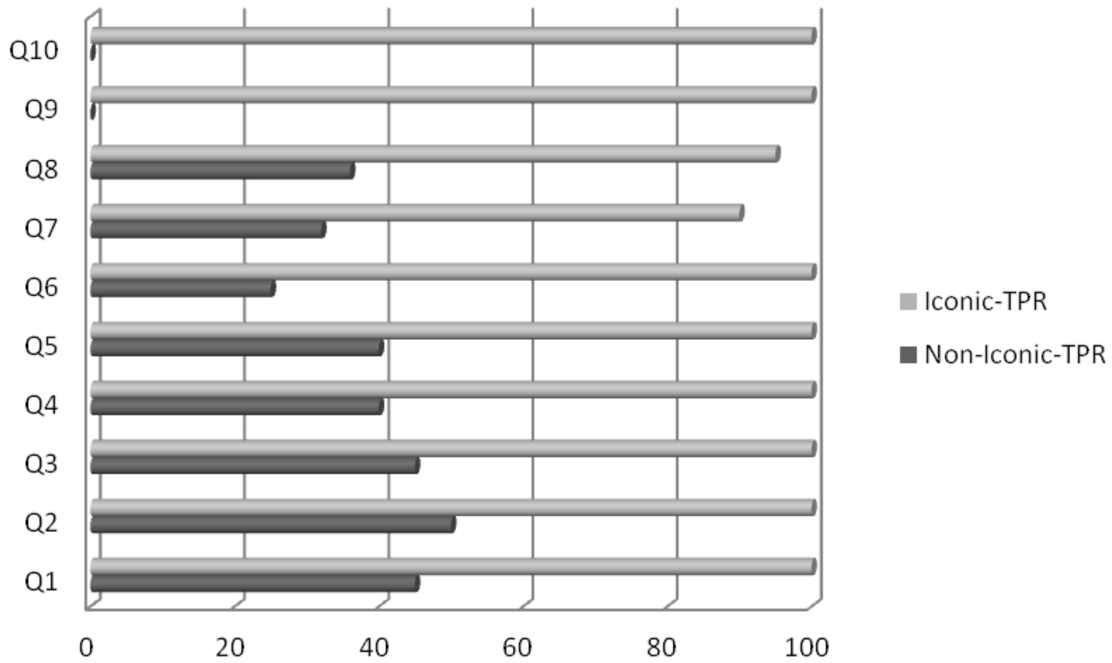


FIGURE 5
A COMPARISON OF STUDENT'S PERFORMANCE IN THE 10 QUESTIONS ON THE QUADRATIC INEQUALITIES

The methodology proposed and proven in the present study is superior to the Triple Representation Model of Schwartz *et al.* (1990) in being inexpensive, student-friendly, and adoptable by any school in the world-rural, suburban, or urban, in any language. Whereas Schwartz *et al.* (1990) used only three representations, the present method employs six different sensorimotor representations for cognitive practice. Further, the present methodology (a) completely eliminates the requirement for special software, (b) is not solely a computer-based event, (c) promotes cooperative learning, (d) enables peer observation and peer teaching, (e) incorporates visual-spatial learning, (f) accords a pleasant feeling of accomplishment in the minds of the students, who are interested in some form of physical activity and oral voice in the class, (g) enhances aural/oral skills of students, and (h) strengthens math vocabulary as well as processing and writing skills. The greatest of all these advantages is the cost-effectiveness, given that the world is in an economic order in which even a developed nation like the USA is finding it hard to run the public K-12 education.

8. Multiple Representation and Quadratic Forms: Inspirations from Ramanujan's Pioneering Work

Multiple representation is gaining more attention in the mathematics curriculum and in teaching methodologies (Brenner *et al*., 1995). The term *representation*, according to the National Council of Teachers of Mathematics (NCTM), "refers to both process and to product- in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself" (NCTM, 2000). However, mathematics educators have been holding a view that multiple representation is possible only with technology, viz., the computer medium and with special software. They have thus explored the computer medium extensively only to realize meager success. Overall, there is a rigid view that multimedia technology and computer alone would offer multiple representation modalities. However, this is not true. Human endeavors and human participation can produce more effective multiple representation modalities; human created art forms both fine arts and performing arts are testimony to this conviction..

The remarkable discoveries made by Srinivasan Ramanujan have made a great impact on several branches of mathematics, revealing deep and fundamental connections. It is astonishing to learn that functions especially quadratic, cubic, and quartic functions, predominated Ramanujan's childhood days and Ramanujan's Ternary Quadratic Form is being continuously explored by many notable mathematicians of the world; despite the availability and utilization of super computers, they have not been able to unravel the reason behind the interesting property of the ternary quadratic equations, discovered by Ramanujan, who used no computer or multimedia, but has created a world record of discoveries, on which all the countries of the world are currently working on.

It can be said that Ramanujan's early exploratory work with functions alongside with numbers could have provided to him the impetus to explore other areas of math. This is just an indication to show how important functions are in general and quadratic functions are in particular. As the simplest and the fundamental polynomials, students ought to get the best foundation in quadratic equations and inequalities; if this is assured, we can hope to make more Ramanujan's in our students.

Ramanujan is a pioneer in multiple representations. In 1916, Ramanujan invented simple yet unique vertex–edge graphs, which are called after him as Ramanujan Graphs. Till this date Ramanujan graphs remain mysterious: Not only are there very few constructions for such graphs, but it is also not even known whether they exist for any *d* e" 3. Friedman (2008) shows that almost every *d*-regular graph on *n* vertices is nearly Ramanujan's. Similarly Petersen graphs are Ramanujan in behavior. Most people wonder from where did Ramanujan get the insight and inspiration for his graphs. Those were days when computers did not even exist. So Ramanujan employed solely human faculties of observation and spatial visualization to cognize, predict, and correctly show to others the uniqueness of numbers, their geometrical weirdness, and the underlying mathematical phenomena. Looking retrospectively at Ramanujan's findings, it seems reasonable to say that Ramanujan's keen observation of day to day objects and mundane events of the natural locality and normal environment have given him the vital glimpses for his discoveries. He may have made cross-disciplinary analogical comparisons of ordinary physical objects and their spatial projections to the metaphysical concepts he was evolving. The perfect match of some of the mundane materials of Ramanujan's household and neighborhood with his products of discovery (Figure 6) is a testimony to this proposition. It is also understandable that Ramanujan may not have remained observing them but may have handled them, analyzed their composition/working, and admired their relevance by experiencing them (Dewey, 1925). It is also inferable from his work that Ramanujan saw a a mechanobiochemical and mathematical universe (Aristotle, Galileo, and Descartes saw a mechanical and mathematical universe) and anchored his discoveries on intuition, demonstration, and experimentation. His prediction of black holes is a time-tested testimony to this inference (Ono, 2012).

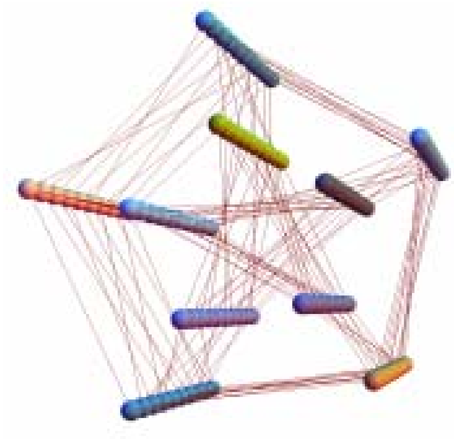


Figure 6 (a)

Left: A yarn-winder a common household item in the rural streets of Tamil Nadu, India in the days of Ramanujan; Right: 3-regular Ramanujan graph on 10 vertices (right). Courtesy: Google images (left) and http://research.microsoft.com/en-us/um/people/eyal/papers/randlifts.pdf (right), respectively; reproduced with permission.

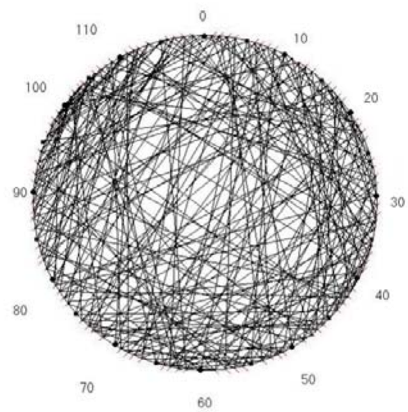


Figure 6(b)

Left: A ball of yarn; Right: Ramanujan graph of prime pairs. Courtesy: Google image (left) and Kevin Powell, http://math.arizona.edu/~kpowell/RTG.pdf (right) respectively; reproduced with permission.

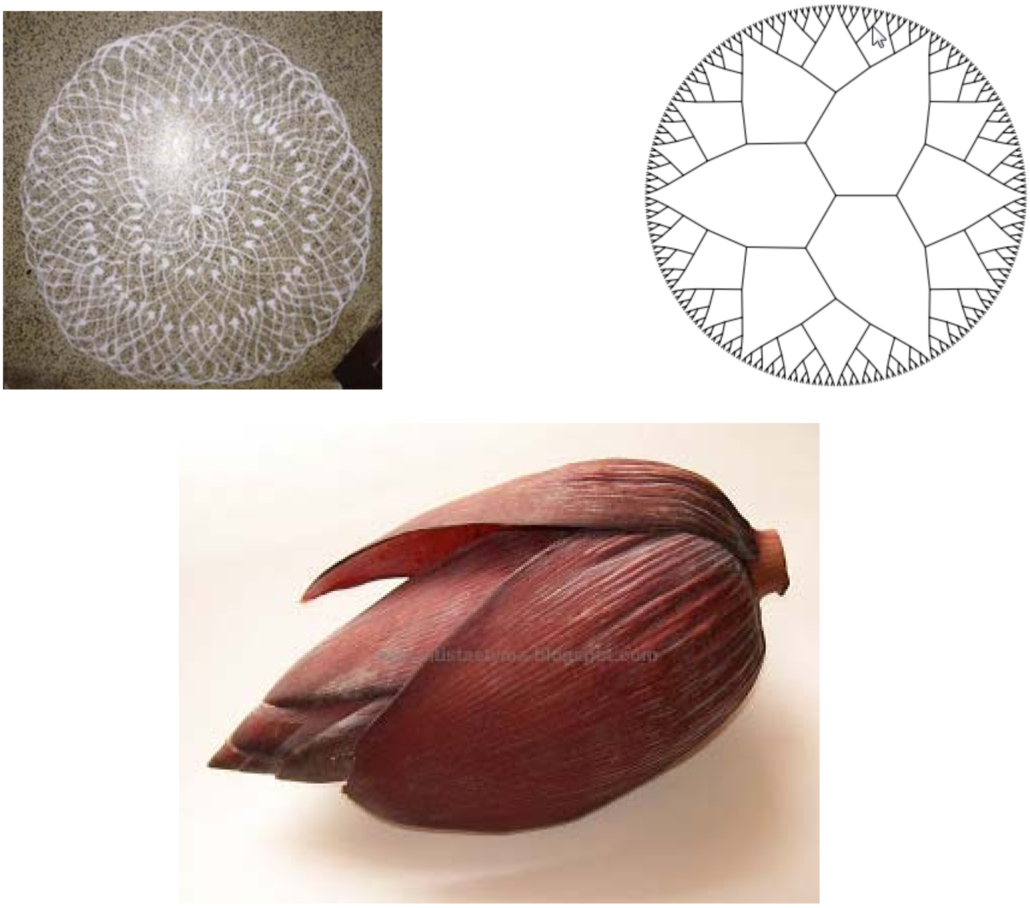


Figure 6(c)
Left: A kolam[3] generally drawn at the place of worship in the homes of Tamil Nadu, a banana flower; Right: Ramanujan graph of a growing tree. Courtesy: Google images; reproduced with permission. A combination of both the kolam and the floral array are reflected in the growing tree graph.

"An equation for me has no meaning, unless it represents a thought of *God*" is the famous quote of Ramanujan. Yes. Ramanujan drew insight and inspiration from the culture he belonged to. It is in the everyday concepts, events, and rituals of the culture "Ramanujan experienced such extraordinary insights in an innocent way, simply appreciating the beauty of the math" (Ono, 2012). Although his discoveries at that point of time appeared mere expressions without application, people are discovering practical applications for them now.

"No one was talking about black holes back in the 1920s when Ramanujan first came up with mock modular forms, and yet, his work may unlock secrets about them " (Ono, 2012).

As said by Jackson (2002), our teaching must nurture the whole learner. Porzio (1997) said there is a need for a set of new topics and problems that emphasize multiple representations, connections between representations, and appropriate uses of technology. Math is full of such topics; the limitation has been our not identifying them. The topic of quadratic inequalities has been there in the high school curriculum for long. Similarly multiple representation

[3] A kolam is a daily drawing of an art work done at the front of the house and at the place of worship inside the house. It is done twice daily one time before dawn and the second time, before dusk. This is a cultural practice in India, especially in Southern States like Tamil Nadu, the native Sate of Ramanujan. Kolams have mathematical and spiritual connotations.

as a tool of math communication has been in vogue for long. Unfortunately, the meeting of the two in the correct format had to wait so long.

Conclusion

History tells us that Newton, Einstein, and countless other scientists have made great discoveries/inventions from mundane, everyday observations because they had the prepared mind. To illustrate this: there are very few people in the world on whose head something or the other has not fallen. Yet it was Newton who immediately grasped the meaning of gravity when an apple fell on his head. Similarly, when Einstein heard that a painter had fallen from the roof, he went to see him in the hospital and asked him how he felt while falling down. The painter's reply that he felt weightless propelled Einstein into formulating his General Theory of Relativity. Ramanujan too had such a wonderful prepared mind: his spontaneous reply about the taxi-cab's number, 1729 (called Hardy–Ramanujan Number) to Hardy's remark is a world record.[4]

What is a Prepared Mind? A Prepared Mind is the mind that is ready for spontaneously identifying and harnessing a novelty or a challenge. It is certainly an outcome of a very deep thought on a particular subject for a very long time, but it is anchored on the point of departure from what is known to what is new and revolutionary. In other words, an astonishing discovery, irrespective of what the subject is, relies solely on a pristine element of unique human intervention, besides subject mastery (Renuka, 2010*b*). Ramanujan and other scientists have been able to make great discoveries/inventions because of the special human element they exercised in their experience with things, acts, and events in their normal environment and culture. It is hoped that the present work on quadratic inequalities through human exercised multiple representation will open up many vistas of exploration of the benefits of utilizing the human element, cultural components, traditional games (abundant in Eastern and especially Indian culture), and materials/experiences of natural inquisitiveness in the challenging problems of math education.

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[4] From quotations by Hardy: "I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavorable omen. "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways." Source: Wikipedia, the Free Encyclopedia.

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RAMANUJAN'S INSIGHTS: IMPLICATIONS FOR MATHEMATICS EDUCATION

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Abstract

Ramanujan (1987-1920), a mathematics prodigy from South India, stands among some of the greatest mathematicians of this century. Although Ramanujan worked with a number of renowned mathematicians at the University of Cambridge and held fellowships at the Cambridge Philosophical Society and the Royal Society of London, his lack of formal training in pure mathematics and in an environment far different to the schools where mathematics education is a priority evokes reflections on both the learning of mathematics and its associated assessment processes. Evidence suggests that Ramanujan self-taught guided by Carr's (1886) 'A Synopsis of Elementary Results in Pure Mathematics' that provided well-sequenced propositions, theorems, formulae and methods of analysis. Ramanujan's mathematics thinking evolved through mathematical analysis, number theory, infinite series, continuous fractions and towards probabilistic functions and models.

This article examines firstly the contents- and subject-matter knowledge embedded in the mathematics text Ramanujan used and its value for mathematics learning, and secondly the need to assess mathematics processes through the lens of a probabilistic model. The article also highlights the work of Georg Rasch (1901-1980) who developed the Rasch model for dichotomous data, which can be applied to response data, derived from intelligence and attainment tests. Importantly, an extension of the Rasch model through the Partial Credit Model, provides important insights as to why methods and processes in mathematics (like the ones adopted by Ramanujan) need to be examined to provide for better learning and diagnostics. A number of exemplars and illustrations are provided.

Keywords: *Ramanujan, content-knowledge, subject-knowledge, instructional sequence, Rasch model, partial credit model, assessment*

Introduction

Mathematics and mathematical thinking interrogate conceptions of quantities, space, structure and change through logical reasoning and abstractions. Mathematics provides the base for a number of fields including the physical sciences, medicine, engineering and human/social sciences. Current developments in games theory and computer games utilise mathematics both in the design, engagement and interactive phases. International studies, and government's commitment to education continue to position and highlight the importance of mathematics. Hence, in a number of jurisdictions, mathematics is compulsory at the formal years of schooling.

Intertwined with mathematics is numeracy. Stephens (2009, p.11) highlighted that "across the world, while it is clear that not everyone uses the term 'numeracy', there is a strong consensus that all young people need to become competent and confident users of the mathematics they have been taught. Numeracy is best described as a key outcome of how mathematics is taught and learned – it bridges the gap between mathematics learned at school and the variety of contexts where it needs to be used in everyday life."

Extending numeracy into 'quality of life' indicators, a number of research studies have started to explore financial literacy and related covariates. In the social sciences, researchers continue to challenge broad understanding of quantitative literacy and its implications for understanding and interpreting data and associated analyses. “By numeracy we mean not only the ability to reason quantitatively but also some understanding of scientific method and some acquaintance with the achievement of science” (O’Donoghue, 2002, p.48).

O’Donoghue (2002) acknowledged the centrality of mathematicians and mathematics educators and their role in guiding developments in quantitative and mathematical literacies. So great is the importance of the subject that the University of Northern British Columbia warns “students who choose to ignore Mathematics, or not take it seriously in High School, forfeit many future career opportunities that they could have. They essentially turn their backs on more than half the job market” (UNBC, 2012). The same can be said about ignoring the importance and wealth of quantitative research studies; one can become myopic about educational research and research in general. Schleicher (2005) continues to argue that “without data, you are just another person with an opinion!” Moreover, Alagumalai, Gopinathan and Ho (2009) reiterate the challenges associated with being ignorant of quantitative methods and the inadequacy to interpret data and findings of large-scale studies and reports.

Hence, mathematics is fundamental to life and society, and Ramanujan was quick to acknowledge its overwhelming importance, even at the risk of ignoring other subject in his formal school years (Hardy, Aiyar & Wilson, 1927; Kanigel, 1991). However, the learning of mathematics and his excellence in the subject was spurred on by his motivation and his singular focus through a number of textbooks: it should be noted here that there are no records that highlight the influence of teachers on Ramanujan’s strides into mathematical thinking and abstractions.

One is left wondering about the roles of sequencing of mathematics contents and/or pedagogy that provided important scaffolding to developments in mathematics. Moreover, what role did the activities within the books by Loney and Carr play (Aiyar & Rao, 1927), in the self-regulation and assessment of learning? Did Ramanujan have a broad subject-knowledge (mathematics) with detailed links to the various content-knowledge of the various topics? What was Ramanujan’s motivation of the extension of axioms and theorems into probabilistic space and reasoning? Are there implications for assessment of learning in this probabilistic space? The sections below seek to highlight Ramanujan’s learning of mathematics, and implications for practice in mathematics education and its assessment.

Sequencing and the Learning of Mathematics

A quick preview of introductory mathematics textbooks (including those for mathematics educators), highlight the following basic skill/cognitive sequence:

Addition → Subtraction → Multiplication → Division

Piaget (cited in Heimer & Trueblood, 1977, p.3) indicated that a learner interacts with his social and physical environment in his mental growth processes through four qualitatively distinct stages: (1) sensory-motor, (2) preoperational, (3) concrete operational, and (4) formal operations. Thus, mathematics “as a subject is a highly organised and carefully structured system of interrelated ideas. Growth of a mathematical concept often reveals a foundation of sub-concepts that is hierarchical in nature” (Heimer & Trueblood, 1977, p.13). Moreover, the type and complexity of reasoning guides the development of mathematics (Western & Haag, 1959).

Western & Haag (1959, p.1) argued the

deductive reasoning underpinning all mathematics facilitates the understanding of "properties of the number system, the concept of sets, relations, and functions, the solutions of equations and inequalities, the formal operations on algebraic expressions, and the abstractions into probability, analytic geometry, circular functions, vectors, matrices, and limits."

An examination of the mathematics curriculum, for example at Madison Central School District 39-2, highlights the following sequence, as rationalised by the department of mathematics [Available at: http://www.madison.k12.sd.us/guidance/registration_manual/mathematics.htm]

1. Pre-Algebra
2. Introductory Algebra
3. Algebra l [Pre-requisite: 8th grade math or Introductory Algebra]
4. Applied Geometry [Pre-requisite: Algebra l]
5. Geometry [*Recommended 10th year math]
6. Applied Math
7. Intermediate Algebra [Pre-requisite: Algebra and Geometry]
8. Algebra ll [Pre-requisites: Algebra and Geometry]
9. Trigonometry[Pre-requisite: Algebra ll]
10. College Algebra [Pre-requisite: Algebra ll]
11. Discrete Math [Pre-requisite: Algebra ll and Trigonometry]
12. Advanced Placement Calculus 1 [Pre-requisite: Trigonometry]

It is important to note the parallels of the above sequence and the requirements for prospective mathematics teachers. For example, those keen in "Teacher Certification Endorsement in Mathematics" at the College of New Jersey are required to have successfully completed the following:

Recommended (8 course units):

MAT 200 Discrete Mathematics
MAT 127 Calculus A
MAT 128 Calculus B
MAT 205 Linear Algebra
MAT 305 Abstract Algebra
MAT 316 Probability
MAT 351 Geometry
STA 215 Statistical Inference

Additional Recommendations:

MAT 229 Multivariable Calculus
MAT 301 Number Theory
MAT 255 Perspectives on the Development of Mathematics
MAT 320 Complex Variables

In addition to the appropriate qualification, "Candidates for endorsement must also pass the necessary PRAXIS examinations required by the state" (TCNJ, 2012).

Are the above sequences and heuristics of contents within mathematics positioned randomly, or are there underlying assumptions about the learning of mathematics and mathematical thinking? Were Ramanujan's explorations of mathematical concepts guided by a heuristics of abstractions? Did Ramanujan acquire subject/discipline-knowledge through his insights into sequentially acquired content-knowledge?

Isoda (2012, p.21) noted "any textbook will have the sequence (organized hierarchically by topic) to explain the meaning and for the children to acquire skills." Isoda (2012) was of the viewpoint that these sequences/heuristics were instrumental in developing mathematical thinking consequently. It is evident from Debnath (1987) and Hardy, Aiyar & Wilson (1927) that Ramanujan gained insights to the concepts and abstraction through the sequence of contents in his favourite books by S.L. Loney's (1893) *Trignometry-Second Part*, and George Carr's *A*

Synopsis of Elementary Results in Pure and Applied Mathematics. Carr's book "awakened his genius; he set himself to establish the formulae given therein" (Debnath, 1987, p.822). *It is important to note in the tributes to and publications of Ramanujan that no reference was made to the role of his teachers or the pedagogy used that enabled his acuity in mathematics*.

Barr (1988) acknowledged the 'nature and sequence of topics in the textbook' as a factor in Ramanujan's development. She further purported that because of plausible variation in topical sequence in mathematics, teachers' views of useful progression of topics may differ from that of the textbook (Barr, 1988). As teacher's pedagogical guidance and direction has not been highlighted as an influence in Ramanujan's acquisition of mathematics concepts, it is advanced here that careful sequencing of the contents in the books cited above provided Ramanujan the gradual 'step-up' to relatively complex and abstract mathematics concepts. This confirms Barr's belief that "many areas of mathematics include hierarchically related concepts, and will strongly influence the nature and sequence of topics actually covered" (Barr, 1988, pp.388-389). Hence, the reference materials Ramanujan used, and in particular George Carr's A *Synopsis of Elementary Results in Pure and Applied Mathematics,* provided him insights to the nature of the mathematical axioms and abstractions.

Moore (1967, pp.361-362) asserted that "every proposition follows from the postulates by a finite number of logical steps," underpinned by "orthodox logical sequence in mathematics …which is based upon the notions underlying the infinitesimal calculus taken as axioms." Thus, each preceding concept lays the basis for subsequent abstraction (Biggs & Collis, 1982 – SOLO Taxonomy). These sequential and hierarchical relationships of concepts provide support for the study undertaken by Schneider, Swanson and Riegle-Crumb (1998). They found a student's course sequences at 10th grade to be the strongest predictors of 12th-grade mathematics achievement gains. *This is important, as it highlights the need for careful construction of textbooks and reference materials to facilitate learning and self-regulation.*

Research studies (PISA, TALIS, TIMSS) continue to highlight the influence of family and schools on the educative processes and learning achievements in the formal years of schooling. Importantly, one must not overlook the 92-94% of learning that occurs beyond the classroom-walls and school-gates (Hoxby, 2001; Walberg, 2010). The magnitude of the out-of-class learning/influence suggests the design of mathematics textbooks should be examined by the key stakeholders (mathematicians and mathematics educators) identified by O'Donoghue (2002).

Stevenson et al. (1994) (cited in Schneider et al., 1998) found that course sequences are not highly associated with standard curricular tracks. This supports the hypothesis that Ramanujan's learning was self-regulated directly from his textbooks, and his progression was through the out-of-school study of mathematics rather than by a school-targeted curriculum and teacher-led pedagogy. (Aiyar & Rao 1927, p.xii). It was the mathematics books that 'opened a new world for him and awakened his genius'.

Ramanujan progressed from trigonometry to geometry and then to algebra, through acquiring and abstracting principled knowledge. This learning sequence ran counter to Berman's (1945, p.26) notion that "the traditional mathematics sequence is algebra, geometry, and trigonometry." Perhaps it is not the ritual embedded in the sequence but the principles of understanding and higher order thinking associated with each routine and axioms.

Solomon (1998) advanced the hypothesis that one of the problems of standard teaching practices is that they support merely 'ritual' as opposed to 'principled' knowledge, that is, knowledge which is procedural rather than being founded on principled explanation. *Ramanujan's motivation and acquisition of mathematics concepts provide direct challenge to received pedagogical knowledge and practice, and its position within content and subject knowledge. Is pedagogical knowledge embedded within the specific content-knowledge and the broader subject-knowledge?*

Research studies continue to highlight the problems of drill and practice (ritual knowledge); "Knowledge is derived from experience. It is not passively received but rather actively built up. Therefore, students' concept image is determined by the functions they work with and not by the modern definition of a function that is presented to them, as several studies show" (Even, 1993, p.111). This highlights the importance of the interactions between content and subject knowledge, and the nexus that exists between them.

Content-Subject Knowledge Interactions

Ball (1989, 2003) reiterated that subject matter knowledge had been taken for granted in teaching mathematics. She summarized the following: First, learning to do mathematics in school, given the ways in which it is typically taught, may not equip even the successful student with adequate or appropriate knowledge of or about mathematics. Second, knowing mathematics for oneself may not be the same as knowing it in order to teach it. While tacit knowledge may serve one well personally, explicit understanding is necessary for teaching. Finally, subject matter knowledge does not exist separately in teaching, but shapes and is shaped by other kinds of knowledge and beliefs (Ball, 1989; 2003).

"Teachers' subject-matter knowledge and its interrelations with pedagogical content knowledge are still very much unknown. This is due, in part, to a change in conceptions of teachers' subject-matter knowledge that has taken place throughout the years." (Even, 1993, p.94). This is further supported by the work undertaken by Brophy (cited in Stein et al., 1990, p.639) who argued that although theory and common sense suggest that teachers' subject-matter knowledge will influence their instructional activities, fine-grained empirical work linking teacher knowledge to classroom instruction is in its infancy.

"Even though it is usually assumed that teachers' subject-matter knowledge and pedagogical content knowledge are interrelated … there is little research evidence to support and illustrate the relationships" (Even, 1993, p.95). A study undertaken by Segall (2004) on conception of function highlighted that limited conception of function (contents) influenced the teachers' pedagogical thinking, and highlighted the 'pedagogical nature of content'. While much of the literature using pedagogical content knowledge sees pedagogy as external to content '*per se*' Segall argued that knowledge is never "*per se*, never for itself" (Segall, 2004, p.491). Hence, "pedagogy cannot be considered simply a method, an afterthought applied to content. Rather, pedagogy and content become one" (Segall, 2004, p.495).

Conventional wisdom says that deep subject matter knowledge is not required to teach elementary mathematics. Duckworth's study supports the growing recognition that, in fact, ***most elementary topics are far from easy or obvious*** (Duckworth, 1987, cited in Stein, Baxter & Leinhardt, 1990). Stein *et al.*, findings point to the unique problems that arise when an unprepared teacher communicates ill-understood concepts to elementary students (Stein, Baxter & Leinhardt, 1990, p.660). These underlying evidences about content-knowledge and subject-knowledge directly challenge the arbitrary hypotheses like "in speculating on what is needed in the future, the teacher will have to have expertise but this will be based on knowing where – not necessarily knowing what" as advanced by Jasman (2009, p.33). Thus, Segall (2004, p.501) noted "understanding that content has pedagogical dimensions that are important when teachers consider instruction in classrooms raises additional questions about the idea that more content courses will make better teachers."

Stein, Baxter & Leinhardt (1990) advanced the importance of 'knowing what' and in relation to 'why and how' through their study of teacher's knowledge of functions and graphs. They concluded that teachers were "missing several key mathematical ideas and that it was not organized in a manner to provide easily accessible, cross representational understanding of the domain, and these limitations were related to a narrowing of their instruction" (Stein, Baxter & Leinhardt, 1990, p.639).

It is evident that Ramanujan was not perturbed by the influences of teachers or the 'pedagogic' knowledge underlying the delivered contents. He had worked independently, and investigated all crucial concepts and links between them, rather than learn isolated math facts, foolproof calculations, and straightforward applications. Mathematics educators now advocate that students work on complex, multi-step problems in order to develop flexibility, inventiveness, and confidence as problem solvers (Stein, Baxter & Leinhardt, 1990, p.640).

Again, an incomplete (teacher) understanding of mathematics (both contents and subject) may result in instruction that fails to lay the groundwork for future utilization and relationships associated with a rich concept. Boulton-Lewis (1987) confirmed this in his study of Australian students on the importance of sequential knowledge and the application of cognitive theories in predicting the sequence and hierarchy of development of knowledge. This lends further support to Stein, Baxter & Leinhardt's (1990, p.641) vision that "mathematics education will not be realized unless systematic thought and research is devoted to questions of teacher subject-matter knowledge." It must take into cognisance the fact that understanding of mathematics is intricately tied to the process of assessment. What assessment process did Ramanujan adopt? Do his theories and thinking of mathematics provide insights to current developments in the assessment process, and to mathematics in particular?

Assessment of Learning in Mathematics: A Probabilistic Model?

Brown (1995, p.36) lamented about the problems associated with 'isolated non-contextualised mathematics investigations' and argued that children should "learn to know when they know and not satisfied until they understand." She asserted the importance of pattern recognition and generalisation in the learning of mathematics (Brown, 1995, p.38). Assessment of learning has to look beyond the 'ritual knowledge and process' associated with selected contents. This implies examining the individual path taken to arrive at a solution, and looking beyond text numbers written in the 'working section/column'. Assessment process needs to examine latent schema and if possible pictorial representation.

The phenomenon of boundary extension also raises interesting issues about the nature of pictorial representation and the relation of the physical picture to the subject's mental schema of the scene that the picture represents (Intraub, Bender & Mangels, 1992, p.191). The author of this paper is not aware of partial marks awarded for relevant pictorial representation or concept map of heuristics in a mathematics examination offered at the formal schooling levels.

Assessment in mathematics is, thus, not a process of awarding raw scores to 'teacher recognised/selected working' and answer. Ramanujan himself acknowledged the deterministic-probabilistic differences. "One of the most remarkable applications of the Hardy-Ramanujan partition theory deals with the problems of statistical mechanics" (Debnath, 1987, p.843). Schaben (2007) in discussing Kanigal's book on *The Man Who Knows Infinity*, highlighted

Ramanujan interest in probabilistic theories through the advancement of Hardy-Ramanjuan asymptotic formula and the provision of a base for the formulation of non-interacting Bose-Einstein systems. Hence, Ramanujan's operated beyond the traditional conception of mathematics, and extended his abstraction into statistics. Moore (1992) reiterated that statistics involves a unique mode of thinking in addition to being a field of mathematics.

This highlights a shift in thinking towards principled knowledge, and away from ritual knowledge (Solomon, 1998). Moreover, there is a need to examine the understanding of mathematics holistically, i.e. through broader numerical laws. Much of Ramanujan's work had broader applications beyond specific, contrived and routine problems. Perhaps, researchers should examine broader interconnecting models of problem solving in mathematics and advance models of interactions of cognition and the conditions optimal for these interactions. Educational measurement needs to advance models for testing through advanced multivariate procedures.

Hardie (1942, p.128) highlighted that the "field of educational measurement is one important part of the educationist's work that has been entirely ignored." He challenged educators (educationists) to demonstrate that the properties which they measure are measurable and meaningful, and acknowledged the works of Thorndike (1927) and Thurstone (1925). Hardie (1942, pp.143-144) challenged the education community that researchers and those involved in the education process need to identify and test 'numerical laws' (also called 'equation of condition') that relate the various 'magnitudes' (intensive, fundamental, and derived) and to verify theoretical conjectures. Just as Ramanujan's thinking evolved into advancing probabilistic models, the modelling of learning of mathematics and its assessment process must continue to abandon deterministic arguments, and examine micro-level processes that will provide meaningful insights for diagnostics and enrichment of learning. Figure 1 highlights the process underlying selected mathematical processes discussed in the above sections, and provides the base to the nexus of assessment of learning.The next, second section examines the application of a probabilistic model, embraced by Ramanujan, and expanded to by Georg Rasch in responding to Hardie's (1942) challenge to educators.

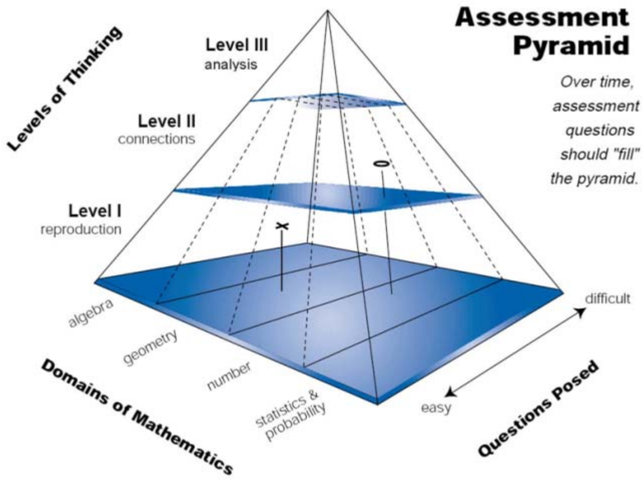


FIGURE 1.
THE SEQUENCE LEARNING-THINKING-ASSESSMENT PYRAMID
(Shafer & Foster, 1997, p. 3, cited in Webb, 2009)

Assessment in Mathematics

Assessment seeks to determine how well students have acquired the desired contents, processes and learner attributes to demonstrate shift in learning and broadly value-add to their education. Research studies on student learning have shown that assessment is a key factor in determining students' approaches to learning. Ramanujan continued his progress in mathematics through self-assessment, guided by worked examples and scaffolded activities in his favourite textbooks. Getting a problem correct (or incorrect) may have been secondary, but the underlying reasons to how the solution worked (or did not work) could be the pivotal motivating factor for Ramanujan. Thus, more than just a raw score assigned to an individual student or a group descriptive statistical report are needed to identify and promote better learning. This section explores the intricacies in item and test development, and how both items and tests can be evaluated for their effectiveness in gauging student learning.

Keeves and Masters (1999, p.14) highlight the salient difference between evaluation and assessment, and argue that "measurement is not undertaken as an end in itself. It is a useful operation in the processes of evaluation, or for research where characteristics must be measured, or as part of the tasks of assessment of student performance."

The descriptors in both the updated Anderson and Krathwohl's Taxonomy and SOLO taxonomy can be considered 'latent traits' of the broad categories, for example 'Comprehension, 'Analysis', 'Multistructural' or 'Relational'. These variables are operationalised on the basis of theory and observable indicators. We can use the Classical Test Theory (CTT) to examine the measurement properties of a test (as gauged through the Item Difficulty and Discrimination Index – please see Izard, 1977). "Applying the CTT model requires several assumptions about error distributions namely, errors are normally and uniformly distributed in persons, have an expected value of zero, and are uncorrelatd with all other variables. The CTT is limited in several ways" (Embretson, 1999, p.5).

Similarly, Alagumalai & Curtis (2005, p.10) indicated that "CTT has limited effectiveness in education measurement," and can be described as a weak measurement theory. Although a number of useful models of item response theory exist, they reiterate that "the Rasch measurement model is uniquely compatible with axiomatic measurement" (Alagumalai & Curtis, 2005). Moreover, Alagumalai, Curtis & Hungi (2005, p.345) advance the notion that "the Rasch model enables a better understanding of measurement and assessment through questioning determinstic judgements, the use of a flawed rubber-ruler and the problems associated with raw-scores."

Keeves & Masters (1999) cautioned classical concepts like reliability and validity need to be examined carefully. "The terms, 'bandwidth' and 'fidelity', have some overlap in the meaning with the more technical terms of 'validity' and 'reliability', but they are not synonymous with these more familiar terms. Moreover, they are increasingly being used in situations where validity and reliability **do not suffice,** since they are more directly related to combining of observations and the making of more precise measurement" (Keeves & Masters, 1999, p.5). Thus, it is important for those who are keen in evaluating their assessment tasks, especially processes associated with mathematics, to have insights into current test theories, and in particular the Rasch Model.

The Rasch Model

The raw scores (or number-right/wrong) do not have the properties of measures, i.e., the scores do not provide information on whether the test is easy relative to the students or the items are distributed 'well' to meaningfully gauge a specific content or the levels within it (Wright & Stone, 1979). A measure, as used in tests, adheres to the following criteria:

- estimates of locations on a single variable (unidimensional)
- expresses in a constant unit of measurement (interval-level)
- freed of the particulars of the instrument used (objective)

Masters (1998, p.14), indicate that "although educational measures can be constructed from responses to test items, not evey set of test items meets these requirements and is capable of yielding unidimensional, interval, objective measures." The following summary reflects the work of Keeves & Alagumalai (1999, pp.41).

The response of a person to a particular item or task is never one of certainty, i.e., there exists an element of probability in the response of the person involved. There always exists an error about any response a person makes to an item or task. In Rasch measurement, probabilities are introduced through consideration of the odds that a person gives a correct or favourable response to an item.

Imagine the odds of a high-jumper going over the horizontal pole! There is an interaction between the person's ability (â) and the difficulty of the task (ä) (see Figures 2 and 3).

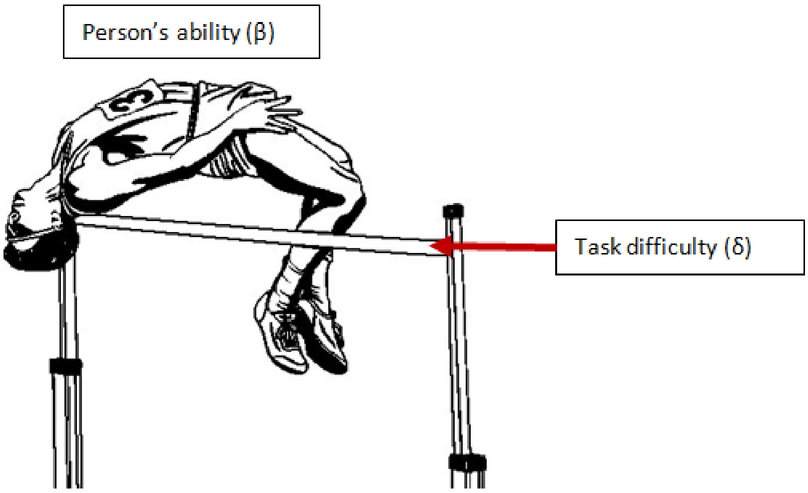


FIGURE 2.
ILLUSTRATION OF 'LOCATION OF' A PERSON'S ABILITY (â) AND TASK DIFFICULTY (ä)
Source: www.vectorjunky.com

We can extrapolate this to a test/task/exam (used interchangeably with assessment) situation. There is a parallel interaction between person's ability (â) [can be content knowledge/IQ/EQ] and the difficulty of the task (ä) [difficulty of an item/task/situation]. It can be conceptualised that the ability of the person (â) and the difficulty of an item (ä) can be positioned in a continuum, i.e., the same continuum. Both the person ability (â) and item difficulty (ä) can be considered as 'latent traits' as they can only be estimated through repeated measures.

The more items the person attempts, the better estimate (and confidence) of his ability (â). Parallel arguments can be presented for the item difficulty (we can postulate that the difficulty of a 'synthesis' item > 'comprehension item' > 'knowledge' item). With regards to the SOLO taxonomy, the difficulty of 'relational level' item > 'multistructural level' item > 'unistructural level' item. However, where the item is precisely positioned can only be ascertained through the item (or test) being administered to a group of students.

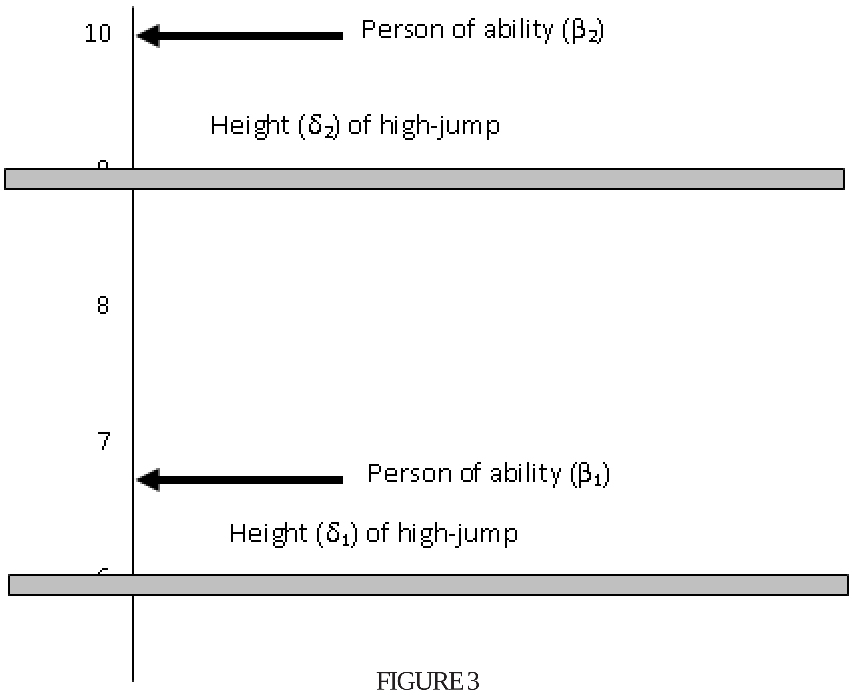


FIGURE 3
SCALE INDICATING THE ABILITIES OF TWO HIGH-JUMPERS

Different combinations of the above person ability (â) and item difficulty (ä) can provide outcomes of success or failure, as illustrated in Figures 2 and 3.

Thus, if person ability, â, is greater than (>) task/item difficulty, ä, then the person will respond successfully.

However, if person ability, â, is less than (<) task/item difficulty, ä, then the person will respond unsuccessfully or incorrectly.

Let's translate the high-jump example to one for responding to an item in a test.

1. person ability â / item difficulty ä > 1, a correct response is expected
2. person ability â / item difficulty ä < 1, an incorrect response is expected
3. person ability â / item difficulty ä = 1, a 50% chance of a correct response is expected

The above combinations can be summarised thus:

Probability (Item Correct) = Function [(person ability â) – (Item Difficulty ä)]

⇨ Probability that the item is correct P (X_{ij} = 1) = Function ($â_i$ - $ä_i$)

Expressed mathematically, the Rasch Model gives the probability P_1 of a person with ability â succeeding on a test item of difficulty ä as:

P_1 = exp (â - ä) / 1 + exp (â - ä) (1)

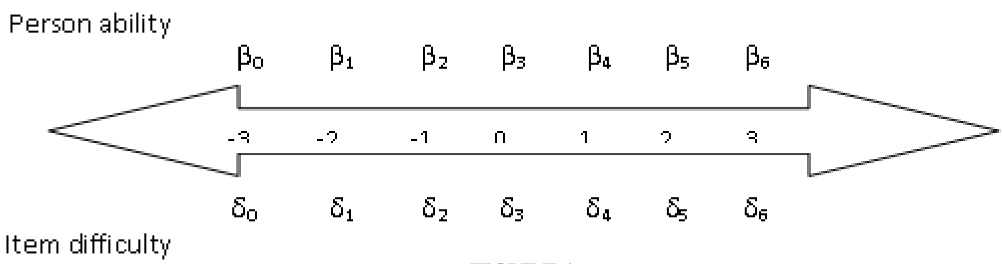


FIGURE 4
THE RASCH SCALE (Keeves & Alagumalai, 1999)

Equation (1) can be represented as a characteristic curve (see Figure 5) that has a probabilistic function.

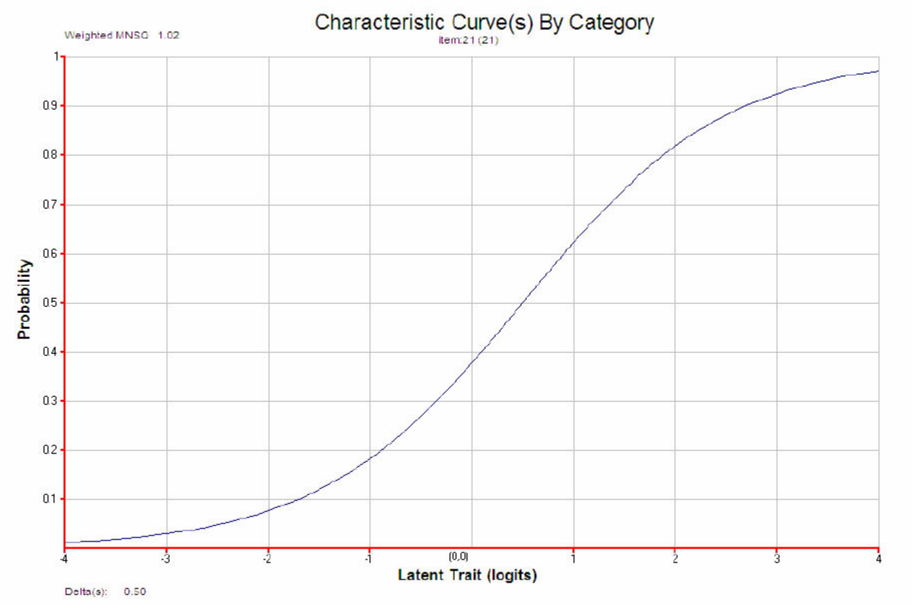


FIGURE 5
CHARACTERISTIC CURVE FOR A DICHOTOMOUSLY SCORED ITEM

Figure 6 represents the characteristic curve for a polytomous item (or an item scored with partial credit) with three scoring categories.

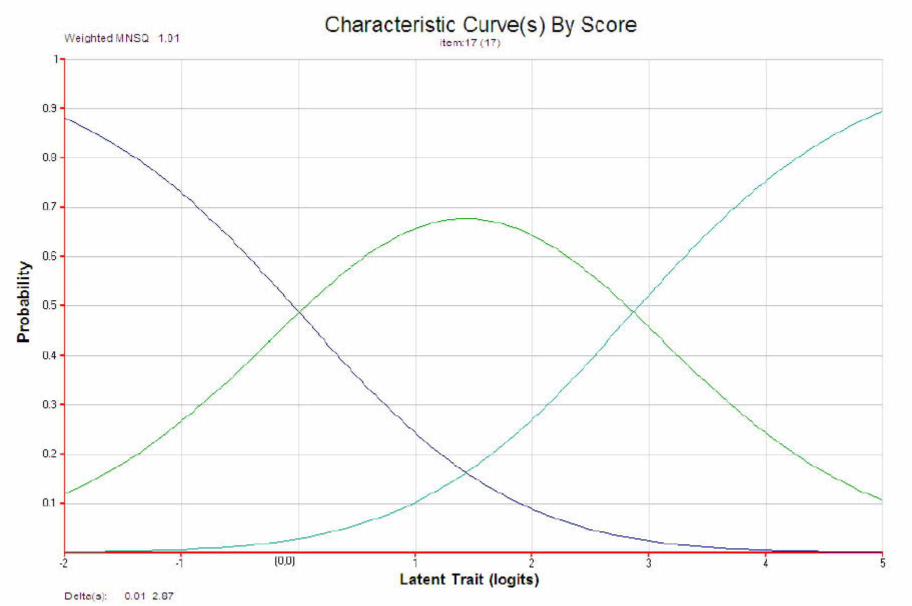


FIGURE 6
CHARACTERISTIC CURVE FOR A POLYTOMOUS ITEM WITH THREE SCORE CATEGORIES

Embretson (1999, p.8) indicated that "the two major advantages that result from the more complete IRT model are: (a) Person trait level estimates are controlled for the properties of the items that were administered, and (b) item difficulty estimates are controlled for the trait levels of the particular person in the calibration sample. In this sense, item-free person estimates and population-free item estimates are obtained." Table. 1 highlights the advantages of the use of IRT (Rasch) model compared to what CTT offers.

TABLE 1
COMPARISON OF SOME PRE-IRT (RASCH) AND IRT FORMS OF INFORMATION ABOUT TESTS.
(Daniel, 1999, p.38)

Information	Pre-IRT	IRT
Item difficulty	Propotion passing (Sample dependent)	location on scale of difficulty/ ability(sample free)
Item validity and accuracy	item-total correlation (sample dependent)	discrimination parameter or fit statistic (sample free)
Person ability*	raw score (test dependent)	location on scale of difficulty/ ability (test free)
Score accuracy	reliability (sample dependent), SEM (usually averaged across ability levels)	SE of ability estimate (sample free, and specific to the ability level)
Population distribution (norms)	no difference between pre-IRT and IRT methods	

* Ability is used generically to refer to any trait

This is the strength of the Rasch model, and the key to objective measurement. The next sub-section examines the test items for the various jurisdictions, and how the Rasch model could be utilised for item and test analysis.

Linking Assessment of Mathematics and the Rasch Model

The SOLO taxonomy (Biggs & Collis, 1982) is useful in designing assessment tasks in mathematics. Figure 7 highlights the usefulness of ordering the tasks/ items.

Thus, a mental sum or problem, as illustrated in Table 2, can be re-written as a superitem (Figure 7) incorporating the various levels of the SOLO taxonomy.

TABLE 2
NUMBER: WRITTEN/MENTAL PROBLEMS

WRITTEN Problem		Mental Computation Problems	
% correct	**Task**	**% Correct**	**Task**
47%	16 + 19	46%	17 take away 13
50%	10 + 30 - 5	60%	13 plus 12
55%	__ + 11 = 23	63%	add to 4 to make 11
56%	3 X 3	74%	3 groups of 2
60%	15 - 9	77%	9 plus 8
62%	12 - 6	85%	8 take away 5
70%	7 + 17	90%	20 plus 10
79%	9 + 8		
80%	9 + __ = 12		

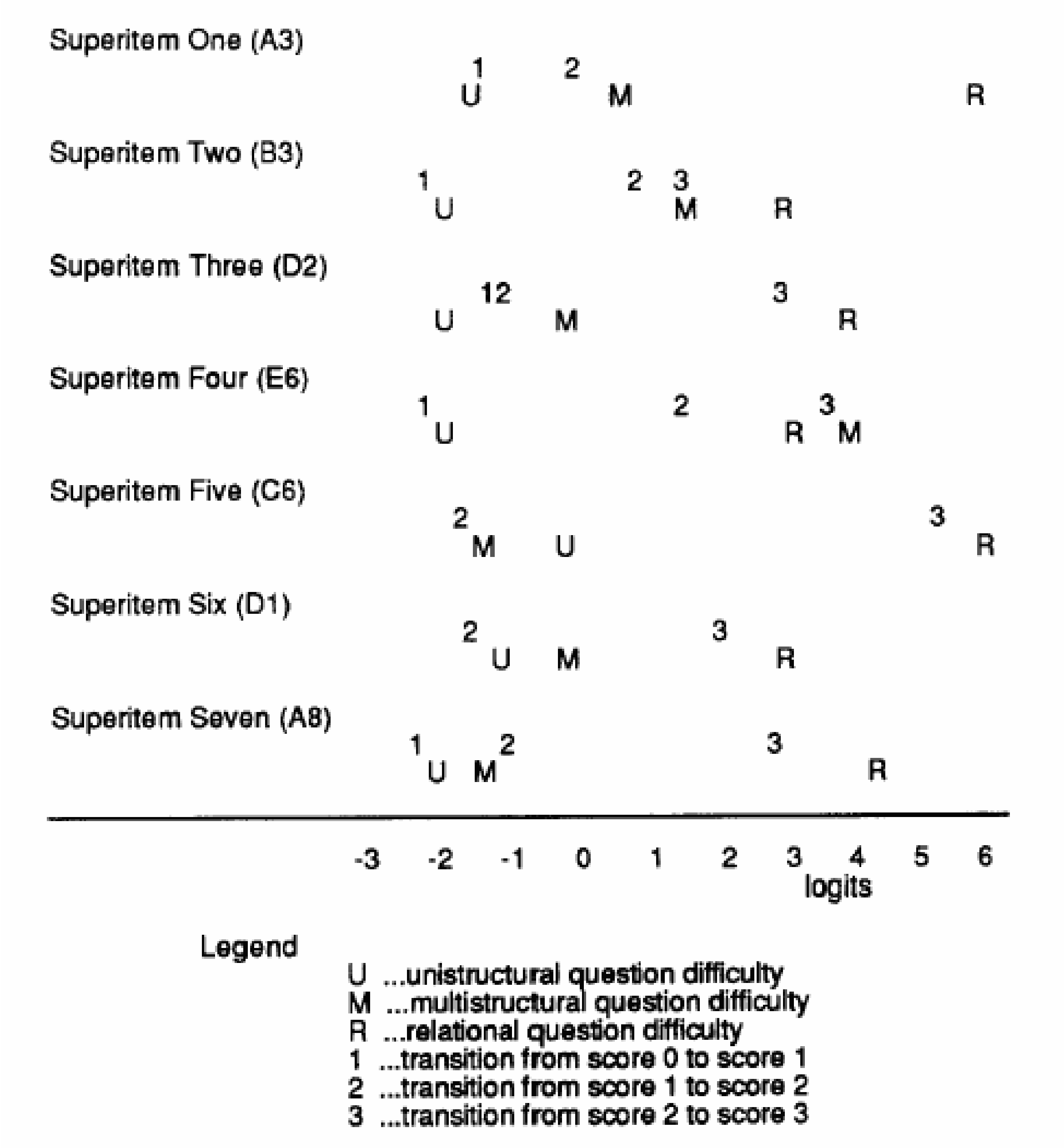


FIGURE 7

THE RASCH AND SUPERITEM LOGIT SCALE (Wilson & Iventosch, 1988, p.328)

To assist in the ordering and identification of the processess in an item, Keeves & Kotte (1995) developed the Content-Process categories for the mathematics items in nine tests. Their findings (Table 3), parallel the model presented by Wilson & Iventosch (1984) in that it offers an insight into the 'difficulty and demand' of the various items. Although the Rasch analysis will provide further confirmatory evidence of the hierarchy of the items in a test, it is pertinent for teachers and test-developers to understand what cognitive processes will be invoked by the items, and what possible diagnostics need to be in place for learning support.

This is pertinent to those keen in identifying areas of learning strengths and weaknesses that require diagnosis and remediation. One is left to wonder how Ramanujan achieved this self-monitoring/assessment and correction processes!

TABLE 3
SCALE VALUES OF CONTENT AND PROCESS CATEGORIES FOR ITEMS IN NINE TESTS
(Keeves & Kotte, 1995, p.32)

Item No	Calibration Scale Value	Content category	Process category	Finn category	Mayer category
A1	330	arithmatic	computation	computation	selection
2	288	arithmatic	interpretation	understanding	identification
3	319	arithmatic	computation	computation	selection
4	386	arithmetic	computation	computation	clarification
5	478	arithmetic	interpreation	measurement	clarification
6	491	arithmetic	computation	computation	selection
7	639	arithmetic	analysis	understanding	interpretation
8	535	arithmetic	computation	computation	selection
9	462	arithmetic	computation	computation	selection
10	550	geometry	computation	computation	selection
11	281	algebra	knowledge	understanding	clarification
12	573	algebra	computation	computation	selection
13	508	geometry	computation	measurement	selection
14	611	algebra	computation	computation	selection
15	549	algebra	interpreation	understanding	clarification
16	624	geometry	analysis	understanding	evaluation
17	719	new mathematics	computation	computation	selection
18	419	arithmetic	analysis	understanding	evaluation
19	583	geometry	application	understanding	applicataion
20	629	algebra	computation	computation	selection
21	-	geometry	comprehension	understanding	clarification
22	715	arithmetic	knowledge	measurement	selection
23	413	arithmetic	computation	computation	selection

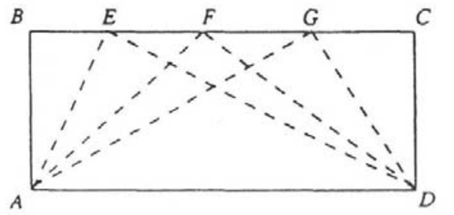
Keeves & Kotte (1995) highlight the underlying 'traits' and difficulty of the items in a mathematics assessment. Thus, mathematics teachers need to appreciate and understand the intricacies of connecting processes for each concept, and its scoring.

Scoring and Partial Credits

In order to manage this rich information one needs a strong supporting framework that can provide a basis both for interpreting the students' reasoning and for making decisions with regard to further instruction. The scoring scales used for correcting open-ended items range from general scoring scales to task-specific ones (van den Heuvel-Panhuizen, 1996, 2001; Webb, 2009). General scoring scales may be understood as categorizations in general levels of cognitive development or levels of problem solving ability, such as Bloom's or SOLO taxonomies. Other general forms of scoring are those labeled 'analytic' scoring and 'holistic' scoring.

Moderation is also an important process for teachers (and assessment specialists) to agree on score, partial credit and grade allocation. Most importantly moderation and specification of the criteria also mention procedures of monitoring and adjusting the scores of different raters, training, and procedures to double-check scoring (Heuvel-Panhuizen, 1996). Figure 8 illustrates how partial credits can be assigned to various responses. This facilitates the charting of progress and diagnostics to be offered that will be discussed in the next sections.

In the rectangle *ABCD* below, explain why triangles *AED*, *AFD*, and *AGD* have equal areas.



Correct Responses

8 All triangles share the same base and have equal altitude (height). Therefore, their areas are equal.

Incorrect Responses

1 Incorrect answer other than partially correct answer.

2 Response implies all triangles share the same base but contains nothing regarding altitude (height).

3 Response implies all triangles have equal altitude (height) but contains nothing regarding same base.

FIGURE 8
SCORING RUBRIC FOR ANOTHER NAEP EXAMPLE

Here a code of 8 generally denotes a correct answer and codes 1,2,3 etc, generally denote incorrect answers. The different incorrect response codes are used to track interesting incorrect or partially correct alternatives. (Braswell & Kupin, 1993, p.174).

The rubrics thus derived will provide directions for the possible 'banding' that could be elicited. Figure 9 showcases how individual items can be identified to the various bands in order to gauge performance at both the individual and cohort levels and to target diagnostics, remediation and extension activities.

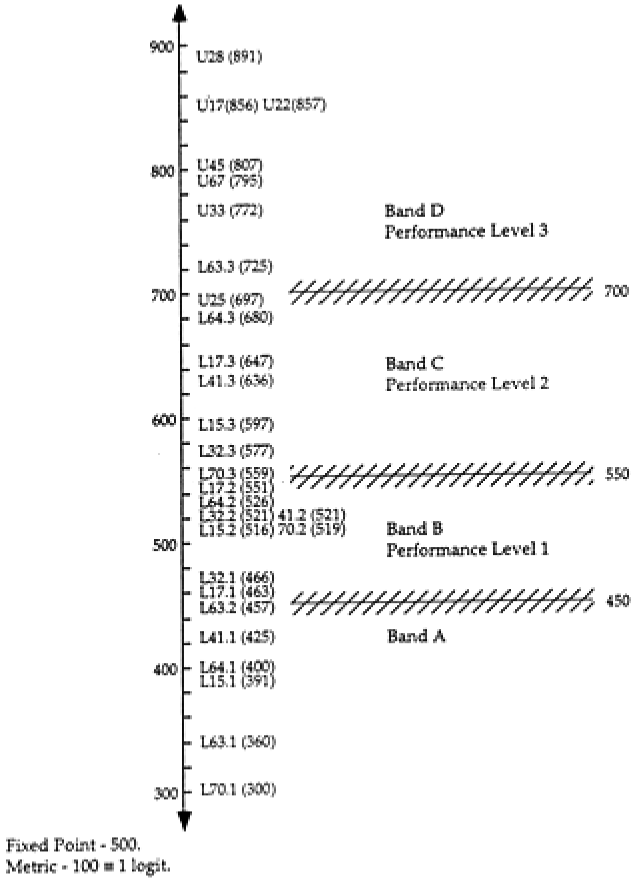


FIGURE 9

ITEM RESPONSE THRESHOLD SCALE LEVELS IN THE MATHEMATICS PERFORMANCE SCALE

(Keeves & Kotte, 1995, p.25)

It is evident from Figure 9, that item #U28 is relatively more difficult than other items in the test. Similarly, item #L70.1 is relatively easy with most students getting that item correct. Both the distribution of students and items present an important picture for those developing, administering and reporting on the test. As Woodcock (1999) indicates (Figure 10), the ability-difficulty maps provide a ranking of items and persons on a common scale, and facilitates a rethink for course restructure.

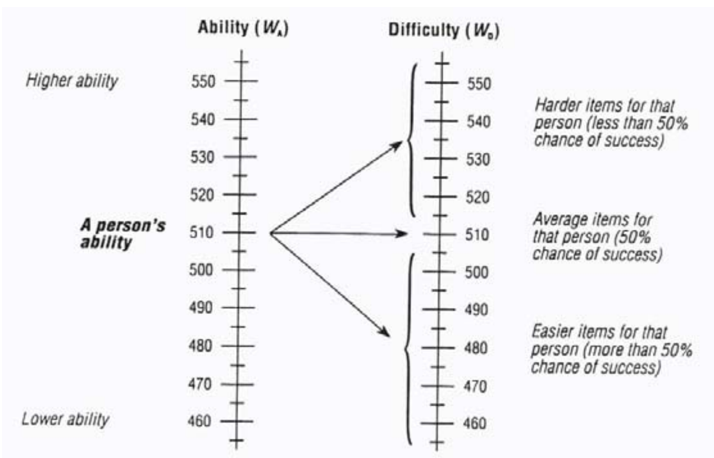


FIGURE 10

RELATIONSHIP OF PERSON ABILITY TO ITEM DIFFICULTY. (Woodcock, 1999, p.109)

An important process of the scoring is the propensity to chart progress of students in the various articulated bands. Figure 11 presents a progress/developmental map with the associated banding. Similarly, the processes and skills can be charted as advanced by Wilson & Iventosch, (1988, p.328), and represented as follows:

```
MORE ABILITY                          HARDER ITEMS
   2            |
                |
                |
                |
                |
               X|
                |
               X|
              XX|
              XX|C 10Q32b.3 Reads & interprets time-distance graph
               X|
              XX|
   1         XXX|D 10Q32a.2 Able to round time to decimal part
             XXX|B 10Q19.3 Solves problems with decimal hours
             XXX|C 10Q32b.2 Mis-interprets time-distance graph
            XXXX|
          XXXXXX|D 10Q19.2 Rounds down to nearest hour, resists decimal
          XXXXXX|A 8Q30.4 Reads clock time (a quarter to)
          XXXXXX|C 10Q32b.1 Misreads scale on time-distance graph
         XXXXXXX|A 8Q30.3 Confuses hour on clock with next hour
       XXXXXXXXX|A 8Q30.2 Confuses hour and minute hand
        XXXXXXXX|B 8Q16.1 Treats hours and minutes as separate
      XXXXXXXXXX|D 10Q19.1 Rounds up to nearest hour, resists decimal
   0   XXXXXXXXX|D 10Q32a.1 Rounds to nearest hour, resists decimals
         XXXXXXX|
        XXXXXXXX|B 11Q12c.1 Subtracts time using 1hr=100min
        XXXXXXXX|A 8Q30.1 Reads quarter to as quarter past
       XXXXXXXXX|
      XXXXXXXXXX|
         XXXXXXX|
         XXXXXXX|
            XXXX|
            XXXX|
           XXXXX|
             XXX|
  -1          XX|B 12Q13.1 Adds times as if 1hr=100mins
              XX|
              XX|
               X|
               X|
               X|B 13Q14.1 Treats hrs/minutes as decimal
               X|
LESS ABILITY                          EASIER ITEMS
```

FIGURE 11

DEVELOPMENTAL MAP FOR SOME TIME ITEM-RESPONSES, n=14420 (1500 per year group), age 8-13. [Wilson & Iventosch, (1988, p.328)]

A number of underlying principles in test construction is illustrated in the output in Figure 11

1. The items should be arranged in order of increasing difficulty, ie., relatively easier items at the start of the test with the more difficult items at the end;

2. It may be desirable to group together items which cover the same topic of learning outcomes; and

3. Useful for test developers to indicate in the marking key, all the possible processes, skills and knowledge required, so as to enable the mapping of items onto the ToS. The test analysis will provide insights into which topic/area had been done well by the students and that needs further examination (please see Alagumalai, 1996, 'SOLO, Rasch, Quest and Curriculum Evaluation' at http://www.aare.edu.au/96pap/alags96406.txt]

Hence, it is important for mathematics educators/teachers to gauge the statistics of the item analysis and map it to the various level indicated in the instructional objectives. Importantly, the use of the Rasch Model, appreciating its underlying assumptions and applications for learning highlights the shift from a deterministic to a probabilistic context espoused by Ramanujan in his journey in mathematics.

The Rasch Model has been shown to work within the limits of statistically determined error. Objective measurement is possible in examinations, classroom assessments and for teacher-made tests, and its future depends as much on those who use the results as on those who construct examinations (Wilmott & Fowles, 1974, p.59).

The Rasch Model, through various available item analysis software, has been utilised for a number of studies (Alagumalai & Curtis, 2005; Embertson & Herschberger, 1999; Masters & Keeves, 1999). Alagumalai (1996 – 2006) has highlighted the challenges of not evaluating assessment, whatever form it may be. Although the Rasch Model provides useful information about items and tests, and users, there is not much evidence that teachers utilise these information for both gauging what is to be reported to students, and also for correcting any oversights in a test (Alagumalai, 2006).

Whether an item is 'faulty' or whether the distractors are behaving differentially or whether the overall test is not discriminating needs to be examined carefully. The Rasch Model offers educators and teachers greater insights into various aspects of assessment and learning, and includes:

1. Adaptive testing

2. Measurement and articulation of developmental levels

3. Test equating (both high stakes to high stakes and high stakes to low stakes)

4. Constructed response tests, projects, essays and partial credit scoring

5. Individualised testing in classroom, including project work involving maths

6. Item analysis and item banking

7. Item bias, identification and correction/ modification

8. Measurement of judgement and raters (markers) effects

9. Charting of progress (developmental assessment)

10. Diagnostics and remediation

11. E-portfolio and progress maps

However, it must be noted here that <u>all assessment tools and analyses are useful if and only if</u>

they improve learning, and this should activate a rethink of teaching strategies. Thus the diagnostic, remediation and assessment feedback loop should be the highest priority and a fundamental aim/outcome of comprehensive item analysis.

Diagnostics, Remediation, Assessment and Feedback Loop

The Rasch analysis provides students value-added learning and motivates students to enjoy mathematics as a subject.

After a test is administered and the associated evaluation undertaken, either through the Rasch modelling software or some other test analysis system, it is pedagogically appropriate for teachers to examine the overall performance of students in the test, how individual students have performed, the utility of the test and of individual items, and also a reflection on the teaching method adopted for the delivery of instructions.

Adams (1988, p.354) indicated, "to develop a framework for interpreting the information, the responses to each item were examined to determine if a hierarchy existed in the error types. That is, does the selection of one incorrect answer indicate a higher degree of competence than the selection of another correct answer?" He highlights that item misfit can itself be useful in diagnosis, and indicate probable misconcepts students may hold. He concludes that "response spaces and maps provide an important mechanism for extracting diagnostic information from the test concerning specific student competencies and strengths. They provide a frame for making diagnosis" (Adams, 1988, p.360).

The marking key for a test item could be designed around the SOLO taxonomy. The scoring could be in the partial credit form:

0 mark – no sense in response (pre-structural)

1 mark – student has basic understanding of velocity and acceleration (unistructural)

2 marks – student makes a number of connections, eg. formulae use (multistructural)

3 marks – student demonstrates understanding of connections and problem solving (relational)

A summary table (Table 4) could be drawn for each item or groups of item (same topic or process skills).

TABLE 4
SUMMARY OF PERFORMANCE AT THE VARIOUS SOLO LEVELS (Item #1)

Marks	SOLO Levels	% of students - correct
0 mark	Pre-structural	0%
1 mark	Unistructural	10%
2 marks	Multistructural	30%
3 marks	Relational Level	60%

The conceptions in Table 4 can be operationalised, both for specific assessment tasks or charting learning over time. A parallel classification can be undertaken at the individual student's level, and specific learning difficulties identified. Moreover, the diagnostics of tests at the various levels (topic, by cognitive levels, by sub-groups – boys/girls) will help provide insights into the type of remedial strategies that could be employed. Figures 12 and 13 highlight the outputs available to examine both individual progress as well as the class/cohort developments.

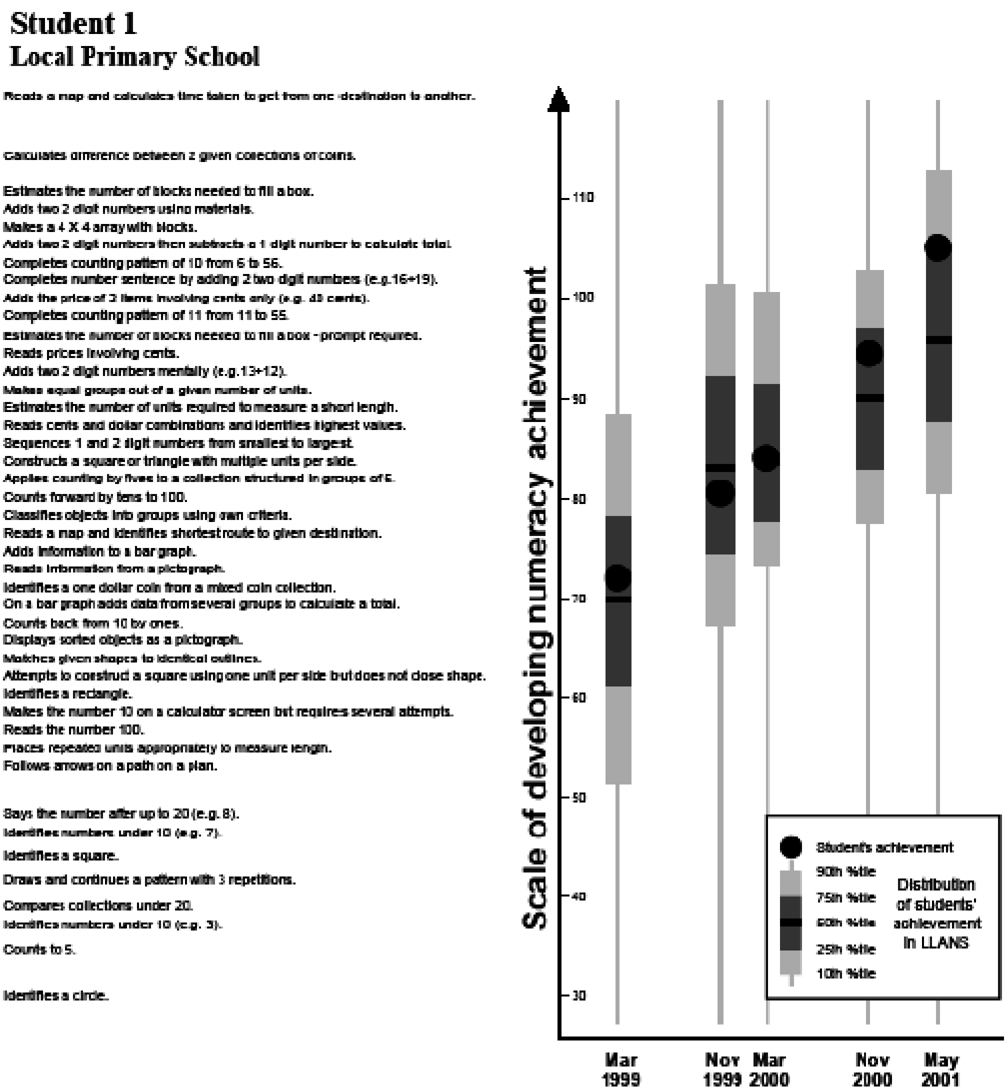


FIGURE 12
AN INDIVIDUAL NUMERACY PROGRESS MAP
(Meiers et al., 2006, p.78)

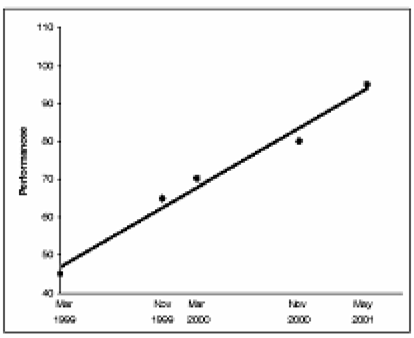
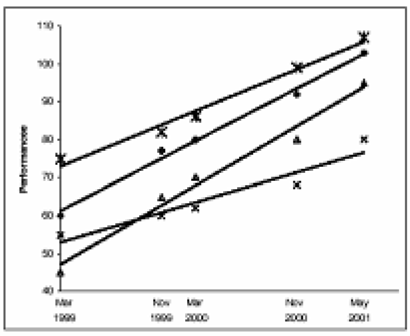



FIGURE 13
INDIVIDUAL GROWTH TRAJECTORY AND INDIVIDUAL VARIATION IN GROWTH TRAJECTORIES
(Meiers et al., 2006, p.80)

Rasch analyses can provide leads into diagnostic learning and the direction for curriculum modification and upgrades, and a number of useful indices have been advanced by Alagumalai (1996). Moreover, Forester & Masters (1998), and Masters & Keeves (1999) highlight the use of progress maps to provide students appropriate developmental support.

The step after diagnostics is remediation. Although the term 'remediation' has different connotations in different jurisdictions, remediation here refers to finding an optimal solution to assist students overcome learning difficulties associated with a specific concept or topic. It is believed Ramanujan was mindful of the challenges each axiom and concepts presented, and his persistence and determination in wanting to understand each micro-step/process allowed him to self-correct and remediate challenges. This monitoring of specific steps in solving mathematics problems, and at the same time having a broad understanding of subject knowledge, is vital for teachers of the subject, and also for enabling learners (to self-check and correct).

In mathematics, students may have difficulty in translating a word problem to its fundamental concept parts or even pictorially representing a work problem. Furthermore, students may have difficulty in manipulating the appropriate formulae to solve a problem. The diagnostics, done over time, may highlight that the student may have difficulty in algebra

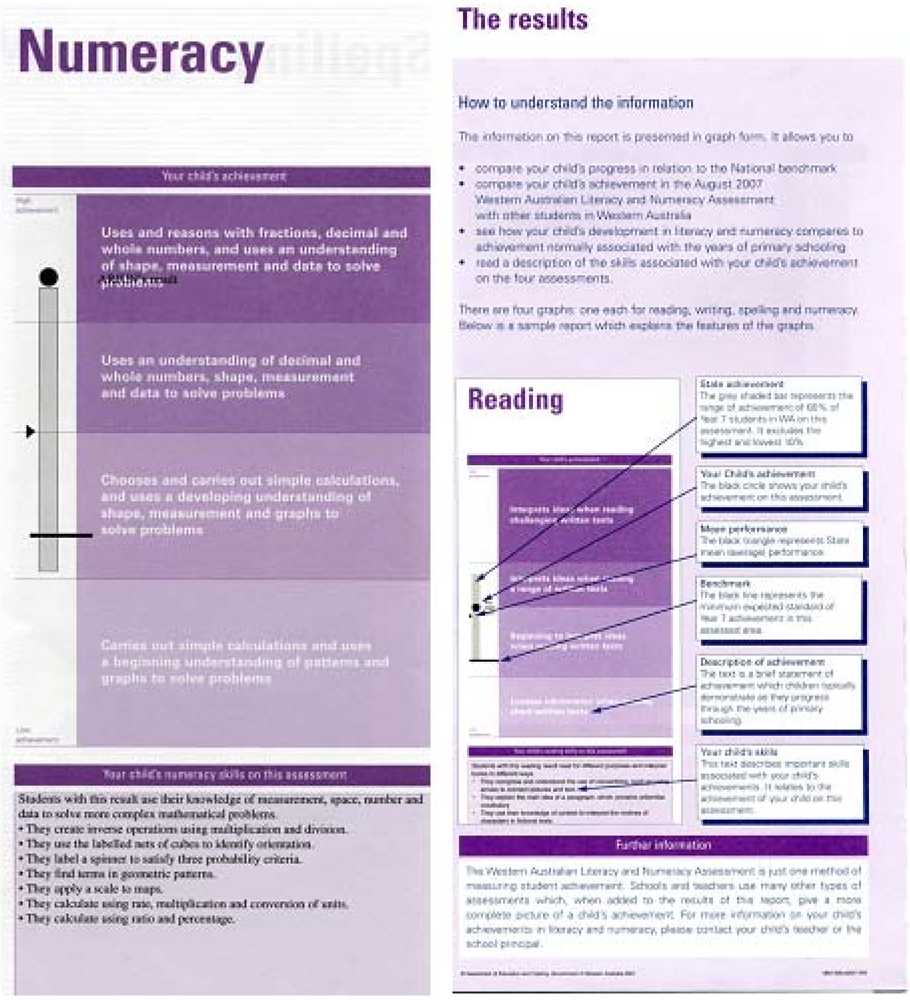


FIGURE 14
PROGRESS MAPS AND INTERPRETATION OF RESULTS
(WA Literacy & Numeracy Test)

and perhaps in understanding key concepts in mathematics. Remediation will involve providing the students with the fundamentals, to empower them to manipulate the formulae to solve the problem.

Moreover, students may have conceptual difficulties in understanding key concepts, say in functions, trigonometry or statistics. If the diagnostics indicate a lack of knowledge or if the student is operating in a pre-structural level, it may be useful to organise and examine where the student's misconception lay.

Thus, feedback to students is not just about presenting a 'faulty' raw score and an 'estimated' grade, but will have information for the student to reflect and correct as required. Moreover, students in the upper spectrum could be provided with extension activities to propel them further. Figure 14 presents a possible solution by which assessment reports can be made meaningful to students.

Further diagnostic details can be provided through a kidmap (see Figure 15). The kidmap presents details about individual items in the assessment task and provides insights into what the student had achieved and where they may have underperformed.

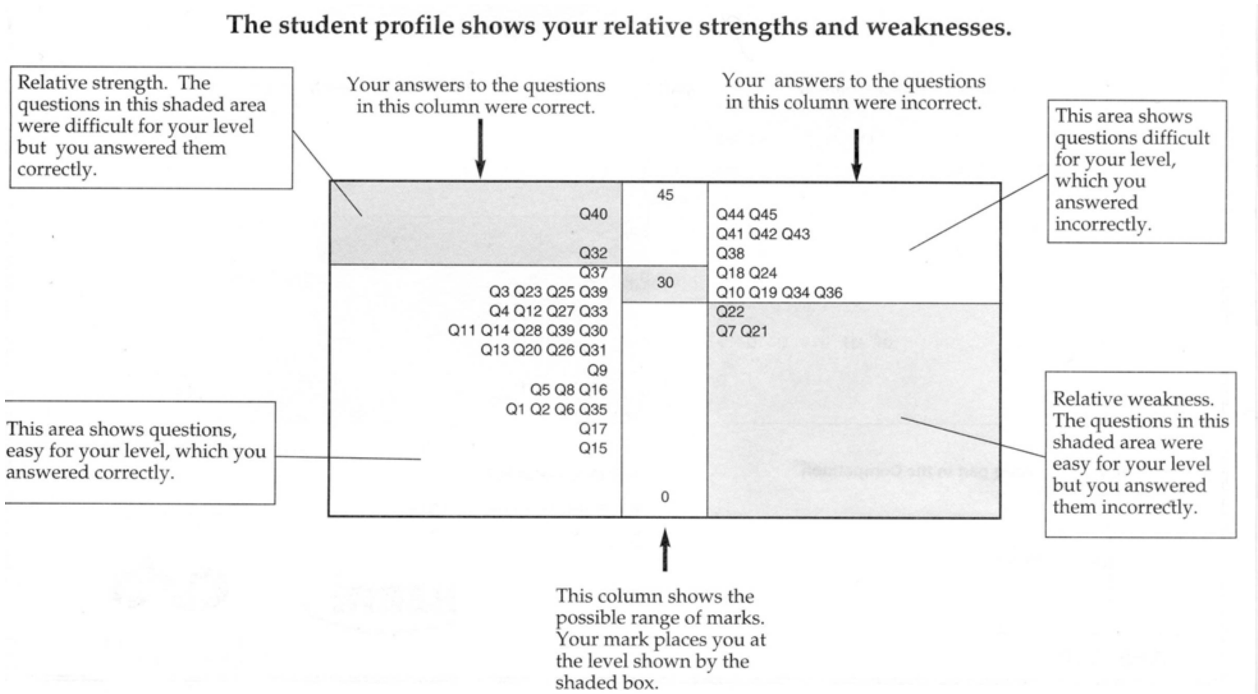


FIGURE 15
KID-MAP INDICATING PERFORMANCE IN TEST FOR INDIVIDUAL STUDENT

Figures 14 and 15 highlight and stress the importance of individualised feedback, and the opportunities to home-in on areas of strengths and weaknesses. The individualised feedback enhances learning for those who need ground-up support and optimises learning for those wanting extensions.

These valuable outputs from Rasch analyses (or comparable objective probabilistic models) provide an interface for the teacher (or school) to decide where extra time will need to be spent (both in teaching and extensions). For example, if students are faring badly in the topic of 'Limit Theorem' it will be useful to examine if appropriate remedial lessons are being undertaken to allow students to internalise their learning.

Conversely, if students are doing well in a set of topics, teachers could accelerate the learning and provide extensions or spend an extended period in areas where difficulties are anticipated.

This highlights the longitudinal nature of keeping records of test analysis and making appropriate curriculum modifications/upgrades. Professional development in this area is pertinent for teachers' understanding of the assessment-evaluation cycle, and also provides the base for teachers and curriculum developers to undertake research into the teaching-learning-assessment process.

An exposition and understanding of Ramanujan's work has opened up for consideration and reflection the nature and importance of sequence in mathematics contents, and also the need to monitor closely the steps one takes towards a viable solution. We will never be able to see or read one's mind when problem-solving happens; however, having an understanding of the heuristics and having the skills to magnify the process through the Rasch model is an important professional and academic requirement of teachers of mathematics. Sampang & Moseros (2005) highlighted the relevance of these professional skills in their study (Figure 16).

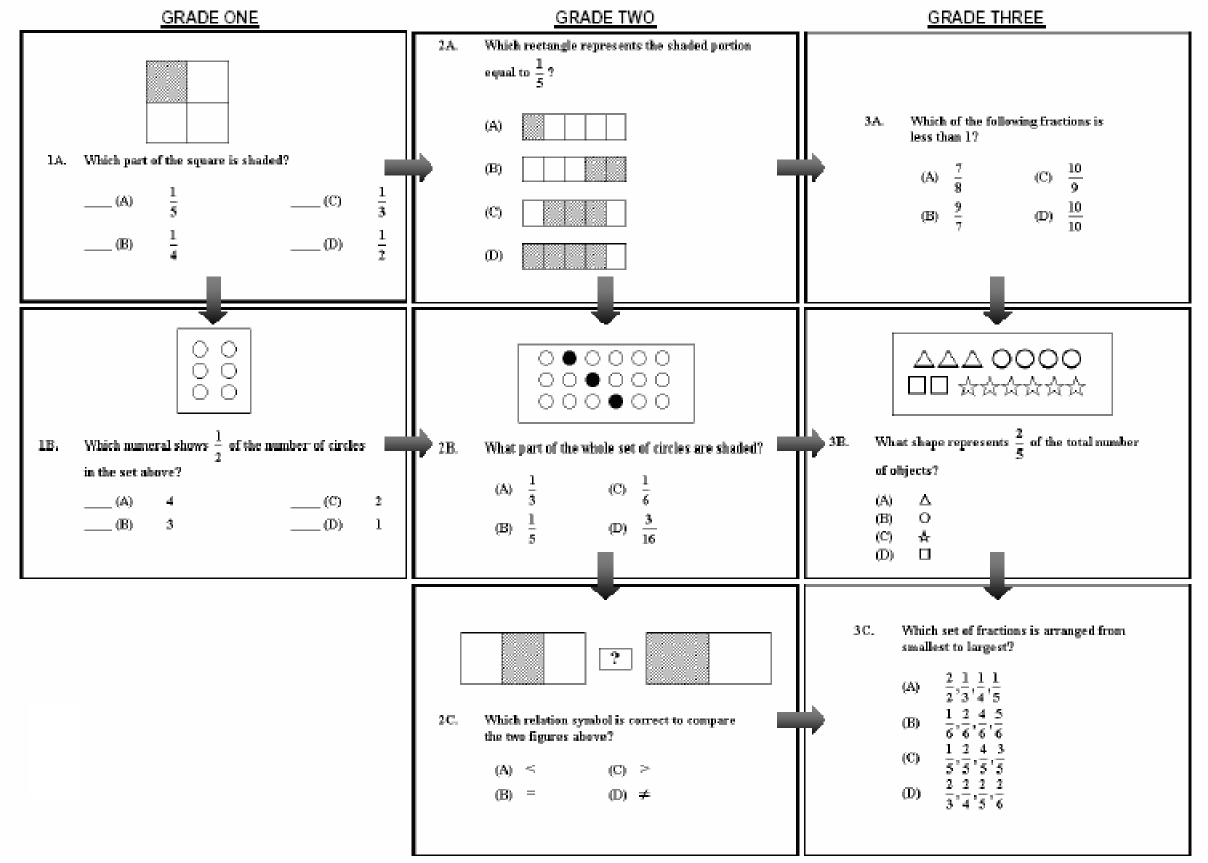


FIGURE 16

SAMPLE ITEMS FOR FRACTIONS – MAPPING (Sampang & Moseros, 2005)

Hence, the sequence discussed in the first part of this article is a necessary foundation to the planning and delivery of lessons to enhance learning. Good learning and excellent achievement can happen for a select group of students. However, if we as educators and teachers want all to benefit in the learning process, we need to rethink the importance of the contents-subject links. Importantly, objective measurement, through the use of Rasch probabilistic model, allows for identifying possible gaps in knowledge (and also possibly of instruction), and to scaffold learning.

Discussion and Conclusion

An examination of Ramanujan's study of mathematics illustrates the importance of textbook and reference materials in enabling the learner to understand the specifics of content-knowledge for the various topics. It is equally important to develop and reinforce conceptions of the subject-knowledge. Ramanujan demonstrated the effective interactions between the macro-subject knowledge and the specific (micro-) content-knowledge. This interaction framework emphasises the importance of teachers of mathematics having a comprehensive subject-knowledge (of mathematics), and how the contents within this subject overlap and interact through the various mathematical processes.

The core principle for designing learning is to acknowledge the "learners as its core participants … and to develop in them an understanding of their own activity as learners" (Istance & Dumont, 2010, p.319). Hence, it is crucial that horizontal and vertical connectedness across contents, areas of knowledge and subjects are facilitated. Further scrutiny and research is needed to explore and articulate how students with high intellectual potential operate in mathematics and associated disciplines.

This article further highlights the centrality of meaningful assessment practices in order to enhance learning and facilitate teaching. Educators, and in particular mathematics educators need to optimise probabilistic reasoning and models of objective measurement, assessment and reporting to demonstrate to students the application of mathematical thinking in the learning of mathematics. The Rasch Model is fundamental and robust in demonstrating its use in enhancing learning. Thus, educators and teachers of mathematics have to demonstrate to the teaching profession meaningful use of data (Education Queensland, 2010), and data derived from objective measurement procedures, and to advocate the use of the data obtained through assessment to inform further instruction (whether it be for diagnostics or remediation) towards the sole aim of enhancing learning (Bernhardt, 2009; Forester, 2009).

It follows that the current practice of locating subject matter mastery in the various academic departments and pedagogy in the schools of education is not only an artificial division but a potentially harmful one (McEwan & Bull, 1991). They argue against "conceiving of teaching as a distinct profession with a unique knowledge base. Rather, they imply that teachers and scholars must be counted as part of the same community, with the additional burden of responsibilities that such membership entails" (McEwan & Bull, 1991, p.333).

Mathematics is an enabling discipline, and every mathematics educator's/teacher's commitment to professional practice and assisting student learning has long reaching positive returns. Rightly, assessment practices need to be revisited and energised to move learning and teaching into the next level.

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Siva's undergraduate training was in physics and mathematics. His masters' and PhD research studies examined problem-solving in mathematical sciences, with the application of measurement principles. He has taught physics and mathematics in high schools in four countries, and had provided senior leadership directions in schools, Department/Ministry of Education, tertiary sectors and two listed companies. He has developed a number of software (standalone and online) applications to gauge, track and diagnose learning. Siva has close involvement with large-scale international studies such as TALIS, TIMSS, TIMSS-Advanced and PISA. He has been consulted on projects sponsored by UNESCO and the World Bank, and innovations in curriculum, assessment and evaluation in Singapore, Malaysia, Canada (Manitoba), USA (Boston), New Zealand, Japan, Thailand and the Philippines.

EFFECT OF MASTERY LEARNING STRATEGIES ON CONCEPT ATTAINMENT IN GEOMETRY

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Abstract

The present study was aimed at finding out the effect of mastery learning strategies viz. Bloom's Learning For Mastery (LFM) and Keller's Personalized System of Instruction (PSI) on concept attainment in geometry among high school students: a random sample of 105 students studying in 9th class was selected and "Three Groups--Randomized Matched Subject Pretest-Posttest Design" was employed. The sample students were divided into three homogeneous groups on the basis of their non-verbal intelligence level by administering Raven's Standard Progressive Matrices (SPM). The first group and the second group were taught through Bloom's LFM and Keller's PSI respectively and were termed experimental groups. The third group was imparted instruction through conventional method of teaching and named the control group. The data were collected by administering self-developed concept attainment test in geometry. The statistical technique of 'Analysis of Co-Variance (ANCOVA)' was employed to analyze the data. The results revealed that both Bloom's LFM and Keller's PSI were significantly more effective in promoting attainment of geometrical concepts as compared to the conventional method of teaching. It was further inferred that Bloom's LFM was significantly more effective in promoting attainment of geometrical concepts in comparison to Keller's PSI.

Keywords: Mastery Learning Strategies, Bloom's Learning For Mastery (LFM), Keller's Personalized System of Instruction (PSI), Concept Attainment, Geometry.

Introduction

Low academic achievement of students has emerged as a major obstacle in achieving the objective of intellectual development. This may be attributed mainly to lack of emphasis on conceptual learning and more stress on rote memorization by schools. Even in a subject like Mathematics, which is entirely based on scientific calculations, the stress is laid mostly on memorizing the concepts and formulae and not on conceptual understanding as well as their application. As a result, Mathematics teaching in schools has become stereotyped. In order to come out of such peculiar situation, a large number of instructional strategies have been developed and tried out by teachers and educators. Among a host of such instructional strategies used in the classrooms till date, each claims to be capable of performing certain functions, though no strategy can boast of being the best and capable of achieving all the educational objectives. A set of major instructional strategies have been developed under the rubric of 'Mastery Learning'.

There are two genotypic approaches to the use of mastery learning strategies. The first approach is 'group-based and teacher-paced' or Bloom's Strategy of Learning for Mastery and the second approach is 'individual-based and learner-paced' or Keller's Personalized System of Instruction.

In Bloom's approach of 'Learning For Mastery (LFM)', students learn cooperatively with their classmates and the teacher controls the delivery and flow of instruction. The theoretical basis for this strategy was provided by a conceptual model of school learning developed by Carroll (1963, 1965).

Keller's approach was first described in his 1968 paper "Good Bye, Teacher ..…". This strategy is an 'individual-based and learner-paced' approach to mastery learning where in a student typically learns independently of his/her classmates. Personalized System of Instruction (PSI) allows students to move through course material at their own rates, and requires that they show mastery of all major course objectives.

Rationale of the Study

The teaching of Mathematics plays a significant role in developing problem-solving attitude, reasoning power and critical thinking among students. Mathematical concepts are given top priority at the school stage because of their wider applicability in future and in learning other subjects. Out of many branches of Mathematics, Geometry is considered most valuable because of its great utility and vocational value. But at present, Geometry teaching is in a miserable condition in schools. This may be due to great emphasis on simple drilling of computations and little emphasis on conceptual understanding of mathematical processes in schools. So, there is a need for using such instructional methods and strategies which will ultimately result in development of mathematical concepts among children because the development of concepts is basic to growth of learning capacity.

Practices and research studies in India and abroad viz. researches by Drake (1988), Abadir (1993), Aviles (1996), Sharma (1998), Lang (2001), Havranek (2002), Mishra & Basantia (2003), Chauhan (2007) and many others revealed that use of innovative teaching strategies and mastery learning strategies in classrooms is very useful in conceptual learning as well as in enhancing students' achievement not only in Mathematics but also in other school subjects. However, some researchers like Brace (1992) and Mckenzie (1999) have reported contrary results. They found that students in the traditional classroom scored significantly higher than students in the self-paced mastery learning strategies. The review of the previous studies indicates that although a sufficient number of researches have been conducted to assess the effects of mastery learning strategies on academic achievement, retention and other psychological variables such as level of aspiration, achievement motivation, study habits etc there is paucity of researches undertaken in the field of mastery learning strategies and their effect on conceptual learning in Mathematics, particularly in the Indian school environment.

Hence, the present study was undertaken to compare the relative effectiveness of

i. Bloom's LFM Strategy and the conventional method of teaching on concept attainment in geometry among high school students.

ii. Keller's PSI and conventional method of teaching on concept attainment in geometry among high school students.

iii. Bloom's LFM Strategy and Keller's PSI on concept attainment in geometry among high school students.

HYPOTHESES

i. The students taught through Bloom's LFM Strategy will not differ significantly from the students taught through the conventional method of teaching with regard to concept attainment in geometry.

ii. The students taught through Keller's PSI will not differ significantly from the students taught through the conventional method of teaching with regard to concept attainment in geometry.

iii. The students taught through Bloom's LFM Strategy will not differ significantly from the students taught through Keller's PSI with regard to concept attainment in geometry.

Design of the Study

"Three Groups—Randomized Matched Subject Pretest-Posttest Design" was adopted with the following variables:

Independent variables: Bloom's LFM Strategy, Keller's PSI and conventional method of teaching.

Dependent variable: Concept attainment in geometry.

Intervening variables: Intelligence level of students, teacher effect and level of concept attainment in geometry before application of treatment variables.

1. **Study-Guides**

For imparting instruction through Keller's personalized system of instruction, study-guides were developed on the first two chapters of 9th class geometry textbook viz. *Basic Geometrical Facts and Some Angle Relations.* These two chapters were then divided into eight sub-units for preparation of study-guides. Each study-guide comprised of five parts namely; introduction, instructional objectives, suggested procedure for achieving instructional objectives, suggested reading material and questions for self-evaluation. The reading material given in study-guides was validated by seeking the views of Mathematics experts, language experts and technical (research) experts. Further, for evaluating the structural accuracy of study-guides, the experts from the field of educational technology were consulted.

2. Formative Tests and their Parallel Forms

For assessing mastery of the students over different sub-units, formative tests and their parallel forms were developed for each learning unit. The main purpose of these tests was to identify the learning difficulties of those students who were not able to achieve pre-specified mastery criterion of 80/80 and to provide them with remedial instruction on the un-mastered content. The students who were not able to achieve pre-specified mastery criterion were provided remedial instruction and parallel form of formative test of the same sub-unit was re-administered on them to check their mastery. The students could proceed to the next sub-unit only when 80/80 criterion was achieved by them either on the formative test or its parallel form. Each formative test and its parallel form was validated in terms of its content by employing the same procedure as in the case of the development of study-guides.

3. Concept Attainment Test in Geometry

In order to evaluate the attainment of geometrical concepts by the students at the end of experimental treatment, a concept attainment test in Geometry was constructed and standardized. First of all, preliminary draft of the test (comprising 125 items) was prepared which consisted of various types of items viz. multiple choice , completion, matching and true-false. At this juncture, the assistance of language experts was sought to remove linguistic ambiguity in the test items. In addition, the assistance of subject experts was taken while creating an item pool of 125 items. It was ensured that each instructional objective had due representation in the test by assigning at least one item for each instructional objective. While preparing the preliminary draft, the items of the test distributed along different levels of Bloom's taxonomy of cognitive domain as well as among all four levels of the concept attainment model given by Klausmeier et al. (1974). Afterwards, concept attainment test in Geometry was evaluated in terms of criterion difficulty (Dc) and index of sensitivity to instructional effects (S) of the test items and thus the final draft was developed. The final draft of concept attainment test in Geometry comprised of 95 test items (22 T/F type, 40 MCQ type, 31 completion type and 2 matching type items). Then the concept attainment test in Geometry was standardized with reference to its reliability and validity. The test-retest reliability was found to be 0.856, which was quite high. The content validity of the concept attainment test in Geometry was ensured by employing the procedure suggested by Tuckman (1979) i.e. by examining subsequent performances of students in short time, in terms of 'gain' from 'pre-test' to 'post-test'. It was observed that a group of 30 students of 9^{th} class obtained an average score of 42.93 on the pre-test. On the post-test, the students obtained an average score of 65.73, thereby showing a gain of 22.80 from pre-test to post-test. This pre-test to post-test gain not only showed the success of instruction but also indicated the content validity of the test.

Raven's Standard Progressive Matrices (SPM) was selected for subject to subject matching on non-verbal intelligence level (pertinent control variable).

SAMPLING

The procedure of multi-stage sampling was adopted. At first, a sample of 50 students was selected to carry out item analysis of the preliminary draft of concept attainment test. At the second stage, a sample of 42 students was selected to calculate the test-retest reliability of the concept attainment test. At last, a cluster sample of 203 students was selected for distributing the students into three different groups for conducting the experiment. These initially sampled students were matched on their non-verbal intelligence level. The group-wise mean intelligence scores for the three treatment groups i.e. Bloom's group, Keller's group and Control group were 34.17, 34.17 and 34.14 respectively. The significance of differences among the means for the three groups was tested using the technique of analysis of variance (ANOVA). The calculated F-value came out to be 0.0002, for df 2/102, which was not significant even at 0.05 level. Hence, subject-to-subject matching on the variable of non-verbal intelligence was considered to be satisfactory. Thus, three groups with 35 students in each group were randomly assigned to the three different experimental treatments. The remaining 98 students were dropped.

Description of the Experiment

PHASE –I (PRE-TESTING)

During the first phase of the experiment, concept attainment test in Geometry was administered on the students of the three treatment groups. The obtained scores were named as 'pre-test scores'.

PHASE –II (EXPERIMENTAL PHASE)

All the three groups were exposed randomly to different experimental treatments for a period of seven weeks. The first group was taught with the help of Bloom's LFM Strategy (Bloom group), the second through Keller's PSI (Keller group) and the third group was taught through the conventional method of teaching (Control group). All the three groups were taught by the investigator himself so as to preclude differences in effect thanks to differences in teacher competence (intervening variable).

PHASE –III (POST-TESTING)

After completion of instruction to all the three groups, the concept attainment test in Geometry was re-administered on all the three groups. The obtained scores were named as 'post-test scores'.

ANALYSIS OF DATA

In order to test the significance of difference among the means of the scores on concept attainment at the time of post-test and to adjust the initial mean differences, if any, in the pre-test scores of the different treatment groups, the statistical technique of 'Analysis of Covariance (ANCOVA)' was employed. Before starting with the actual procedure of analysis of covariance (ANCOVA), the assumptions of normality, randomness, homogeneity, additivity, correlation and regression were ascertained.

Results

After testing all the assumptions with reference to analysis of covariance, the investigator proceeded to test the significance of the difference between the adjusted mean scores on the concept attainment test in Geometry among the three treatment groups. The summary of the results of analysis of covariance is given in Table 1.

The results in Table 1 showed that the three groups namely; Bloom, Keller and the Control group differed significantly ($F = 126.64$, $p<0.01$, df 2/101) from one another with regard to their mean concept attainment scores in Geometry. The magnitude of the differences in the mean concept attainment scores of the three groups was computed and tested for their significance. In order to find out the significance of difference in the adjusted mean scores of the three treatment groups in different combinations (taking any two instructional strategies at a time) the least significant differences (LSDs) at 0.01 level were computed. The results of means of pre-test, post-test and adjusted mean scores of students of all the three treatment groups on concept attainment test in Geometry are given in Table 2.

TABLE 1
SUMMARY OF THE RESULTS OF ANALYSIS OF COVARIANCE IN THE SCORES ON CONCEPT ATTAINMENT TEST IN GEOMETRY FOR THE BLOOM, KELLER AND CONTROL GROUPS

S.No.	Componentsof Variability	Sum of Squares	df	Variance	F-Ratio	S.D.y.x.
1	Between Treatments	27041.10	2	13520.55	126.64**	
2	Within Samples of Error	10782.40	101	106.75		10.33
3	Total	37823.50	103			

** Significant at 0.01 level.

It is clear from Table 2 that the students taught through Bloom's LFM Strategy and Keller's PSI have achieved 85.94% (81.64 marks out of 95

TABLE 2
MEANS OF THE PRE-TEST, POST-TEST AND ADJUSTED SCORES ON CONCEPT ATTAINMENT TEST IN GEOMETRY OF THE THREE GROUPS: BLOOM, KELLER AND CONTROL

S. No.	Group	N	Mean (Pre-Test)	Mean (Post Test)	Adjusted Means	Difference between Adjusted Means
1	Bloom (A)	35	36.83	81.17	81.64	38.07** A-C
2	Keller (B)	35	40.37	72.20	71.26	27.69** B-C
3	Control (C)	35	36.86	43.11	43.57	10.38** A-B
4	General Means (GM)		38.02	65.49	65.49	

** Significant at 0.01 Level.

For df 101, Least Significant Difference at 0.01 level = 6.49

marks) and 75.01% marks (71.26% marks out of 95 marks) respectively in concept attainment test in geometry. In a similar manner, the concept attainment marks of students [in percentage] taught through the conventional instructional method was found to be 45.86% (43.57 marks out of 95 marks). Further, Table 2 makes it evident that the computed value of difference in adjusted means of the concept attainment scores between Bloom's group and Control group was 38.07 which is much greater than the least significant difference (6.49) at 0.01 level, for df 101. So, the null hypothesis (Ho) stated as, "the students taught through Bloom's Mastery Learning Strategy do not differ significantly from the students taught through conventional method of teaching with regard to concept attainment in Geometry", was rejected. So, it was interpreted that the adjusted mean concept attainment score of students taught through Bloom's Mastery Learning Strategy (81.64 marks or 85.94%) was significantly higher than the student taught through conventional method of teaching (43.57 marks or 45.86%). Hence, Bloom's Mastery Learning Strategy was significantly more effective in concept attainment in Geometry as compared to the conventional method of teaching.

In a similar manner, the difference in adjusted means of concept attainment test scores between Keller's group and Control group was found to be 27.69, which was again greater than the least significant difference (6.49) at 0.01 level of significance, for df 101. So, the null hypothesis (Ho) stated as, "the students taught through Keller's personalized system of instruction do not differ significantly from the students taught through conventional method of teaching with regard to concept attainment in Geometry", stood cancelled. Therefore, it was interpreted that the adjusted mean concept attainment score of students taught through Keller's PSI (71.26 marks or 75.01%) was significantly higher than the adjusted mean concept attainment score of those students who were taught through conventional method of teaching (43.57 marks or 45.86%). Hence, Keller's PSI was found to be significantly more effective in the attainment of geometrical concepts in comparison to the conventional method of teaching.

Thus, it was concluded that both Bloom's LFM Strategy and Keller's PSI were significantly more effective in concept attainment in Geometry compared to the conventional method of teaching. The present findings are supported by the results of Kohli (1999) who reported that the students, when taught through mastery teaching strategies, attained more geographical concepts compared to the students taught through non-mastery teaching strategies. The reason for such results may be attributed to the fact that in both of these strategies, objectives are clearly stated, instructional/study material is properly planned, difficulties of students are identified and remedial instruction and corrective feedback is provided wherever necessary. On the contrary, the present results were not in agreement with Brace (1992) and McKenzie (1999) who reported that the students taught through mastery learning strategies did not have significantly higher achievement scores compared with the students taught through the conventional method of teaching.

Table 2 further reveals that the difference in the adjusted means of concept attainment scores between Bloom's group and Keller's group is 10.38 which is significantly higher than the least significant difference (6.49) at 0.01 level, for df 101. Hence the null hypothesis (Ho) that, "the students taught through Bloom's Mastery Learning Strategy do not differ significantly from the students taught through Keller's Personalized System of Instruction with regard to concept attainment in Geometry", was not accepted: it was interpreted that the adjusted mean of concept attainment test scores of Bloom's group (81.64 marks or 85.94%) was significantly higher than the Keller's group (71.26 marks or 75.01%). In other words, it was inferred that Bloom's Mastery Learning Strategy was significantly more effective in teaching concept attainment in geometry compared to Keller's personalized system of instruction. The higher and effective conceptual learning in Geometry among students taught through Bloom's LFM may be attributed to the presence of teacher in the class and his control over the flow of instruction whereas in the case of Keller's PSI, there is very little control for the teacher over the class and the flow of instruction.

Discussion

It was found that both Bloom's LFM Strategy and Keller's Personalized System of Instruction were significantly more effective in promoting concept attainment in Geometry compared to the conventional method of teaching. Hence, teachers should prefer Mastery Learning Strategies in their classroom situations. For this, the teachers should make an extra effort to define the objectives of teaching, present the learning material sequentially and to identify learning difficulties of students. On the basis of such diagnosis, remedial instruction/corrective feedback should be provided to the students for improving their conceptual understanding.

From a different angle, introduction of Keller's Personalized System of Instruction in classroom situations may need lot of finances because for each discipline, additional material in the form of study-guides has to be prepared. The schools which can afford to spend some extra finances in preparing such material (study-guides) can safely make use of this strategy. Otherwise, the task of preparing study-guides for using personalized system of instruction in different school subjects can be undertaken by the NCERT at the national level and SCERTs / SIEs / University Departments of Education at State level. By doing this, the cost of preparing study-guides can be reduced and the use of personalized system of instruction can be made feasible to large number of school-going children. On the other hand, introduction of Bloom's Mastery Learning Strategy needs an extra effort on the part of teachers and no extra finances are required. Hence, it is of vital significance to organize in-service training programmes for teachers to sensitize them regarding the methodology and use of such mastery learning strategies at school level in different school subjects. Further, it is recommended that at pre-service training stage, student-teachers should be thoroughly oriented with mastery learning strategies and they should be imparted practical training in the use of mastery learning strategies in school situations at the time of practice teaching / internship so that they can further use such innovative instructional strategies when they are appointed as regular teachers in schools. Hopefully, this will enhance the quality of school education on a wider scale.

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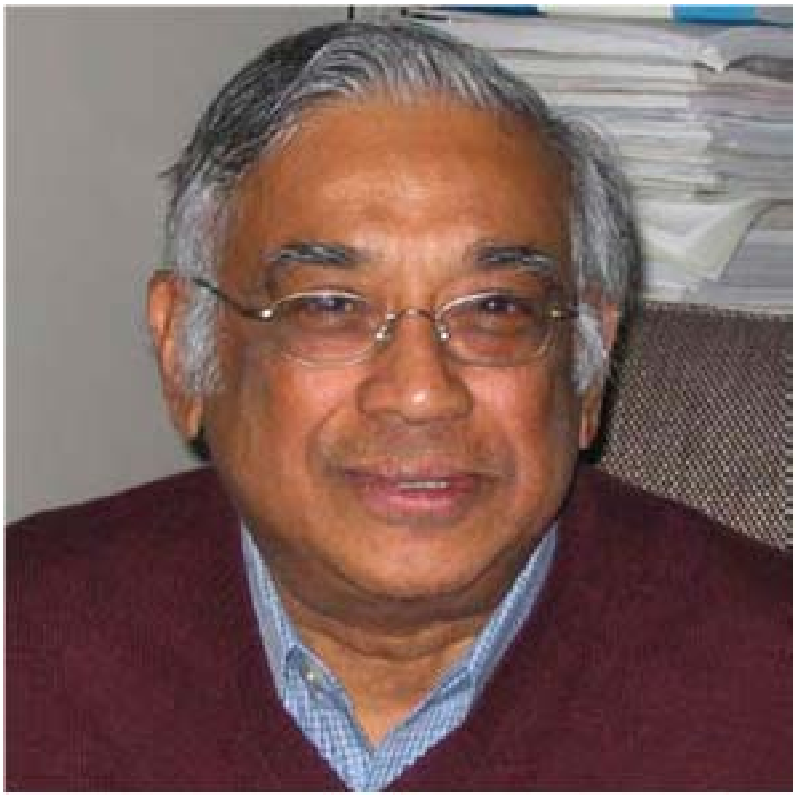

Srinivasa S. R.Varadhan

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